Properly Scaling the Speed of a Model Roller Coaster

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Abstract

The scale-model roller coaster is a niche artform growing in popularity among coaster enthusiasts. These gravity-powered models present unique design challenges compared to, say, model railroads. In particular, gravity does not scale proportionately with a model's size; this commonly produces a cognitive dissonance in which a model coaster's train appears to travel inappropriately "fast." The physics underlying this discrepancy is frequently misunderstood, and most commonly proposed solutions are either invalid or imprecise. This paper will properly assess these proposals, and show definitively how this dissonance is mathematically reconcilable.

Introduction

Roller coaster enthusiasts form a thriving global community of coaster riders, designers, and documentarians. Among these pursuits is the construction of functional scale-model coasters, much in the tradition of model railroads. Model coaster projects have grown more popular and innovative in recent years, largely thanks to tools such as Arduino and 3D printing. Hobbyists build and decorate models using either mass-produced coaster kits (notably from LEGO, K'NEX, and Coaster Dynamix) or original materials; Figure 1 shares examples of both. Like their full-size counterparts, most model roller coasters are gravity-powered: the train rolls freely along the track under the influence of gravity, aside from moments where a chain lift or launch contributes to the train's potential or kinetic energy respectively. This presents a unique engineering challenge: construct the model track, train, and supports precisely enough for the train to reliably traverse the layout, with minimal energy losses to friction and vibration.

Figure 1: *(Left) A huge 1:40-scale recreation of* Hyperion *built with K'NEX [5]. (Right) An original 3D-printed model, with a pulley-based launch system [2]. Images used with permission.*

Consider how gravitational acceleration does not change with a model's size. This affects the *scale* of a model coaster's motion, causing for many viewers a cognitive dissonance in which a model's train seems to travel at unreasonably fast speeds. While talk of this discrepancy exists in fragments across internet forums and comment sections, it does not appear to be documented in literature (in fact, very little proper literature seems to exist on model coasters). In such forums, solutions to this dissonance are frequently posited, but due to misconceptions about physics, these proposals are often imprecise, informal, or incorrect.

The goal of this paper is thus to properly assess the viability of these proposals. First we will clarify the problem at hand, and enumerate the most common suggestions. We then determine the mathematically correct implementation of one such solution, which has typically just been discussed informally. Finally, we consider aspects of model coaster physics that warrant further study.

The Gravity Problem

Suppose we want a 1:48-scale model railroad to display appropriately scaled motion—in other words, if we inflated the model back up to full size, the trains' motion would match that of the real thing. Because velocity scales linearly with distance, we can simply calibrate the locomotives to travel at $1/48^{\text{th}}$ the speed of the real trains. Thus, over any interval of time, the real and model trains will have the same *apparent* displacement. This approach cannot be applied to a gravity-powered model coaster, because gravitational acceleration is *the same* for both the full-size and scale coasters. Consider a 1:80-scale model of a coaster with an initial vertical drop of 50 meters. The real coaster, subject to $9.8\,\mathrm{m/s^2}$ downward acceleration, will reach the bottom of its drop in about 3 seconds, traveling $31\,\mathrm{m/s}$. The model, *also* subject to $9.8\,\mathrm{m/s^2}$, will traverse its 0.625 m drop in 0.35 s, and reach $3.5\,\mathrm{m/s}$. This means the model will reach an *apparent* speed of $3.5\,\mathrm{m/s} \times 80 = 280\,\mathrm{m/s}$!

So we see that model coasters travel disproportionately fast for their size. Online and in-person observers, both well-acquainted with physics and otherwise, frequently recognize this dissonance with confusion or frustration. I suggest the mental disconnect is a matter of *expectation*. Our brains intuitively grasp how fast something that looks like a roller coaster *should* move, and model coasters violate this intuition. Consider in contrast the related medium of the *marble coaster*—tracks for marbles to roll along, which may take the form of children's toys, physics classroom demonstrations, or elaborate kinetic sculptures. Marble coasters do not tend to induce cognitive dissonance, as our brains do not associate them with a full-scale object.

Comments acknowledging this quirk of model coasters often include ideas to mitigate it. We will now address four ideas which I estimate to be the most common, from personal observations across years of engaging with the model coaster community. The first proposal is to weigh down a model train so it accelerates more slowly. While a heavier train can have different motion characteristics, it will not accelerate differently under gravity, which unlike most forces affects all objects equally. The second idea is to introduce friction to counteract gravity. Friction *would* reduce the train's acceleration, but would do so non-conservatively, draining the momentum the train needs to traverse the layout.

The third idea is to fully motorize the train and simulate a full-size coaster. This is promising as it eliminates the gravity problem entirely, but also introduces significant technical challenges which have only been tackled by a few recent projects (Figure 2). These motorized models are remarkable, although they occupy a slightly different category from gravity-powered ones, which are more in the "tradition" of a model roller coaster. Furthermore, motorization can induce a converse dissonance, in which the model train's acceleration appears far too *slow* for an object so *small*.

Figure 2: *Two 3D-printed models, motorized and driven electronically. (Left) An original layout, motion-controlled with a rack-and-pinion [3]. (Right) A 1:45-scale recreation of* Stunt Fall*, controlled with a loop of cable [4]. Images used with permission.*

The final idea, which will be the remaining focus of this paper, is to film a model coaster and play back the footage in slow motion. This is appealing because it can be applied retroactively to any model. Indeed, YouTube commenters often recommend this approach to fellow viewers, seemingly by guessing a playback speed (e.g. ×0.25) that feels appropriate. At the outset, I was unsure whether this idea could yield physically correct results. We will see in the following section that if employed appropriately, viewing a model coaster in slow motion can in fact yield apparent motion that matches a full-size coaster.

Slow-Motion Model Mechanics

For slow-motion footage of a model roller coaster to "scale correctly," we should imagine comparing it to a full-size coaster with the same track layout. If both the full-size train in real-time, and the model train in slow-motion, traverse corresponding track elements at the same time and with the same *apparent* speed throughout their layouts, then we have chosen an appropriate playback rate for the model. Because a coaster train is locked to the track by wheels atop, below, and beside the rails, its singular degree of freedom is *along* the track; therefore, we will describe the train's motion with scalars for its position and speed along the length of the track, rather than with 3D motion vectors. Importantly, we assume for simplicity that the train has only one car, and thus sits at a single position on the track at any given time.

To derive the appropriate playback rate, we leverage conservation of energy, while disregarding small influences like friction and drag. Consider a full-size segment of track, whose height at "track-wise" position X is $H(X)$. Say the full-size train has mass m, begins at position 0, and has speed $V(X)$ when at track-wise position X. The initial energy (kinetic + gravitational potential) of the train is $\frac{1}{2}mV(0)^2 + mgH(0)$. Energy is conserved throughout the layout, so for any X, $\frac{1}{2}mV(X)^2 + mgH(X) = \frac{1}{2}mV(0)^2 + mgH(0)$. This rearranges to equation (A): $V(X)^2 = V(0)^2 + mg(H(0) - H(X))$.

Next consider a 1:σ-scale model of this track, with height $h(x)$ at position x, and a model train with speed $v(x)$ at x, where we will choose an initial speed $v(0) = cV(0)$ with some coefficient c. Since distance scales linearly, the model must have the property $h(x) = \sigma^{-1}H(\sigma x)$. We find, again through energy conservation, equation (B): $v(x)^2 = v(0)^2 + mg(h(0) - h(x)) = c^2V(0)^2 + mg\sigma^{-1}(H(0) - H(\sigma x))$. We seek a playback rate r so that the *apparent* speed of the model in the footage corresponds to that of the real train along every point of the track; precisely, $v(x) = (r\sigma)^{-1}V(\sigma x)$. Substituting this into (B) and rearranging yields $V(\sigma x)^2 = (r\sigma c)^2V(0)^2 + mg\sigma r^2(H(0) - H(\sigma x))$, which combined with (A) creates a system with sole solution $r = c = 1/\sqrt{\sigma}$. In other words, if we scale down the initial speed by $\sqrt{\sigma}$, and watch footage of the model slowed down by a factor of $\sqrt{\sigma}$, the motion of the full-size and 1:σ-scale coasters will correspond.

We can demonstrate this result with a physics simulation. Figure 3 shows a full-size segment of track, and a 1:4-scale model of this segment. The initial speed of the full-size train is 6 m/s, so we choose $6/\sqrt{4} = 3$ m/s

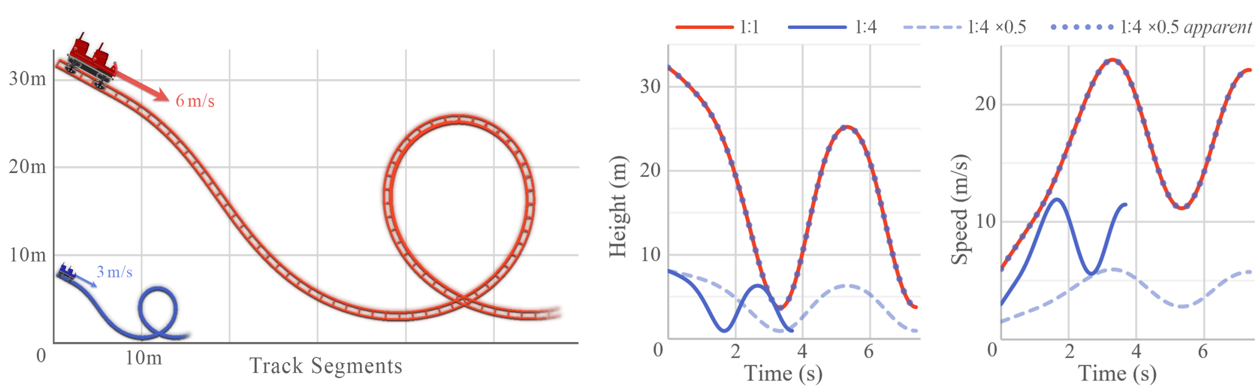

Figure 3: *Full-size and 1:4-scale track segments in red and blue, with simulated height and speed plots. Note how the model's apparent (scaled-up) motion at ×0.5 playback matches the 1:1 motion.*

for that of the model. We then simulate both trains subject to gravity by accumulating their track-wise speeds and positions over small time-steps; on a track segment with an ascension angle of θ, the trains experience $-g \sin\theta$ m/s^2 of track-wise gravitational acceleration. We plot the simulated height and speed of both trains, as well as the apparent (i.e. scaled-up) motion of the 1:4 model at a footage playback rate of $1/\sqrt{4} = 0.5$. The plots verify that indeed, the apparent motion of the model matches that of the full-scale coaster!

Conclusion

We have shown that viewing a model roller coaster in slow motion is a viable technique to achieve realistic-looking motion. To employ this approach on an actual model, the initial speed coefficient $c = 1/\sqrt{\sigma}$ should be applied to all components with controlled speeds, like chain lifts and launches. This is important to note not just for slow-motion playback but in general. For instance, the world's tallest roller coaster, *Kingda Ka*, launches to 57 m/s to crest its 139-meter hill [1]. To build a 1:80-scale (1.7 m tall) model, a *linearly scaled* launch speed of $57/80 \approx 0.7$ m/s will not be remotely fast enough; rather, $57/\sqrt{80} \approx 6.4$ m/s is needed.

The solution we have derived for slow-motion playback is idealized compared to the real mechanics involved. For one, our simplified model does not accommodate a multi-car train, for which the gravitational potential energy depends on the distinct positions of each car, rather than just one point on the track. Additionally, most model coasters experience significant energy losses compared to real coasters, for reasons including poor tolerances between the wheels and track, and elasticity of the track and supports. Constructing models with precise, low-friction wheel assemblies and rigid track is an ongoing challenge. Lastly, while we dismissed the significance of the model train's mass earlier, it does in reality come into play—not in terms of gravitational acceleration, but momentum. Adding weight to a model train is a common practice to help it overcome friction and preserve more speed throughout a layout, at the expense of imparting more stress on the track and supports. Further mathematical and empirical analysis are needed to fully appreciate how all of these factors contribute to the motion profile of a model coaster.

As a concluding thought, let us consider the two main promising techniques for properly scaling the speed of model coasters: slow motion and motorization. The former is highly accessible, though it cannot address any in-person model-viewing dissonance. The latter, as discussed earlier, is technically complex and satisfies a different niche from gravity-powered models. It may seem frustrating that no one approach checks every box, but I find it exciting. It means the model coaster community may never converge on one truly "correct" way to build their models—there will always be space for creativity and innovation.

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