Algorithms for Generating Crease Patterns from Sewing Patterns

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Abstract

This article discusses two algorithms for generating flat-foldable crease patterns from sewing patterns in fabric origami. The first algorithm requires that pairs of mountain and valley creases are parallel. The second algorithm, however, does not require parallelism in any pair of mountain and valley creases.

Introduction

Unfolding a piece of origami, one will discover a crease pattern that consists of mountain and valley folds that are imprinted on the paper. Origami crease patterns serve many purposes by providing a structural representation of the artwork, a visual guide on how to fold, and an alternate way of looking at the folded work. One of the early recordings of the crease pattern was to show how to fold a female poet in a collection of paper-folding instructions printed in 1845 [1]. With the development of mathematical folding and systematic design in origami since the end of the 20th century crease patterns have been widely investigated as the blueprints of all folded works [5].

Previously the author explored a tessellation grafting technique to generate sewing patterns consisting of polygons for fabric origami [6]. To make fabric origami, the corners of the polygons are sewn together, and then the gathered fabric is pleated and flattened. In this article, the author demonstrates a couple of algorithms for generating flat-foldable crease patterns from these sewing patterns. Understanding crease patterns will help artists create more aesthetically pleasing sewing patterns and come up with ways for pleating the fabric for fabric origami. In addition, these crease patterns generated from the sewing patterns can also be used directly to fold paper origami. Due to the limited size of this article, only local flat-foldability of the crease patterns from sewing patterns, such as sewing triangles or quadrangles, will be discussed.

Background Information on Local Flat-foldability of a Single Vertex

Local flat-foldability of a single vertex has been very well studied [5]. Several important theorems regarding the local flat-foldability of a single vertex are reiterated here, including the Kawasaki-Justin Theorem, the Maekawa-Justin Theorem, and the Big-Little-Big Angle (BLBA) Theorem. Kawasaki-Justine Theorem states that in order for a crease pattern to be locally flat-foldable, the number of lines connecting to a single inner vertex (the points where crease lines meet) must be even and the sums of alternating angles must be 180 degrees [2][4]. While the Kawasaki-Justin Theorem deals with the sector angles, the Maekawa-Justin Theorem and the Big-Little-Big Angle Theorem determine the mountain and valley fold assignments in a crease pattern. The Maekawa-Justin Theorem states that for any flat-foldable vertex, the difference between the numbers of the mountain folds and the number of valley folds is two [3]. And the Big-Little-Big Angle Theorem states that for any flat-foldable vertex, the crease on either side of any sector whose angle is smaller than those of its neighbors must have opposite crease assignments [5].

In this article, the author will present two algorithms for generating the crease patterns from the sewing patterns. In all of the crease patterns presented, the solid lines are used to represent mountain folds, dashed lines are used to represent valley folds, and the original sewing patterns are superimposed in lighter lines and noted with uppercase letters. Unless otherwise noted, all of the crease patterns presented can be proven to satisfy the aforementioned theorems regarding local flat-foldability. The author will leave it to the reader to provide proofs in each of the cases.

Generating Crease Patterns from Sewing Triangles and Quadrangles

For sewing a single triangle in fabric origami, corners of the triangle are sewn to a point on one side of the fabric and then the fabric is pleated and flattened on the other side. There are indeed many different ways to pleat and flatten the fabric of a single triangle fabric origami, each of which corresponds with a crease pattern. Given a sewing diagram of a single triangle ABC with a circumcenter of O as in Figure 1, the algorithm for generating the crease pattern in Figure 1 is following: Draw perpendicular bisectors of line OD, OE, and OF. At point A, draw line AA″ so that AA″ is parallel to line OD. Let line AA″ intersect with line OF at point A′. Repeat at point B with line BB″ and point B′ and point C with line CC″ and point C′. Erase lines OA′, OB′, and OC′. Vertices A′, B′, and C′ satisfies the Kawasaki-Justine Theorem. Use the Maekawa-Justin Theorem and Big-Little-Big Angle Theorem to assign the mountain and valley folds at vertices A′, B′, and C′. Figure 1(a) shows the diagram of the process generating a crease pattern from a single triangle fabric origami using the aforementioned algorithm. Figure 1(b) shows the resulting crease pattern and the folded form. Figure 1(c) shows an alternative crease pattern generated from sewing triangle ABC and its resulting folded form. It should be noted these crease patterns are not new as Robert Lang also called these close-back twists [5]. Indeed, there are many alternative twisted crease patterns that can be generated from a single triangle sewing pattern by altering the above algorithm slightly. Figure 2 lists all six possible crease patterns and their respective folded forms.

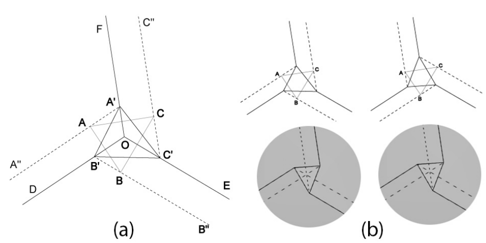

Figure 1: *Crease Patterns from a single sewing triangle.*

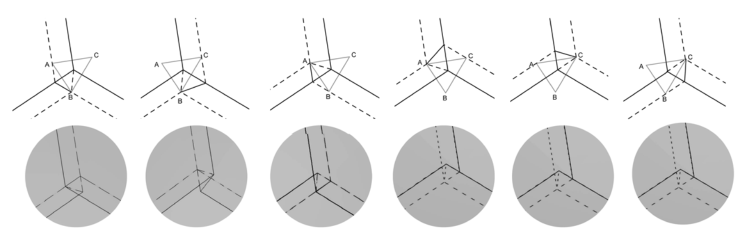

Figure 2: *Six alternative crease patterns from a single sewing triangle and their respective folded form.*

The above crease patterns share a common feature in that there are parallel pairs of mountain and valley creases. Due to this quality, crease patterns can be generated from triangular sewing patterns that were created based on the triangular tessellation grafting techniques [6]. Since crease patterns can interact with each other in folded forms, determining the global flat-foldability of crease patterns is very difficult and beyond the topic of this paper. However, since there are many possible flat-foldable crease patterns that can be assigned to each vertex, it is possible for an artist to pleat and flatten a piece of fabric origami so that the pleats can be organized in a visually consistent way without intersecting with each other. Figure 3 shows an example of one of the many possible crease patterns generated from sewing six triangles in fabric origami. Figure 3a shows the original triangle tessellation and with its reciprocal Voronoi diagram. Figure 3b shows the resulting grafted tessellation. Figure 3c and 3d show one of the crease patterns and the its resulting folded form. Figure 4 shows an example of fabric origami that is sewn from a sewing pattern that is generated from the triangular tessellation grafting technique. The artist chose the pleating patterns based on both aesthetic and technical considerations.

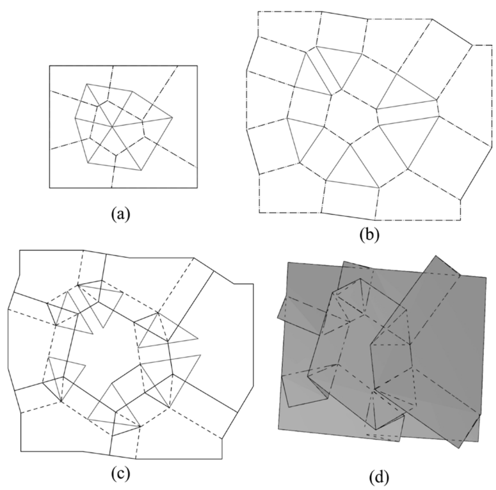

Figure 3: *An example of sewing six triangles in fabric origami.*

Figure 4: *An example of 48” by 30” fabric origami in muslin fabric. Artist: Jiangmei Wu.*

Though the parallelism found in the pairs of mountain and valley folds might be a good feature for creating tessellated crease patterns as shown in Figure 3, the parallelism is not a necessary condition for local flat-foldability for sewing one triangle. Given a sewing diagram of a single triangle ABC with an arbitrary interior point O, an alternative algorithm for generating a crease pattern from a sewing pattern is following: Find circumcenter A′, B′, C′ for triangles AOB, BOC, and AOC respectively. Connect points A and A′ and then extend to create line AA″. Repeat this process with point B and B′, and C and C′ to create lines BB″ and CC″. Draw perpendicular bisector line DA′ of edge AB, EB′ of edge BC, and FC′ of edge CA. Erase lines triangle ABC and lines OA, OB, and OC. Vertices A′, B′, and C′ satisfies the Kawasaki-Justine Theorem. Use the Maekawa-Justin Theorem and Big-Little-Big Angle Theorem to assign the mountain and valley folds at vertices A′, B′, and C′. Figure 5(a) shows a diagram of the process of generating a crease pattern from a single triangle sewing pattern using the above algorithm. Figure 5(b) shows the resulting crease pattern and the folded form. Indeed, there are infinite possibilities for flat-foldable crease patterns generated using this algorithm. While the crease patterns generated using this algorithm might not be useful in creating triangular tessellation, the algorithm is useful for generating crease pattern from other sewing patterns such as a rhombus sewing pattern. While there are infinitely many different crease patterns resulting from sewing a single rhombus, Figure 6 shows an example of using a similar algorithm as shown in Figure 5 for generating a flat-foldable crease pattern from sewing a rhombus. Given a rhombus ABCD with point O as the intersection of lines AC and BD, draw perpendicular bisectors of triangles ABO, AOD, BCO, OCD. Figure 6a shows a flat-foldable crease pattern that is drawn based on these perpendicular bisectors and its folded form. Figure 6b shows one of the possible tessellated crease patterns from Figure 6a. Figure

6c shows a half-way folded form of Figure 6b. To sew the fabric origami of the example in Figure 6b, rhombi are sewn on both sides of the fabric.

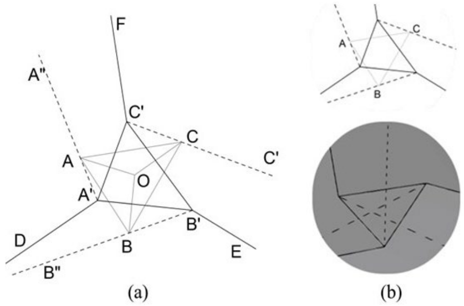

Figure 5: A *crease pattern from a single sewing triangle.*

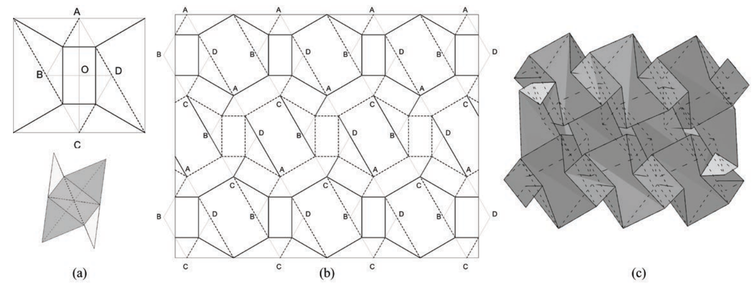

Figure 6: *An example of generating crease pattern from sewing rhombi.*

Conclusions

This article discussed a few examples of algorithms that are used to generate flat-foldable crease patterns from sewing patterns. As compared to sewing patterns, crease patterns are much harder to generate. Generating crease patterns from relatively easily-to-find sewing patterns therefore provides a valuable technique in creating origami tessellations. There are many more crease patterns of flat-foldable tessellations that need to be explored, particularly crease patterns that can be generated from sewing quadrangles and other polygons. In addition, the global flat-foldability of the crease patterns can be further studied.

Acknowledgments

The author's interest and work on fabric origami is seeded from a collaborative grant supported by the Center for Craft. Further support has been provided by the Office of the Vice President for Research, Indiana University, Bloomington. The images of the folded forms are edited from images generated by Origami Simulator.

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