Knitting on Helicoid Scaffolds

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Abstract

We explore the relationship between a knitted yarn network and its "scaffold surface" to address topological, geometric, and mechanical questions about knit textile behavior. The scaffold structure allows for a mean-field understanding of yarn mechanics, reducing the need to simulate direct yarn-to-yarn contacts by focusing on the geometry of the scaffold surface as an intermediary object. The scaffold structure as part of the Schwarz diamond minimal surface further offers insight into the geometric tendencies of knit and purl stitches, as well as their interfaces.

Introduction

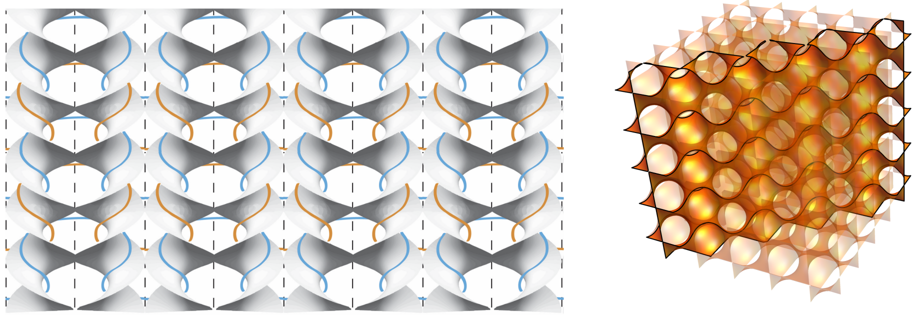
Whether handmade or industrially produced, weft-knit fabric is created by pulling a series of yarn loops through one another in an ordered rectangular array. The resulting material showcases complex geometric and mechanical behavior, and swatches of fabric made from the same type of yarn may boast vastly different properties depending on the stitch pattern. This allows knit patterns to be programmed to generate specific variations of curvature or metric [1].

Although knit loops are not typically knotted in the strict mathematical sense, their local entanglements are prescribed during the fabric's creation by a stitch pattern that dictates the movement of the needles manipulating the yarn, resulting in topological arrangement that cannot be modified later on without breaking the yarn. As the functionality of the resulting fabric can change the yarn geometry but *not* the topology, directly modelling yarns in the fabric requires computing optimal yarn geometry subject to a number of topological constraints on the order of the total number of stitches. For computational efficiency, it would be greatly beneficial to reduce the space of possible yarn paths in a manner that naturally enforces the topology of the knit structure.

Knitting on a Helicoid Scaffold

A serendipitous property of the basic knit pattern is its close relationship with the single-layer *helicoid scaffold* (Figure 1(a)) [2]. The scaffold surface is created by connecting alternating left- and right-handed helicoids in a single layer, and it separates space into two distinct regions. The helicoid scaffold surface is equivalent to a slice of the Schwarz diamond surface, a minimal surface with cubic symmetry that tiles 3D space (Figure 1(b)) [3, 4].

The helicoid surface fully separates the yarns of a knit swatch that has been pulled taut, and the regions that would contain necessary yarn-to-yarn contacts occur near the axes of the helicoids. At all points along its path, the yarn lies along the surface of the scaffold, offset slightly to one side or another. The scaffold thus offers an approximate, two-dimensional parameterization of the yarn path that sidesteps the problem of enforcing almost all local topological constraints.



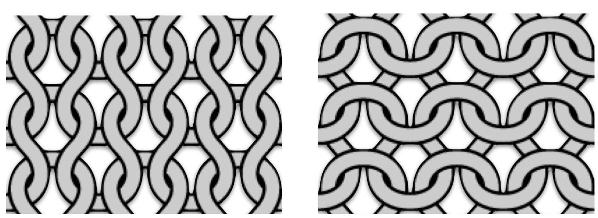
(a) **(b)**

Figure 1: *Structure of the helicoid scaffold: (a) the layout of knit stitches on the helicoid scaffold, which separates yarns of different rows (adapted from [2]); (b) the helicoid scaffold is a layer in the* $(1, -1, 0)$ *plane (non-uniquely) of the Schwarz diamond minimal surface, which can be visualized as a square checkerboard pattern of helicoids.*

Perhaps surprisingly, not only does the helicoid scaffold encode the topological entanglements of the yarn segments twisting around each other, but it also seems to provide insight into the *geometry* of the yarn path as well. Wadekar [5] showed that geodesic paths following the stitch pattern along the helicoid scaffold are in fact the same paths that minimize the total squared space-curvature of the stitch subject to endpoint (and endpoint tangent) constraints, *without* any constraint to lie along the helicoid surface. In general, if the yarn is assumed to be under tension, then yarn paths constrained to the surface of the helicoid scaffold will form geodesics on the scaffold surface, and optimization of the fabric's elastic energy can be parameterized over small deformations of the scaffold itself.

Trading Hands and Switching Planes

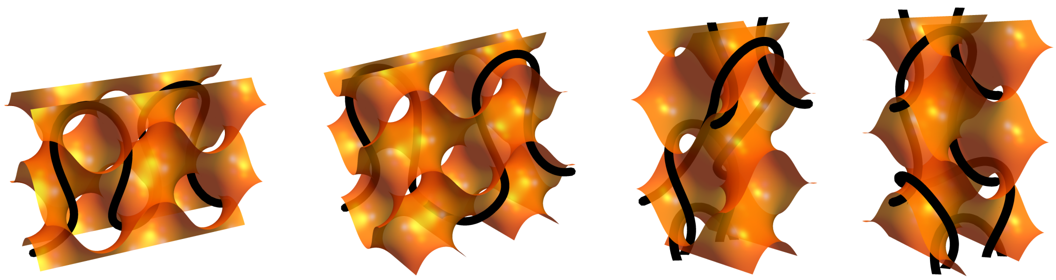
Although a knit structure is defined by its twisted yarn crossings and the helicoid scaffold can be viewed as a superposition of individual helicoids, the knit structure is *not* a chiral material on the whole. However, it does impose a preferred "knitting direction", shown vertically in Figure 2(a) and known in practice as the *wale* direction; the horizontal direction is known as the *course* direction. Although a bulk section of pure knit stitches appears to have up-down symmetry, boundaries or more complex stitches such as transfer stitches will break this symmetry.



(a) **(b)**

Figure 2: *(a) A schematic showing the crossings of nine knit stitches, loops that have been pulled from back-to-front; (b) a schematic of nine purl stitches, the mirror image of knit stitches with loops that have been pulled from front-to-back.*

A purl stitch (Figure 2(b)) is the mirror-image of a knit stitch, where a loop of yarn is pulled from front-to-back instead of back-to-front. As a result, a purl stitch requires reversing the positions of left- and right-handed helicoids on its portion of the scaffold. However, because the scaffold surface is created by a checkerboard pattern of left- and right-handed helicoids, switching from a knit to a purl stitch (or vice-versa) requires changing the plane of the fabric on the helicoid scaffold (Figure 3).

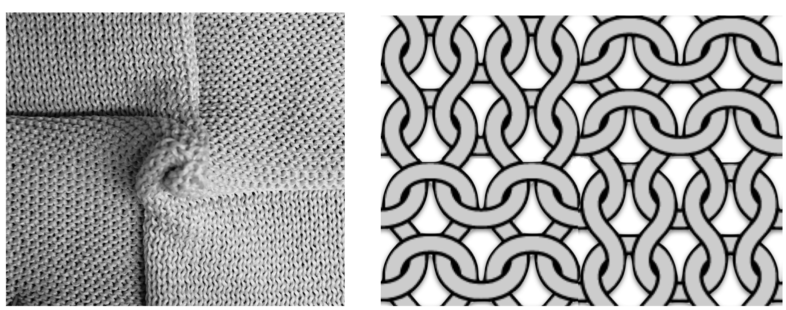


(a) **(b)** **(c)** **(d)**

Figure 3: *(a-b) Horizontal transition from a knit loop (left) next to a purl loop (right) on the helicoid scaffold, requiring a change of planes between the two loops. Surrounding yarns are omitted for clarity. (a) shows a natural configuration requiring less overall yarn and bending energy. (c-d) Vertical transition from two knit stitches (bottom) to a purl stitch (top). (c) shows the configuration requiring less yarn and less bending energy.*

The change of plane required to join knit and purl sections of fabric causes regions of different stitches to be offset from each other slightly (Figure 3, Figure 4(a)). In practice, one finds that joining a section of knits to a section of purls via a vertical boundary results in the purl section getting pushed backward into the plane of the fabric while the knit section is pushed forward, as shown in Figure 3(a). However, when joining sections of knits and purls via a horizontal boundary, the opposite is true (Figure 3(c)). Topologically, the knitted structure should be agnostic toward the direction of this plane change, but in practice changing planes to one direction is always favored in order to minimize both the extra yarn length and bending energy required to do so (Figure 3(b,d)).

One consequence of these interface-dependent planar offsets is that a "four corner" pattern of knits and purls naturally develops spiral features at their junction (Figure 4). Although none of the stitches themselves are chiral, their alignment and interactions result in a chiral feature emerging from the interface of knits and purls.



(a) **(b)**

Figure 4: *(a) Vertical and horizontal interfaces between knit (top-left and bottom-right) and purl (top-right and bottom-left) sections of a fabric are offset from one another perpendicular to the plane of the fabric, where the offset direction is dependent on the direction of the interface. These interfaces result in a chiral structure at the center of the four regions. (b) A crossing diagram of the chiral feature shown in (a).*

Generating Curvature

Another manifest feature of knit materials is their propensity to curl, especially near the edges of the fabric or at interfaces between knit and purl sections (Figure 4(a)) [1]. These curvatures arise because sections of yarn are forced to bend around each other by the knit topology, and can be understood qualitatively from the geometry of the helicoid scaffold.

Because yarn geometry is physically enforced by yarn-to-yarn interactions, we investigate the geometric structure of the scaffold surface near these contacts. From Figure 1(a) we note that the scaffold corresponding to pure knit stitches has saddle-like regions oriented in the same way at every yarn intersection, on average curved positively in the course direction and negatively in the wale direction. This curvature represents the action of two yarn segments forced to bend around each other. However, the flexural stiffness of each yarn segment generates a bending moment in the exact *opposite* direction; the fabric as a whole then energetically prefers a two-dimensional curvature that is opposite to the curvature of the helicoid scaffold at that point.

This results in a tendency for fabrics made of knit stitches to curl forward at their horizontal edges and backward at their vertical edges. For regions made with purl stitches, all curvatures are reversed; both can be clearly seen in Figure 4(a). The curling behavior of knits can therefore be read off directly from the geometry of the helicoid scaffold near yarn-to-yarn contacts.

Summary & Conclusions

The helicoid scaffold offers a unique perspective on the behavior of knit textiles and enforces local topological constraints encoding the yarn crossings dictated at manufacture. Additionally, the extension of the helicoid scaffold to the larger Schwarz diamond minimal surface suggests natural geometric features at the junction of knit and purl stitches.

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