Tricurves: Behind and Beyond Tiling

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Abstract

The tricurve is most familiar as a monohedral tiling shape used in periodic, nonperiodic, and radial patterns. But the shape itself is very interesting. In this paper we look at three characteristics of the tricurve family of shapes: 1) elegance of construction; 2) surprises with the 1:2:3 shape; and 3) interesting issues with scaling.

Introduction and Background

The tricurve is probably most familiar as a monohedral tiling shape. You can buy it as a puzzle [1] or make your own puzzle pieces [7] as shown in Figure 1; this figure also shows periodic, nonperiodic and radial and mixed patterns. Tiling works because the shape has equal amounts of concave and convex arc, and has agreeable proportions of arcs and corner angles. Previous articles have explained the selection of various tricurve angles and proportions, and how tricurves fit together in tilings [3] [5].

Figure 1: *Tiling patterns with puzzles purchased (left) or made (middle); mixed periodic tiling (right).*

The layout of a typical tricurve as shown in Figure 2, with a generic shape on the left and two specific shapes in the middle that are commonly used in puzzles. For any tricurve, by definition, the vertex opposite the long arc C lies on the reflection of arc C about its endpoints.

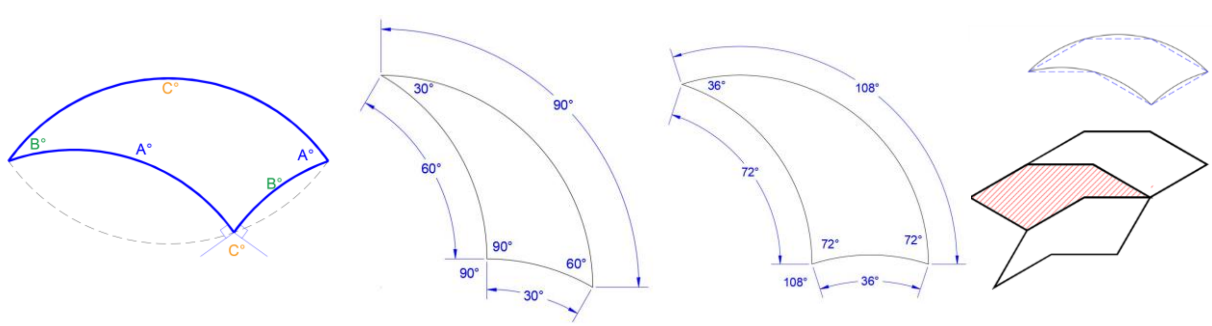

Figure 2: *Tricurve basic layout examples; faceted equivalent at right.*

The tilings achieved with a tricurve are generally the same as those made with a faceted version of the shape [2] [6]. In Figure 2 on the right we see a faceted version of 30-60°-90° tricurve, with facets spanning 30 degrees of the tricurve's arcs. The corner angles A and B remain the same as in the tricurve, and tiling is possible by matching up facets. However the tricurve is a more simple, elegant shape than the faceted version, since it is based on the rich geometry of arcs and circles. In this paper we look at some unique characteristics of the tricurve family of shapes. Other unusual features of tricurves, such as area calculations [5] and phantoms [4], have been covered in previous articles.

Some notes are needed regarding conventions. Radius is considered one unit. Angles are in degrees and indicate either corners (inside) or arc length (outside). Small concave arcs are indicated A or B with C indicating the large convex (i.e., major) arc. Specific tricurves are indicated by all arc lengths in degrees in ascending order, such as 30°-60°-90°.

Simplicity of Construction

The tricurve is a very simple shape. With its unit radius it can be described with only two values: any two of its arc lengths or corner angles; or, say, its major arc length and the ratio of its arcs. Construction is usually shown starting with the large concave arc which is then mirrored and apportioned using the two small concave arcs, and this is the most straightforward. But we can *deconstruct* the tricurve in ways that help us appreciate the relationships. Recalling that the large convex arc is the sum of the two smaller arcs, we can break the tricurve into two pairs of arcs, in two ways, as shown in Figure 3. This breakdown shows us two ways to look at the tricurve: 1) in a type of parallelogram framework with two pairs of identical opposites sides; or 2) in a "kite" configuration with same-size arcs adjacent at each end.

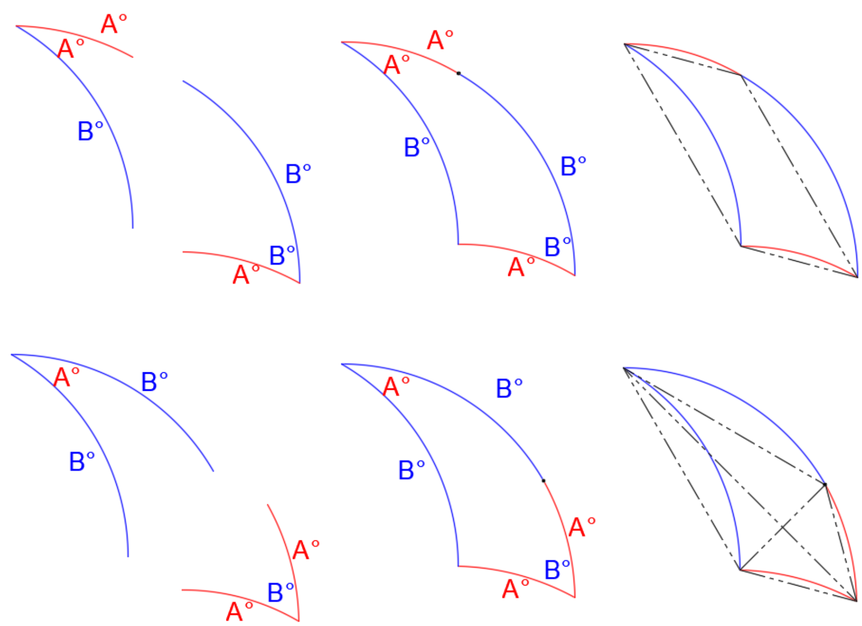

Figure 3: *Making or breaking a tricurve with two sets of two arcs, in two different ways.*

The tricurve can also be deconstructed by considering it as a large lens C, from which two smaller lenses A and B are subtracted, as shown in Figure 4. Each lens can be described by its single angle value of not only the two arcs but also the two corner angles.

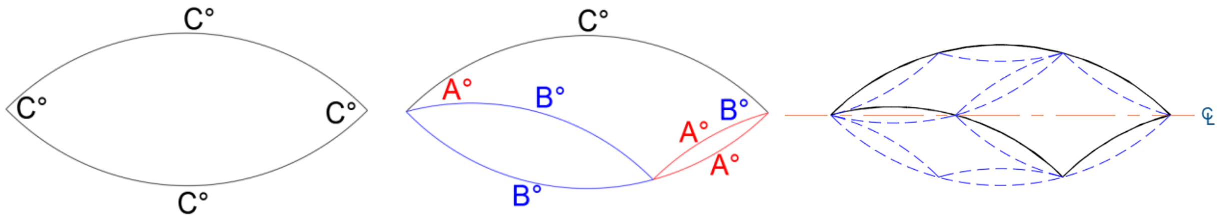

Figure 4: *Construction via lenses, with 1:2:3 breakdown at right.*

Properties of the 1:2:3 Tricurve

A tricurve with arc ratios of 1:2:3 presents some additional interesting relationships, regardless of the size. This type of tricurve is used in the puzzles as 36°-72°-108° and 30°-60°-90° shapes. The relationships can be illustrated by inserting an A lens to outline the shape, as shown on the right in Figure 4. The full C lens can always be filled with the nine A lenses, where 3 groups of three show the same orientation. Note that arc B is always bisected by the long axis centerline of the large C lens.

Among the 1:2:3 tricurves the 30°-60°-90° shape is unique. The Law of Sines states that sides of a triangle are proportional to the sines of the angles opposite: you can't have the sides *and* the angles in agreeable proportions. Tricurves in general violate this law [3], but in the 30°-60°-90° tricurve each arc is the *same* angle as the opposite corner angle, as shown in Figure 3.

Scaling

For most normal triangles, neither side lengths nor corner angles are in pleasant proportions. However, the application of scaling through similar triangles is easy and intuitive, as the corner angles stay the same and the sides stay in proportion. With tricurves you have the opposite case: both arc lengths *and* corner angles are in pleasant proportions, but straight-line size scaling is not only hard but impossible. A form of proportioning of scaling that can be done is more complex and is non-intuitive, as shown in Figure 5. This is because both the corner angles *and* the arcs lengths must change with the scaling, in order for the shape to remain a tricurve. Figure 5 also compares a standard 30°-60°-90° tricurve with is half-scale version (15°-30°-45°) and one-third version (10°-20°-30°).

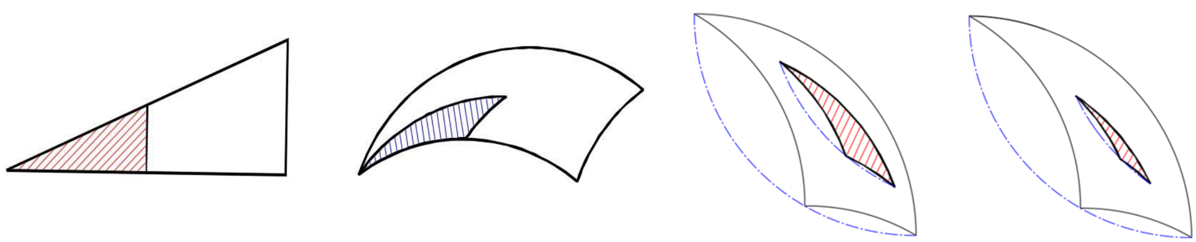

Figure 5: *Comparing half-scale shapes left; one-half and one-third scale versions in comparison to the original 30°-60°-90° tricurve.*

One way to visualize a down-scaled tricurve in relation to the original is shown in Figure 6. Let's assume we want a one-half scale version of a given tricurve. We start by identifying the point E that splits the large C arc into A and B portions; this lets us view the tricurve as a sort of curved parallelogram with two pairs of curved sides, as in Figure 3, with opposite sides equal but not actually parallel. Then we can bisect each arc and copy arcs to make a sort of curved 2x2 grid. Now we can see the new half-size tricurve (shaded) with arcs of size A/2, B/2 and C/2. Note that the point E now also divides the C/2 arc of the half-size tricurve into A/2 and B/2. This also shows how portions of the grid outline two other related tricurves: one with A/2 and B as concave arcs; and one with A and B/2 as concave arcs

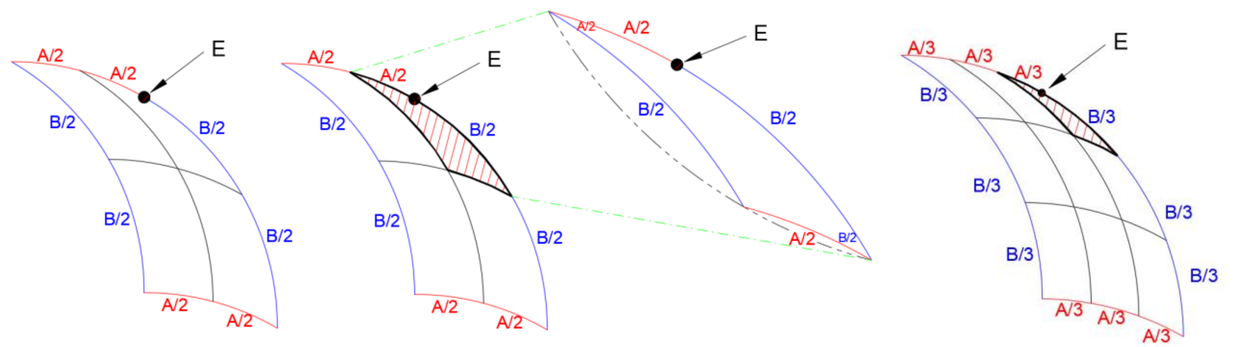

Figure 6: *One-half scaling with 2x2 grid (left); one-third scaling with 3x3 grid (right).*

Figure 6 also shows this works for a one-third scaled tricurve, by trisecting the A and B arcs and a 3x3 grid, to show the one-third tricurve (shaded). Portions of the grid outline several other tricurves, including one with A/3 and B as concave arcs; and one with (2/3)A and (2/3)B as concave arcs for a two-thirds scaled tricurve. All these additional reduced tricurves include the area of the one-third scaled tricurve.

With the above unusual properties, it looks like we might be able to construct a one-third scale version of a tricurve incorporating a given, unknown arc: that is, trisect the angle. This seems impossible of course, but to try is not only interesting and challenging, but educational: most of the author's material in this paper was discovered and developed during just that quest.

Conclusions

It's fun to experiment with the geometry of tricurves; the more we look the more we find. Some of the characteristics shown in this paper and its references illustrate basic geometry and show the structure underlying the many but finite tiling possibilities. On the other hand ideas related to phantoms and scaling seem more open-ended and may invite further exploration.

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