On and On: Infinity in Song

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Abstract

The author, an award-winning songwriter, explores a diversity of references to infinity in the music and lyrics of mathematical and non-mathematical songs. Educational connections to mathematics content include infinite series, limits, recursion, perspective, irrational numbers, transfinite numbers, fractals, and asymptotic behavior of graphs.

Introduction

The main focus of this paper is to explore the varied instances of appearance of infinity in songs in connection with the uses of these songs in mathematics education, such as in a calculus class [1]. Although some comments are offered on the music itself, the emphasis is on the songs' lyrics. There has been attention to examples of infinity in realms ranging from short stories (e.g., [11]) to religion [19], but there is a dearth of scholarship specifically on infinity in songs. Songs are likely to be of interest to our students, and many educational roles, mechanisms, and putative benefits have been identified [8][17][20].

While we acknowledge structures of infinity (suggesting unboundedness or approaching a limit) in classical music (e.g., [9]) or ambient (e.g., [13]) music, our focus here is songs in a popular style with lyrics. We look first at infinity in "non-mathematical songs"—songs commercially released in the context of the music business without the main goal being use in (mathematics) education.

In the subsequent section, we will look at "mathematical songs"—songs written by mathematics instructors/educators to motivate the learning or appreciating of mathematics. Many mathematical songs are parodies of (ideally, famous) non-mathematical songs because they are easier to write (and learn) using an already familiar tune, and offer potential humor from lyric changes made (or not made) [20].

Infinity in Non-Mathematical Songs

On a musical structure level, there are some famous popular songs (particularly in the context of classic rock) that end with an "infinity section," a single repeating musical phrase that opens up the song with material not already heard in the song. For a classic example, see the analysis [24][26] of the endless 4-bar loop outro of English rock band The Beatles' "Hey Jude." Sometimes an entire song is what repeats, as in the Shari Lewis children's marching song "The Song That Doesn't End." Sometimes infinity is evoked not in the songwriting process but in the recording process such as how the song "One Stage Before" from Scottish-born folk-rock artist Al Stewart's 1976 platinum album *Year of the Cat* ends each verse with the word *infinity* sung with a studio effect to make vocals sound distant. According to my first album's co-producer Edloe, that is likely an old-fashioned "telephone" effect, a bandpass filter which drops the highest and lowest frequencies to leave only a narrow middle spectrum around 1-2 kHz.

With lyrics, a March 31, 2024 lyrics.com search yielded 12,743 lyrics containing *infinity*. Because it was impractical to browse 12,743 examples, I categorized the first pages' worth of output until reaching "saturation" for types of usage. The most common occurrences focused on romantic love/lust and/or forever (e.g. "I love you 'til infinity"). In decreasing order of frequency, other themes I identified were: state of mind or personification (e.g., "I am infinity"), idealized sense of home/place (e.g., "take me to infinity"), transcendence (e.g., "infinity and beyond"), and divine attribute ("Lord, You are infinity").

Sometimes the word *infinity* is not just in the body of the lyric but also in the title, such as love songs titled "Infinity" by pop stars Mariah Carey or One Direction.

Infinity can also be evoked without the word *infinity*. Canadian singer-songwriter Nelly Furtado in "Hey Man!" describes her interactions with someone as being in a Möbius strip (which is also the structure of an 8-track tape). "Infinity Sign" is a song by the British rock band Coldplay (from its 2021 album, *Music of the Spheres*) which repeats a Latin phrase for the Holy Spirit, giving infinity a (Christian) religious connection. A song [10] by American folk-pop singer Jonathan Coulton refers to many fractals while focusing on the particular "Mandelbrot Set." The 2014 lyric "Zeno's Paradox" by rapper Clayton Jokythur describes the famous race between Achilles and the tortoise which connects to an infinite geometric series.

A couplet from "Down Here," a 1999 tribute to nonconformists by American songwriter David Wilcox, refers to the tails of the normal distribution probability density function curve, which evoke infinity as they approach the horizontal axis. Stephanie Chou's "Asymptote" and Nicole R. Campbell's "Horizontal Asymptote" explicitly use asymptotic behavior as metaphor for elusive relationship fulfillment.

from: Down Here
by David Wilcox

So you come for a visit, but you lose your nerve
Out at the edges of the bell-shaped curve.

An example referring to perspective (i.e., vanishing point "at infinity") is in the train imagery in the breakup song "Rails" (from my 1992 album *Afterglow*), whose opening verse appears below.

from: Rails
by Lawrence Mark Lesser

Like these rails, looks like we'll meet down the line.
Love that fails seems to just need time.
The way you set your eyes on that horizon
Shows me love is blind. Soon you'll find…

Edutainment songs, especially those written for children's albums, may not obviously fit as either "mathematical" or "non-mathematical" if the recording artist's primary identity is recording artist, not educator. A notable example is the American band They Might Be Giants, who (in addition to their many albums of alternative rock) have several children's albums, including their GRAMMY-winning *Here Come the 123s*, whose song "Infinity" explores the challenge of how to count up to infinity. A more poetic depiction of this challenge is written by American contemporary folk singer-songwriter Christine Kane in the opening verse of her song "Tucson."

from: Tucson
by Christine Kane

Way out west a girl could lose her given sense
In all the truth that old desert sky can tell
That horizon's just too much for me
But if I could count infinity, I'd know the desert well.

Infinity in Mathematical Songs

While most examples in this section happen to be mine (because of my prolific output on this topic and what I have copyright permission for), I note that there is a surprisingly large community of instructors and educators who use and/or write songs for the teaching of mathematics, statistics, and other areas of STEM, as evidenced by, for example, the CAUSE curated, searchable collection [5] of roughly 200 statistics songs, Walter Smith's collection [27] of physics songs, the MASSIVE database of 7000+ science songs [6], and the international network VOICES ([28], where 100+ archived presentations from 5+ years' worth of conferences can be viewed).

A simple example of infinity in a mathematical song occurs with the induction step of mathematical induction [4] since there is a non-terminating process with no "last" statement. Another example is Haverford College computer science professor John Dougherty having his students sing (to the tune of the children's song "It's Raining, It's Pouring") "The Recursion Song" [8]:

The Recursion Song
by John Dougherty

Recursion, recursion, we'll sing about recursion,
Until the case has reached its base, we'll sing about
[*repeat the above two lines a half-dozen times to let students "experience the sense of endlessness" before introducing the following revised second line to allow an exit*]
Now the case has reached its base, ending the recursion.

Another Dougherty computer science parody is "Stuck in an Infinite Loop" (to the tune of the 1972 song "Stuck in the Middle with You" by Scottish band Stealers Wheel) whose lyrics include "the reality of system interrupts from outside the infinite process." [8, p. 108]

A different type of infinity is an unending sequence of digits, as with the number pi. This infinitude is in the chorus of my parody "American Pi" (provided with discussion questions in [18]) of the 1971 Don McLean #1 hit "American Pie." As an aside, a famous song which already has a mathematical word (or homonym or rhyme of one) is ripe for being parodied for class use, especially if it has a repeated chorus.

from: American Pi
by Lawrence Mark Lesser

Find, find, the value of pi, starts 3 point 1 4 1 5 9.
A good ol' fraction you may hope to define,
But the decimal never dies, never repeats or dies.

An example of an infinite set is the set of prime numbers (with Euclid's famous proof from his book *Elements*) as captured here in my previously-unpublished parody of the 1984 Cyndi Lauper/Rob Hyman #1 hit "Time After Time."

Prime After Prime
by Lawrence Mark Lesser

Whole numbers factor uniquely into primes:
Proved by that Euclid in only a pair of lines!
So nice, concise, it's so sublime – *Elements* proving prime after….
Euclid, he formed the product of "every prime"
And one plus that product has a factor that's another prime!

And so we see the big bottom line – there is no final prime!
If you want, you can look and you'll be finding prime after prime.
If you want, you can look and they'll be waiting – prime after prime.

Legendre and Gauss found primes get less dense –
Like 1 over log *n*, that's the Prime Number Theorem.
Things get thinner as we climb, but no end of the line….
If you want, you can look and you'll be finding prime after prime.
If you want, you can look and they'll be waiting – prime after prime.
Is there no end to primes we chart, like primes 2 apart?
If you want, you can look and you'll be finding prime after prime.
If you want, you can look and they'll be waiting – prime after prime.
Prime after prime, prime after prime….

A college algebra writing-to-learn exercise sparked my lyric [16] (to the tune of the same-titled GRAMMY-winning song by Julie Gold) whose last verse involves a fractal with finite area bounded by an infinite perimeter.

from: From a Distance
by Lawrence Mark Lesser

From a distance, some curves look the same as they do when we are close.
From a distance, patterns repeat along a cloud or coast.
From a distance, there is similarity and it echoes at every scale.
Let's find what makes those Koch snowflakes and follow the fractal trail!

A song that does not just mention fractals but also manifests fractal structure is "AABA" by Vi Hart [12], where the music and lyrics of the overall song can be described as having the traditional AABA song form (where B represents a musically and lyrically contrasting bridge section inserted into three musically identical A verse-sections), but where the B section and each A section also has an AABA form! This iterated fractal style may conjure Mike Naylor's instrumental "Abacaba" [22], whose core melody follows a simple binary fractal pattern with notes from the A natural minor scale.

A famous infinite hotel metaphor inspired my parody [10] of the Eagles' 1976 #1 hit "Hotel California." Introduced by German mathematician David Hilbert in a lecture a century ago [14], the metaphor appears in a study [21] and Netflix documentary [23] and has implications for cosmology [14].

from: Hotel Called Infinity
by Lawrence Mark Lesser

CHORUS: "Welcome to the hotel called Infinity —
Where every room is full (every room is full), yet there's room for more.
Yeah, plenty of room at the hotel called Infinity —
Move 'em down the floor (move 'em down the floor) to make room for more."

My mind got more twisted when I saw a bus without end
With an infinite number of riders coming up to check in.
"Relax," said the nightman. "You each will move
to the double of your room number: that frees the odd-numbered rooms."

Renteln and Dundes [25] published an endless version of "99 Bottles of Beer on the Wall" by changing 99 to aleph-null, and Byrd [3] provides further variations. Also going beyond aleph-null is my original historical song, "Cantor's Coat" [15], inspired by the Dauben biography [7] on German mathematician Georg Cantor.

One of the most surreal results involving infinity is the Banach-Tarski paradox [29], which inspired this previously-unpublished version of a lyric to the tune of George Harrison's "Here Comes the Sun." The paradox is a set-theoretic geometry theorem which says a ball can be decomposed into as few as five disjoint subsets, which can then be reassembled to yield two copies of the original ball. This paradox relates to infinity because the pieces of the ball are not typical "solids" but infinite scatterings of points. Another formulation says a solid ball can be broken into pieces that can be reassembled to form a solid ball of a very different size, such as the informal well-known example of a pea and the Sun which sparked my lyric.

The Pea and Sun
by Lawrence Mark Lesser

The pea and sun, the pea and sun, and I say it's all right.
Banach-Tarski, come break a ball into 6 pieces
Reassembled, we now have twice as big a sphere!
The pea and sun, a 2-for-1, and I say it's all right.

Banach-Tarski – it is a mathematics wonder:
Banach-Tarski – world hunger's end is drawing near!
The pea and sun, I'm really stunned! And I say it's all right.
Pea and sun, we are done!

Summary and Conclusions

Having seen diverse aspects and manifestations of infinity (as listed in the abstract) even in our modest samplings of mathematical and non-mathematical songs, we close by also noting there can be a hybrid of overlap. For example, Jason Brown and I each have released recordings of original non-mathematical songs and we each have primary careers as university faculty in departments of mathematical sciences. In 2023, we began co-writing (e.g., [2]) with the goal of having songs follow or be informed by mathematical structure (e.g., fractals), but in a way subtle enough that the song still "works" on its own for general audiences.

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