

Polyhedral Pysanky

Susan D. Jones

Canton, Connecticut, USA; susandjones27@me.com

Abstract

This workshop will show how the structure of polyhedra can be used to divide the surface of eggs before they are made into *pysanky*, an ancient artform from Ukraine. Workshop participants will learn how to divide an egg using octahedron, cuboctahedron, disdyakisdodecahedron, dodecahedron, and icosidodecahedron principles for spherical solids, and how to adjust for an egg being non-spherical.

Introduction

Participants in this workshop will learn to divide an egg into sections, based on the forms of geometric solids, or polyhedra, using pencil and tape measure. Artists who create *pysanky* (often called Ukrainian Easter eggs, from the Ukrainian verb *pysaty*, “to write”) will mark the surface of an egg into smaller areas into which design motifs are repeated to make an overall design, usually with symmetry. The art dates back centuries, and several divisions are very traditional in folk *pysanky*, two being eight triangles (Figure 1), and forty-eight triangles (Figure 2), which correspond to the polyhedra octahedron and disdyakisdodecahedron. These will be taught in this workshop.



Figure 1: *Eight Triangles, or Octahedron.*

Among the divisions that have become popular are the expanded cuboctahedron (Figure 12) and the rhombic dodecahedron (twelve diamonds) (Figure 13) but even more adventurous designers have ventured into using the dodecahedron (twelve pentagons) as inspiration for designs, as well as the many other wonderful geometric solids.

Pysanky are written with hot wax and special wax pens, but this would not be practical in this workshop. Sources for all of the materials needed to write *pysanky* will be listed at the end of this paper.



Figure 2: *Forty-eight triangles, or Disdyakis-dodecahedron.*

A Basic Division

One of the most basic ways to plan an egg design is with two perpendicular vertical lines, and one horizontal line around the middle, called the “equator.” Follow the steps in Figures 3-8 to arrive at the “Basic 8.” *pysanka* division with eight triangles. Figure 3 shows how to find and mark the top and bottom of the egg.

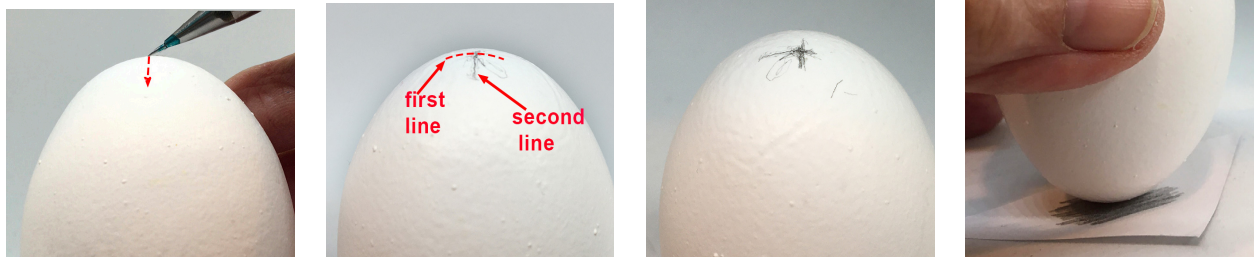


Figure 3: *Find the top of the egg. Hold it straight and make a short mark at the point that looks highest. Turn the egg 90° and make a second mark. Repeat on the bottom of the egg. Or you can rub the top and bottom of the egg on some graphite from a pencil. If you do this, be sure your egg is vertically straight.*

Connect the top and bottom marks, as shown in Figure 4.

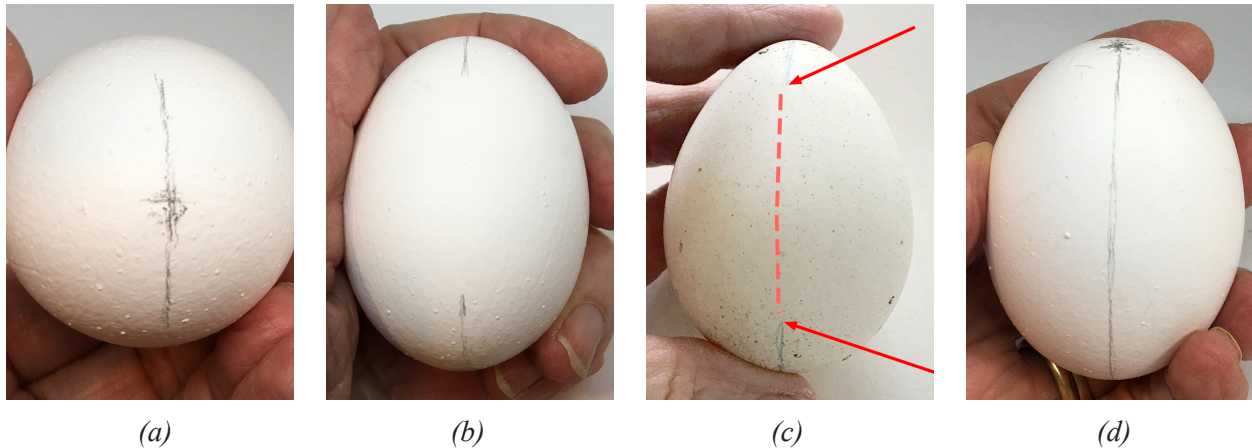


Figure 4: (a) After marking the bottom of the egg the same way you marked the top, look straight at one end and draw a straight line about an inch from the mark in both directions. Repeat this on the other end. (b) and (c) You can see at this point if the lines will line up. If they don't, adjust one until your two lines look like they will meet. (d) Make the connection and complete the line on both sides of the egg.

Figure 5 shows two alternative methods to draw the first vertical line. They have the advantage of giving you a way to trace the line, but both require some dexterity and practice. After you make a good first vertical line, you have to make a second vertical line, perpendicular to the first one. See Figure 6.

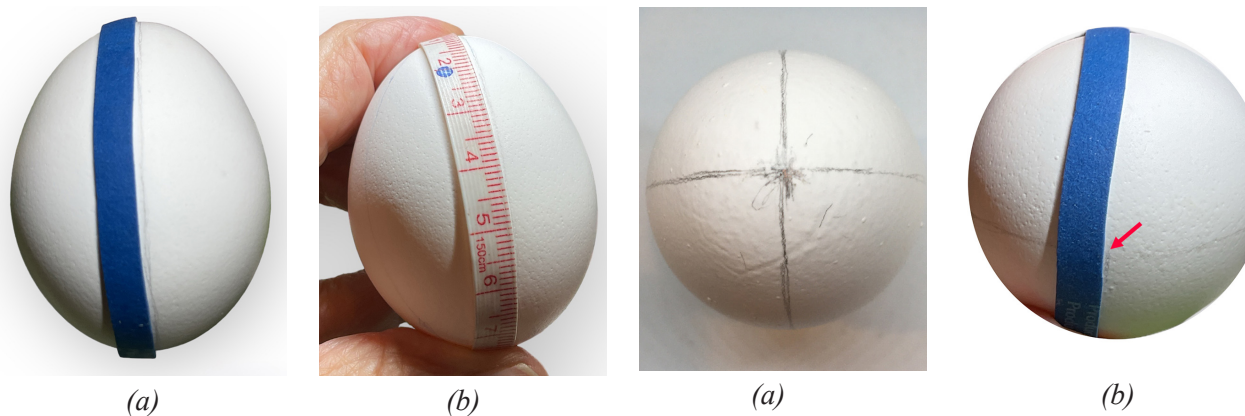


Figure 5: (a) Use a rubber band around the egg, being careful to keep marks for both top and bottom visible, and draw a line along the rubber band. (b) Use a non-slippery, flexible ribbon or tape measure that won't slip on the egg and trace your line along it.

Figure 6: (a) If you made your first line following the steps in Figure 4, do this again with a line perpendicular to the first one. Extend the new lines until you can see them, like Figure 4(c), then connect. (b) You can use a rubber band again to make this second line.

These steps may seem overly fussy at first, but if the first lines are not accurate, the inaccuracies are compounded as the division gets more complex. Using a strip of paper instead of the rubber band or tape measure in these steps is not recommended. Paper slips around on the egg, and it is difficult to hold it in place while you draw a line next to it. However, there are special egg lathes for making these initial important lines, and many pysanka artists use them. Figure 7 shows how using a template can help to mark the egg. Sources for the lathes and templates are listed at the end of this paper.

The egg lathe is not the only special tool available to pysanka artists. There are cardstock templates marked for 45°, 60° and 90° positions; placed over the top and bottom of an egg, you can mark points to connect for vertical lines. You still need to have marked the top and bottom.

The next line to this basic division is the horizontal line around the middle of the egg: the equator. Figure 8 shows you how to find the middle; when you connect them (Figure 9), your “Basic 8” is complete.

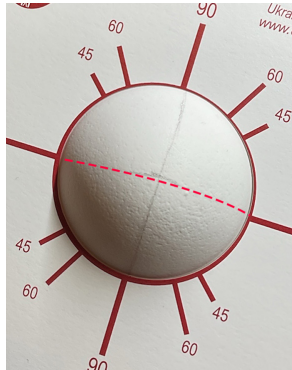


Figure 7: An egg marked with a template. Mark the 90° positions on both ends and connect the marks.

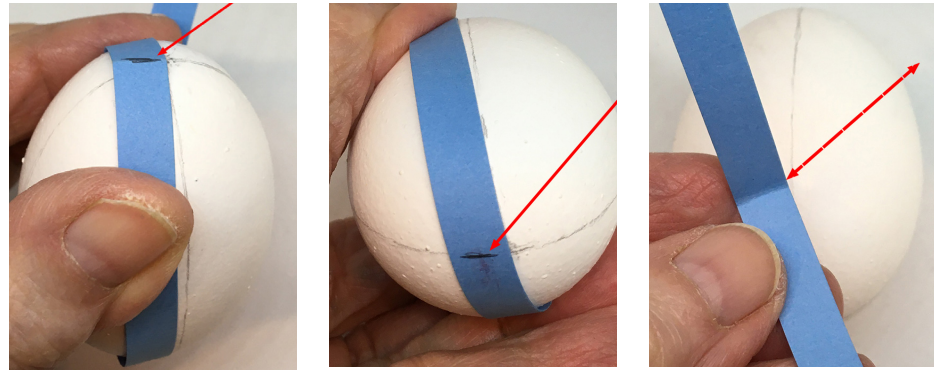


Figure 8: After drawing both vertical lines, as shown in Figures 3-7, make a horizontal line around the middle. Lay a strip of paper along one of the vertical lines and mark it for the top and bottom; remove the paper and fold it to find the halfway point between the top and bottom mark. Then put the paper back on the egg and mark the midpoint of each vertical line. You can also use a tape measure, preferably one with millimeters.

This basic pysanka division—often called the Basic 8 because it has eight triangular areas—creates large spaces, which are usually subdivided into smaller areas to be filled with designs such as flowers or stars. One common way to further divide this egg would be to make a line from the corners of each triangle to the middle of the triangle. In the polyhedron notation of John Conway, this is a “kis” operation. The egg in Figure 1 uses the premise of this operation to break up the large triangles into three smaller triangles, and fills them with a symmetrical design.

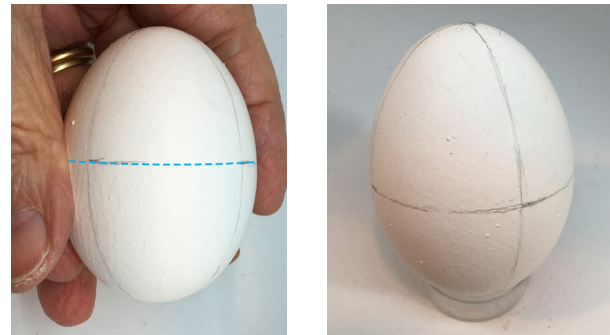


Figure 9: The “Basic 8,” an octahedron!

Subdivision

We will now transform your egg into another common division, the cuboctahedron, by applying a different operation, named “ambi.” The ambi, sometimes called “rectify,” operation connects the midpoints of the edges of each face of the polyhedron. Although an egg is not a sphere, the result of this operation is like a cuboctahedron, but with the squares on the top and bottom much larger than the ones around the middle. See Figure 10, left.

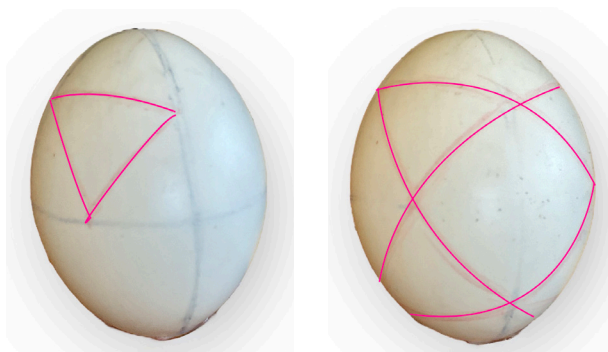


Figure 10: Connect the midpoints of each face’s edges and smooth out the lines to make the squares and triangles as uniform as possible: cuboctahedron!

This is a very popular division for pysanky, with large diamond spaces for designs and surrounding triangles for smaller motifs. Some examples are shown in Figure 11.

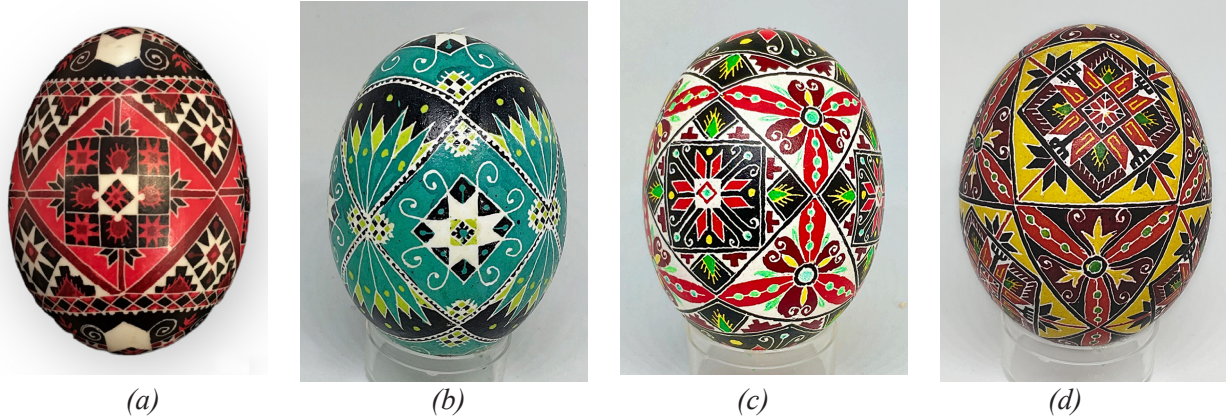


Figure 11: Variations in cuboctahedron eggs: (a) Making the edges of the triangles straight makes the “squares” on top and bottom circles. (b) Making the top and bottom more square creates curves on one edge of the triangles. (c) Putting a second square in the squares—another ambi operation—makes the top and bottom appear much squarer. (d) This egg has triangles on the top and bottom; to achieve this cuboctahedron division, divide the egg into six vertical sections plus equator. If you draw the triangles with the equator as their base, the squares will remain; or draw the squares, and you will get the triangles.

Two other divisions often used by pysanka artists are the expanded cuboctahedron and the rhombic dodecahedron, often called Twelve Diamonds. Both start with the cuboctahedron, shown in Figure 10. Two different operations are used to make these divisions. The expand operator is a double ambi: connect the midpoints of each face of a polyhedron, and then do it again. In Figure 12, the red lines are the first rectify, making the cuboctahedron from the octahedron, and the blue lines the second, which is the expand. Figure 13 shows the dual operator, which connects the midpoints of each face of a polyhedron. The red lines are the cuboctahedron. The blue lines are the dual, and the resulting polyhedron gets its name—rhombic dodecahedron—from the twelve diamonds it creates.

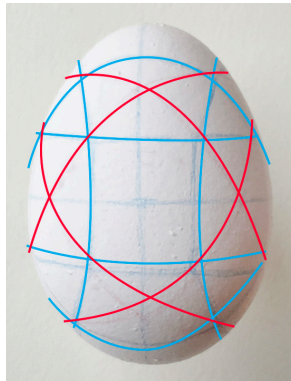


Figure 12: Starting with the cuboctahedron, shown in red lines above left, rectify again, connecting midpoints of each edge (blue lines), to get the expanded cuboctahedron. You have to make adjustments for the shape of the egg, making your resulting polyhedron less accurate. In order to keep the squares around the middle, top, and bottom of the egg square, the squares going from the middle to top and bottom become rectangular.

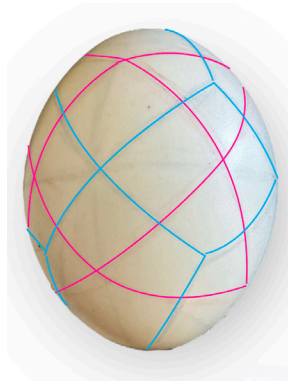


Figure 13: The rhombic dodecahedron is the dual of the cuboctahedron, shown as the red lines, above left. Connect the midpoints of each face to create the new division, which is very popular among pysanka artists today. The one drawback to this division is that the height of the egg often makes the rhombuses around the top and bottom very tall if the diamonds around the middle are square, and squares around the middle are often preferred.

One more very popular, and very traditional, division, also based on the octahedron, is the “48 Triangles.” The easiest way to create this pattern is to divide each of the eight triangular faces into six smaller triangles. The steps to make this are shown in Figure 14.

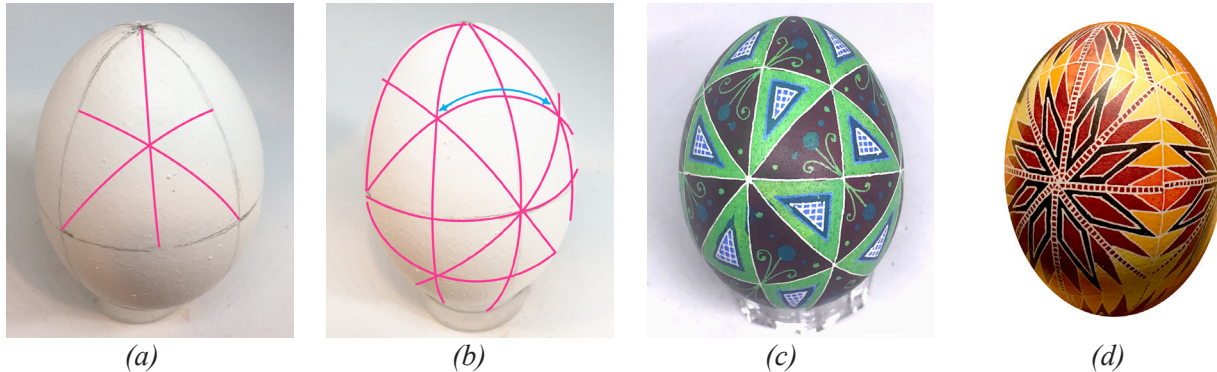


Figure 14: (a) Divide each triangle from apex to midpoint on the opposite side. (b) Connect the lines to wrap smoothly around the egg and adjust lines to make the 48 triangles as equal as possible, given the shape of an egg. It helps if you arc the horizontal lines between the centers of the triangles. (c) Designs for this division usually have alternating designs in the triangles, but (d) also popular is one called “Six Roses,” which centers a design around each intersection of the original “Basic 8.”

There are many other easy ways to modify the octahedron division that make excellent divisions for egg artists. One that uses the truncate operation is shown in Figure 22.

Making The Dodecahedron

There are also several great designs that start with a dodecahedron (twelve pentagon). The rewards of this challenging division are worth the learning curve. This is described in Figures 15-17.

Dividing an egg into twelve pentagons is more difficult than making the eight triangles of the octahedron, but it can be done with the help of a tape measure. The first step is to make one vertical line around the egg, just as you did in Figures 3 and either 4 or 5. Skip the step shown in Figure 6, because you don’t want to divide the egg in fourths. Figure 8 demonstrates a way to find the middle of an egg by using a strip of paper, using it to measure the height of the egg and folding it in half. In this sequence of steps, we will use a tape measure with millimeters. Plastic tape measures don’t slip too much on the egg’s surface.

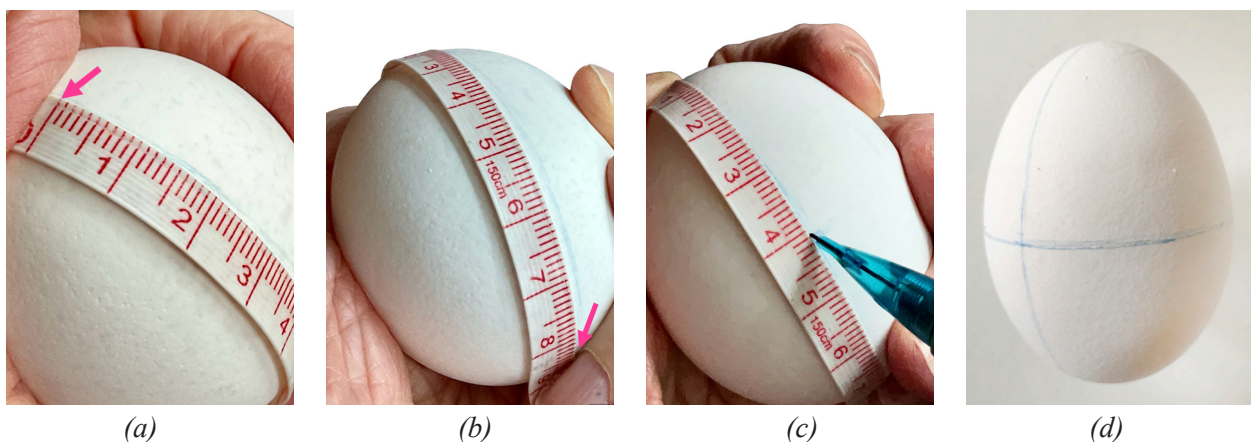


Figure 15: (a) Place the tape measure with 0 at the top of the egg and (b) hold it carefully down to the bottom. (c) Divide this number in half and mark the vertical line. Move the tape around to different places on the egg, always putting the tape on the top and bottom marks, and make marks around the egg’s middle. (d) Connect the dots the draw the equator.

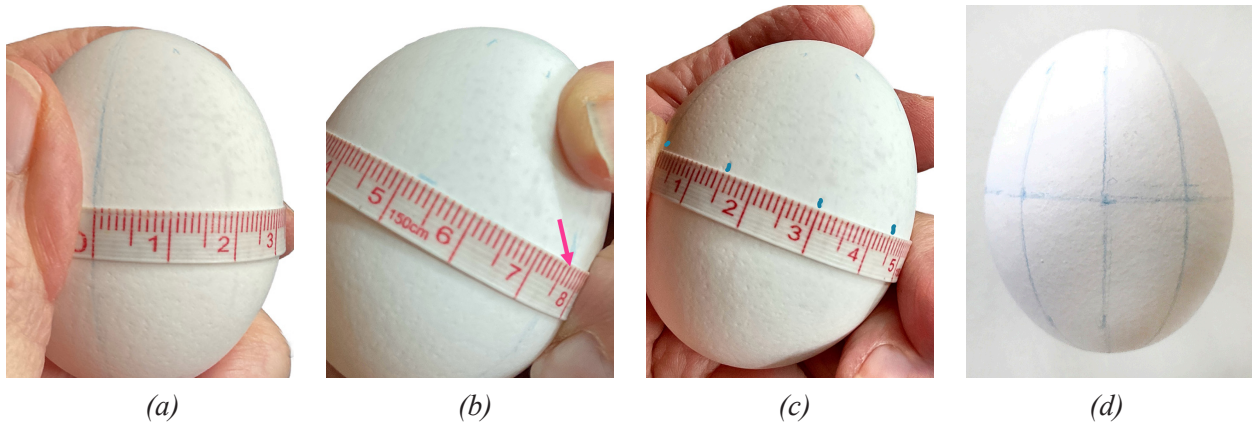


Figure 16: (a) Place the beginning of the tape measure on the intersection of the equator line with one of the vertical lines (b) and hold it along the equator until it reaches the other vertical to measure one half of the circumference. (c) Divide this number by five and make marks on the equator. (d) When you have done this for both sides of the egg, draw lines from these marks to the top and the bottom of the egg.

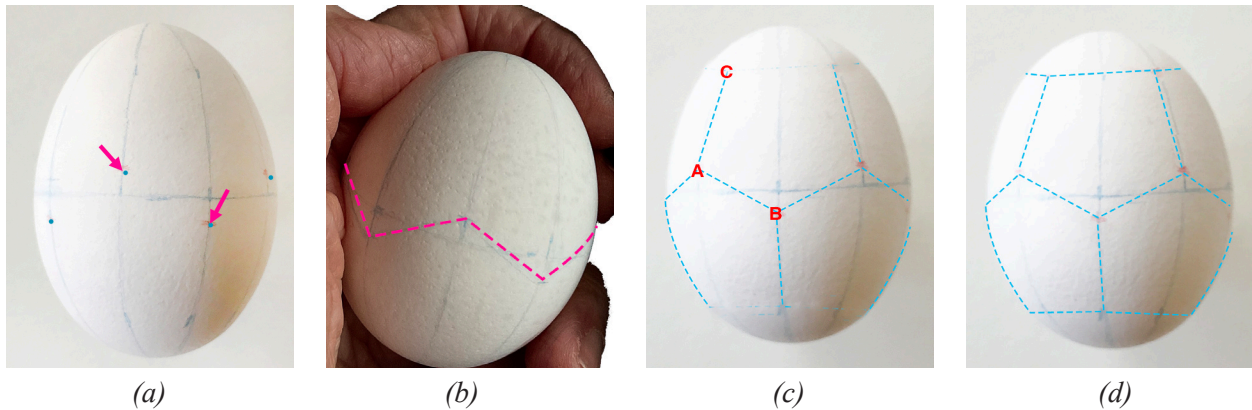


Figure 17: (a) Make marks on the vertical lines, alternating $3/16''$ or 4mm above and below the equator. This measurement is not scientific, but it always seems to work out right. (b) Connect the marks, making a gentle zigzag around the egg. These are the bottoms of ten pentagons. The sides of these pentagons are already on the egg: the lines you drew in Figure 16. (c) If your egg is very tall, the distance from A to B will be less than A to C, creating very tall pentagons. Adjust visually to take into account the height of the egg. (d) Draw lines for the tops of these pentagons. The pentagons on top and bottom will always be larger than the ones around the middle unless you have a nearly spherical egg. They do exist!

The dodecahedron is a gateway to so many variations that look amazing on eggs, and while it is not traditional in any way to Ukrainian folk designs and egg divisions, it has become popular among a small group of artists who don't mind veering away from the ancient roots of *pysankarstvo* (the art of pysanky).

The dual operator applied to the dodecahedron gives you twenty triangles, the icosahedron. An easier operator to apply is the ambi, connecting the midpoints of the edges of the pentagons (Figure 18).



Figure 18: The icosidodecahedron is the rectified (ambi) dodecahedron, producing twelve smaller pentagons and twenty triangles. It can be rectified again (expanded) and still be a workable division for an egg, but the design spaces are small.

Some examples of pysanky using the icosidodecahedron as the division are shown in Figure 19.

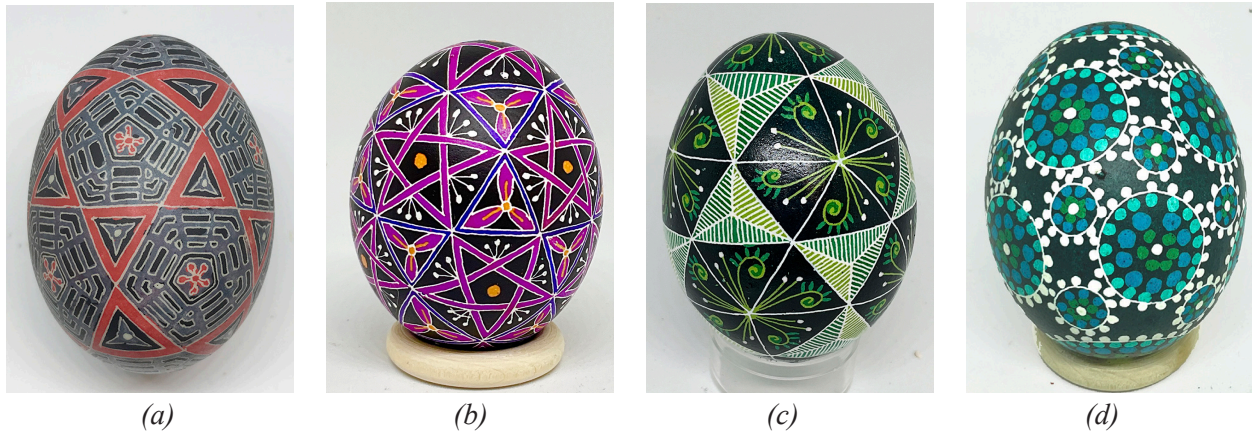


Figure 19: (a) *THE Pentagon*, aerial view. This pysanka was made for a friend who works there. (b) A pentagram in every pentagon and a propeller in each triangle, a lively design. (c) Using the *kis* operator on both pentagons and triangles makes a line from each face's corner to the centers. (d) Using the division lines only for placement, each face contains a circle.



Figure 20: *The rhombic triacontahedron has thirty rhombus or diamond-shaped faces.*

There are many divisions for eggs based on polyhedra that go beyond these covered in this workshop. One of these is the rhombic triacontahedron (thirty rhombuses, Figure 20), which is the dual of the icosidodecahedron. By connecting the midpoints of each face of the egg shown in Figure 18, you get the division shown in Figure 20. However, applying the dual operator on an egg that already has lots of lines can be messy and confusing. The easy way to create this division is to draw lines from the center of each pentagon to the corners, and leave out the outside edge lines of the pentagons themselves. You can also use color and dots to add enough new information to a design; the triacontahedron division on the egg in Figure 21 is delineated with solid white lines, but the dotted lines form twenty triangles.



Figure 21: *The white lines show the triacontahedron; the dotted lines make an icosahedron.*

The four eggs below are examples of another polyhedron operation you might like to experiment with: truncate. Truncate cuts off the corners of the faces to create new faces with different properties.



Figure 22: *L. to R.: truncated octahedron, truncated cuboctahedron, truncated dodecahedron, and truncated icosahedron (also recognizable as the soccer ball).*

Sources

Here are some sources for the supplies you'll need if you want to explore the art of pysanky further. PysankyUSA [8] has beginner kits as well as a full line of supplies, but they don't ship outside of the United States. Ukrainian Gift Shop [11] has a full line of supplies and tools, as well as design books, and they ship worldwide. They do not sell eggshells. Ukrainian EggCessories [10] also has all supplies except for eggshells. They ship to Canada and USA. Studio Karakusa [9] is in Japan. They sell basic pysanka supplies, but no eggshells. Eggshells to Dye For [2] is a source for empty eggs of all kinds, from ostrich to quail, but they ship only to Canada and USA. Some of these vendors have kits for beginners.

If you want to use whole eggs, they will eventually dry up inside if they are stored in a cool place and rotated frequently, but if they explode, the smell is terrible. Most people do drain the egg, either before or after dying it. And most people protect their pysanky with varnish. Because dyes are water soluble, the varnish must be oil based. Look for varnishes that say UV Protection and Non-Yellowing, although eggs displayed in bright light will fade despite the claims on the can of varnish.

You can buy cleaned and empty eggshells, but if you plan to empty your own eggs, be sure to buy egg-blowing tools. If you want to dye emptied eggs like the ones you used in the workshop, you will need to either buy silicon plugs to keep the dyes from going inside the egg, or learn how to make wax plugs.

To help you get started, there are dozens of tutorials for beginners on YouTube, like the one by Watson [12]. There are Facebook groups devoted to pysanky where people can ask questions and get answers and post pictures of their eggs. Pysanky USA, Ukrainian Eggcessories Eggers, and Incredible Eggs - Pysanky and Batik Style Egg Art are private groups that people must ask to join. Ukrainian Gift Shop [9] sells several books with history, lore, and step by step design instructions. I have written several design books that have a mix of traditional and modern pysanka designs; good ones to start with are *Pysanky Beginnings* [4] or *Divide and Conquer* [3]. Petrusha [7]'s website covers everything to do with pysanky.

For more information about polyhedra, Hart [1] explains the Conway notation and operators. McCooey [6] has categories of polyhedra and interactive models, and Levskaya [5] also has interactive models based on Conway's operations on the five Platonic solids. All three of these websites were instrumental in making it possible to translate a polyhedron into a division on an egg.

References

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