Squiggles: Examples and Explanations

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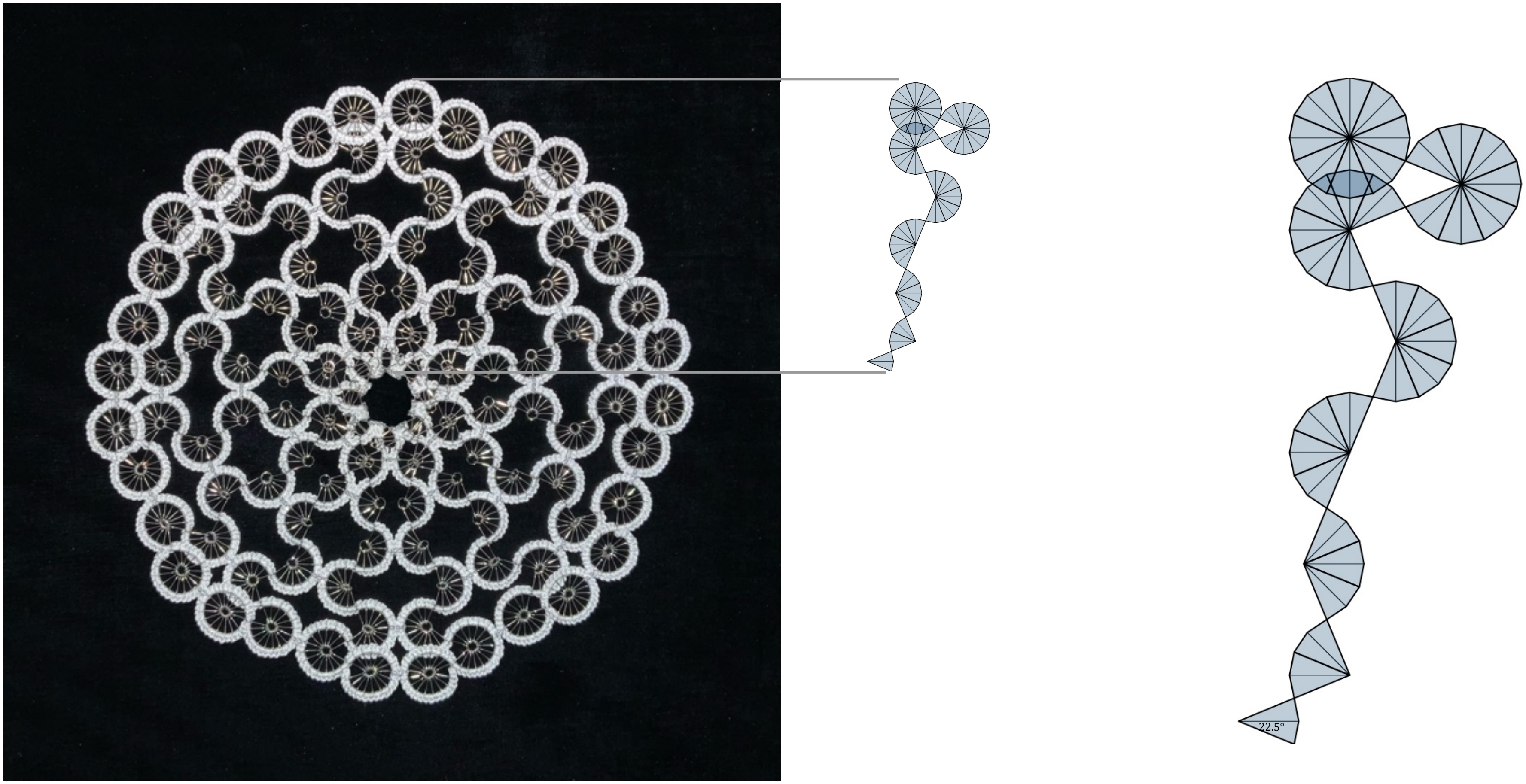
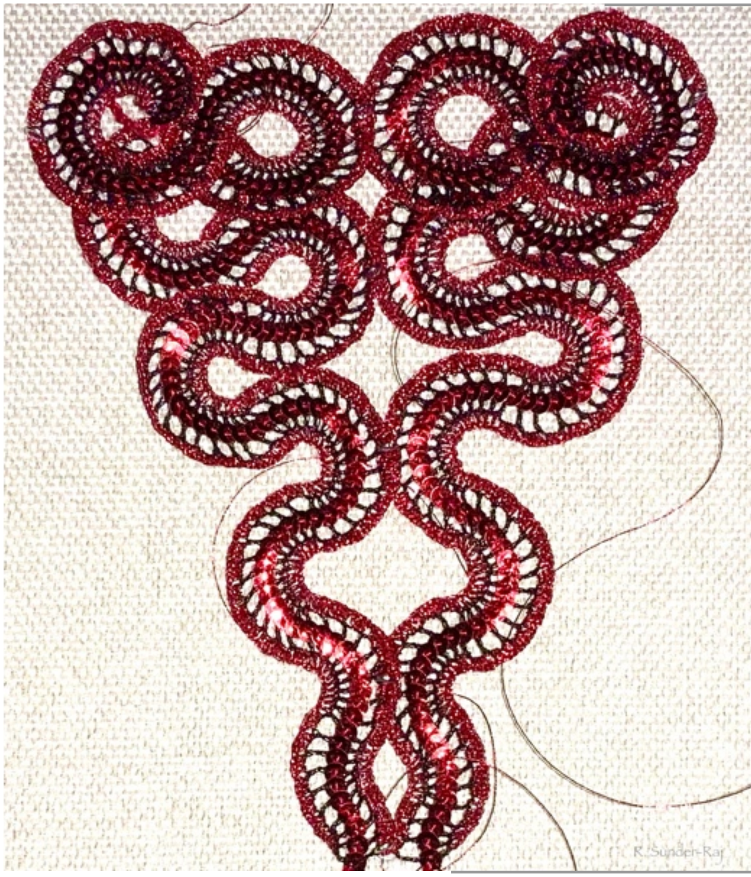
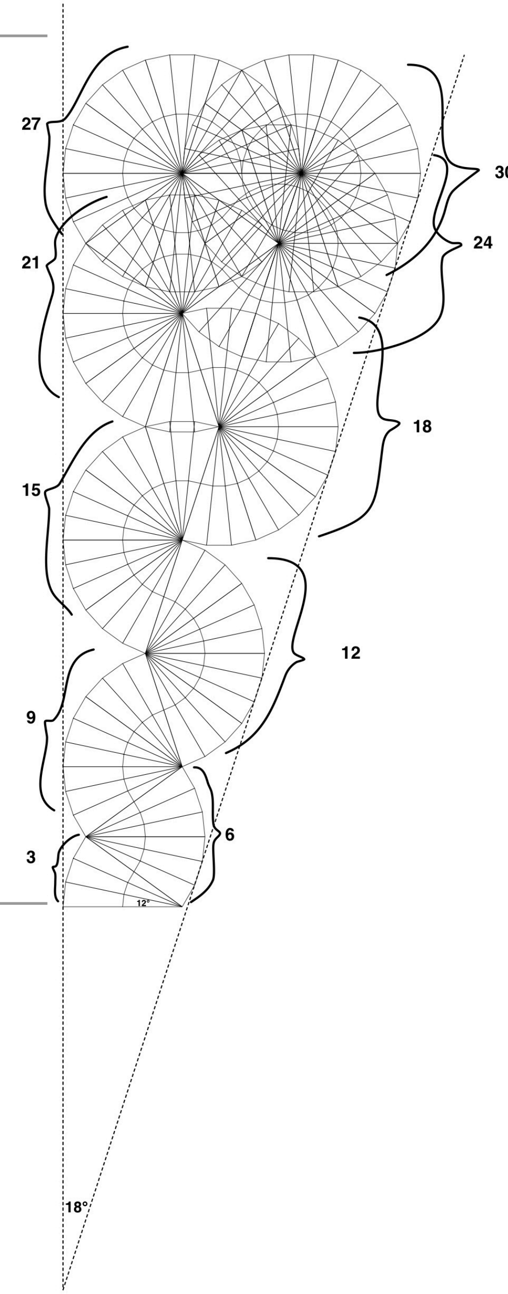
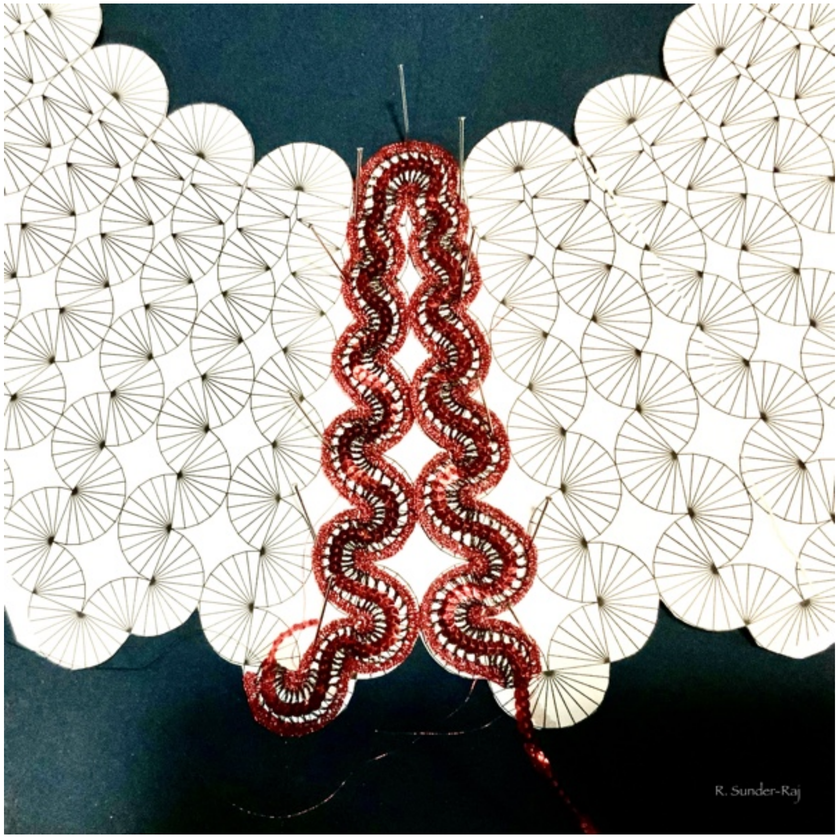


Knot Again!
A tatted piece in the Art Exhibit at Bridges 2023 made of 16 **wedge squiggles** with a 22.5° angle using the sequence 2, 4, 6, 8, 10, 12, 14, 16 (https://gallery.bridgesmathart.org/exhibitions/2023-bridges-conference/rashmi-sunder-raj).



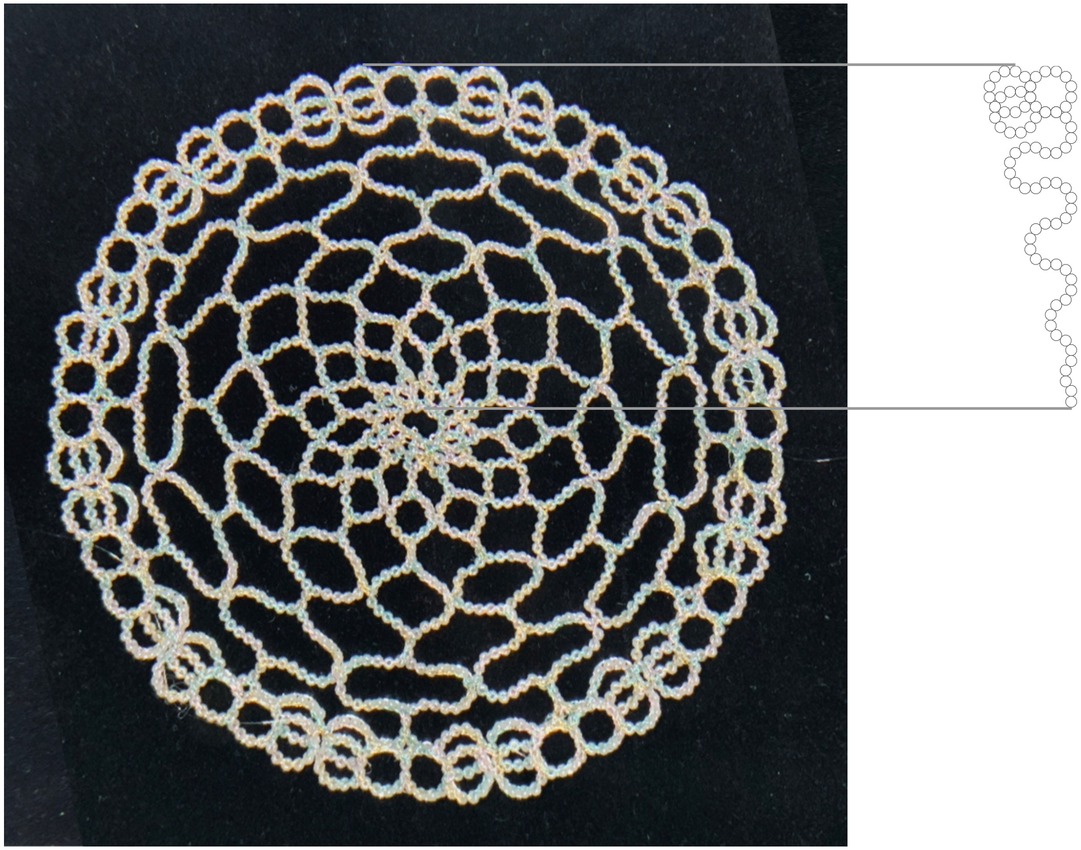


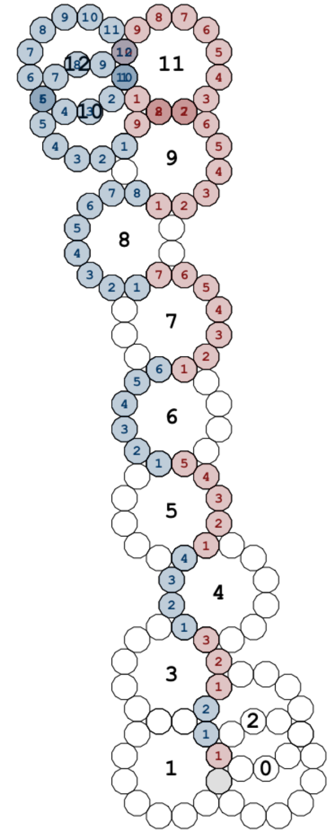
A mirrored pair of squiggles crocheted in thread around a line of sequins. They approximate **extended wedge squiggles** with a 12° angle using the sequence 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Parts of the squiggle around the centers of rotation and outer edge of the wedges have not been included.



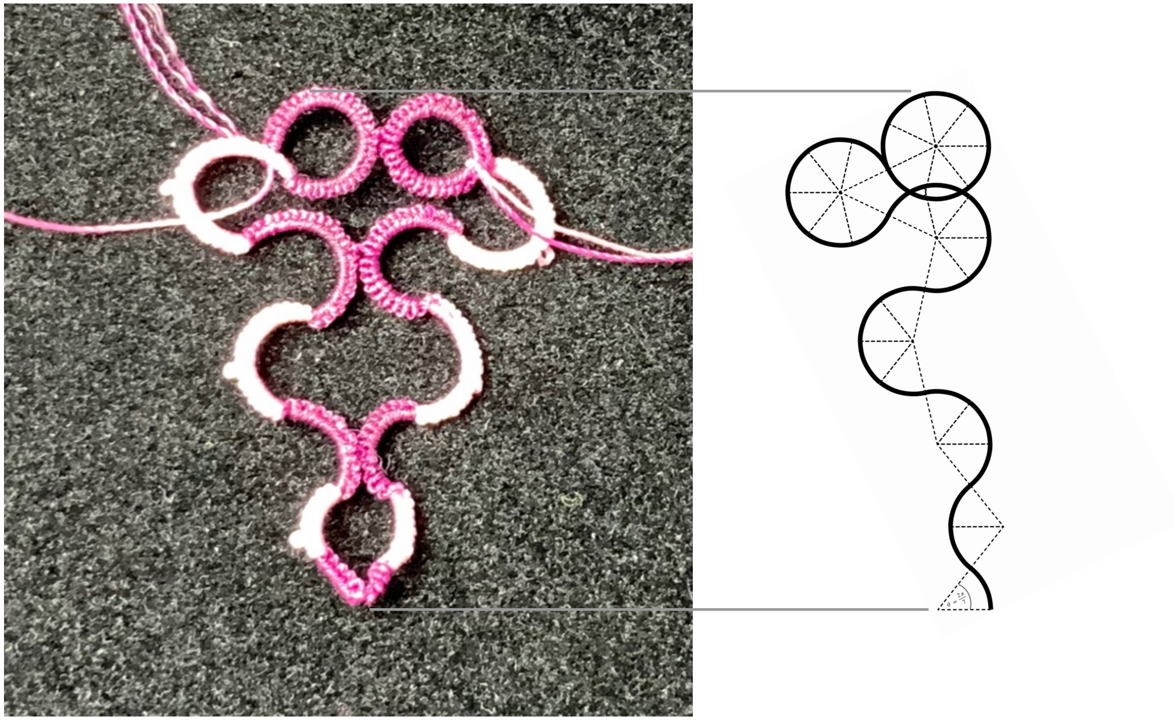
Part of a very long **extended wedge squiggle** pattern based on the repeating sequence 12, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 13, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. The first 12 here represents half of a circle, and the 13 at the middle could be considered to be 1, 0, 11, 0, 1.

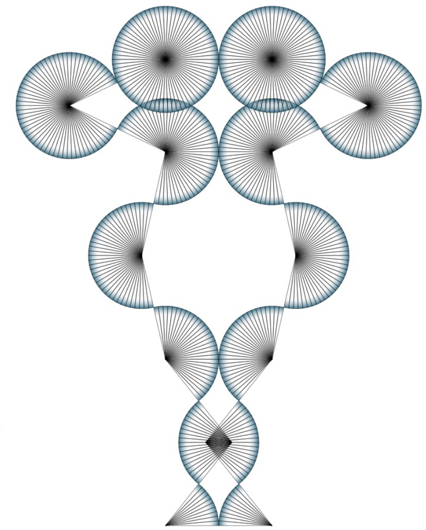
Then we could write it as (12), (12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0), (11), (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12). I have not really settled on a terminology for expressing the sequences yet. It seems that once I do, it may be easier to make calculations to determine which things will fit together nicely.



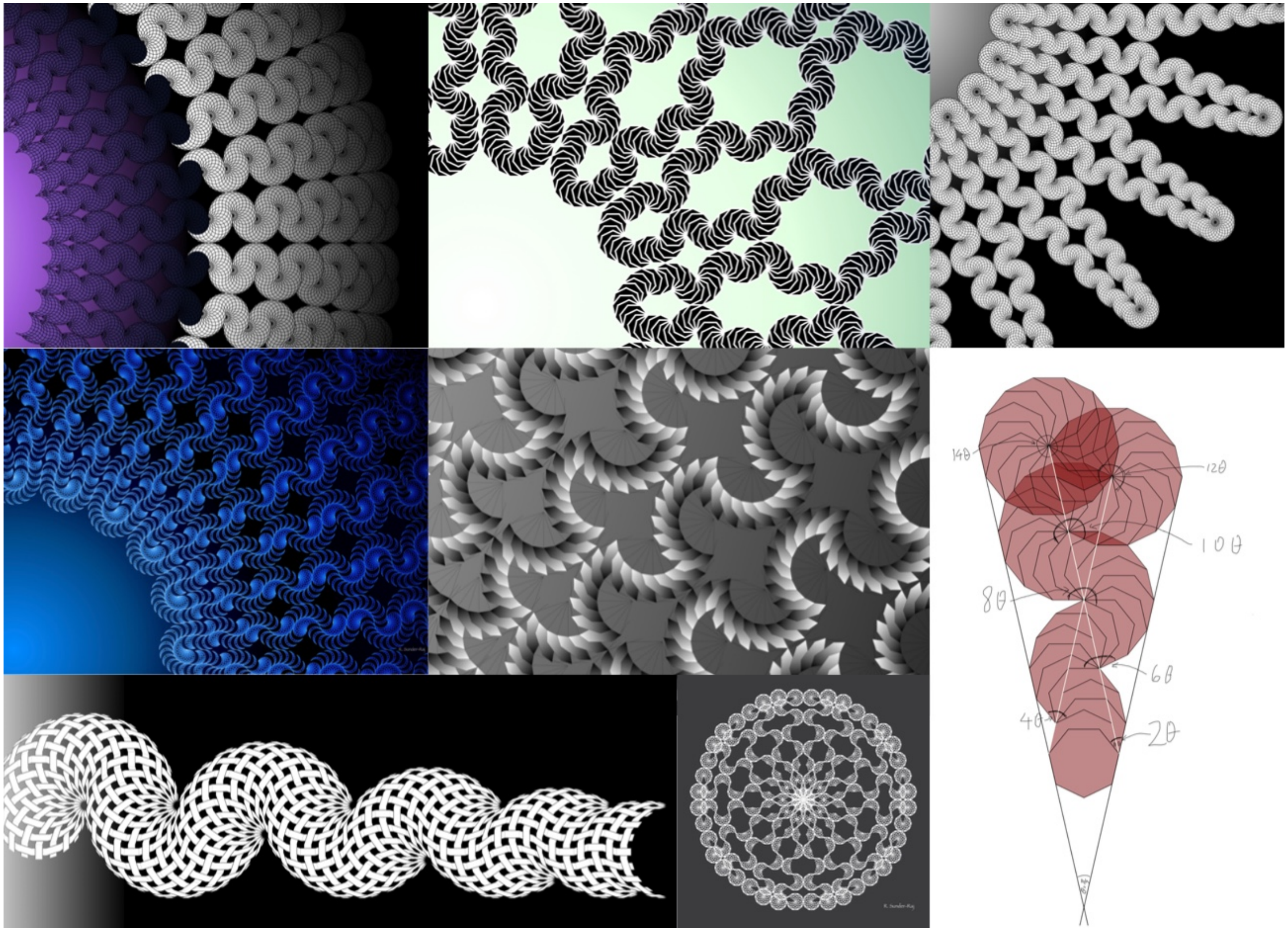


This is a pattern done in a type of bead crochet. It can be thought of as based on overlapping pairs of **polygon squiggles** made of dodecagons, using the sequence 1, 1, 2, 3, 4, …, 12. The large numbers at the centers of the polygon rings are the number of dodecagons used from the ring. The addition of an extra polygon at the bottom (the extra 1 at the beginning of the sequence) was just done for convenience when making the beading pattern. It was originally designed using 24-gons, placed in rings of 12, but that makes little difference when translated to round beads.

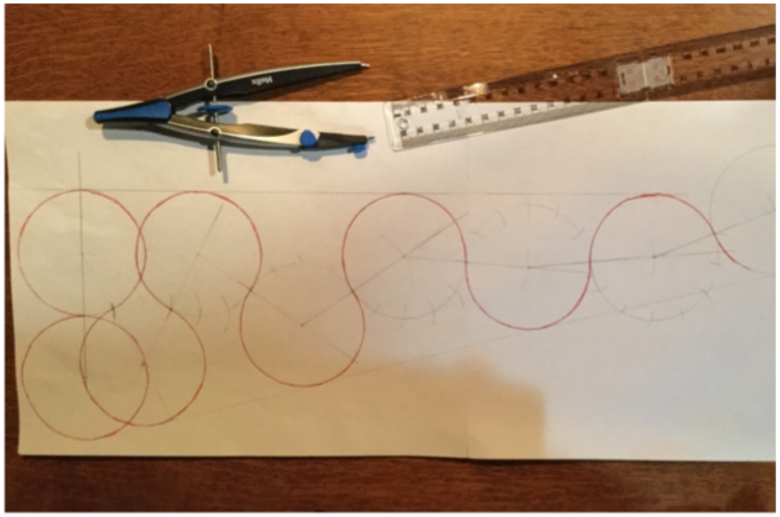


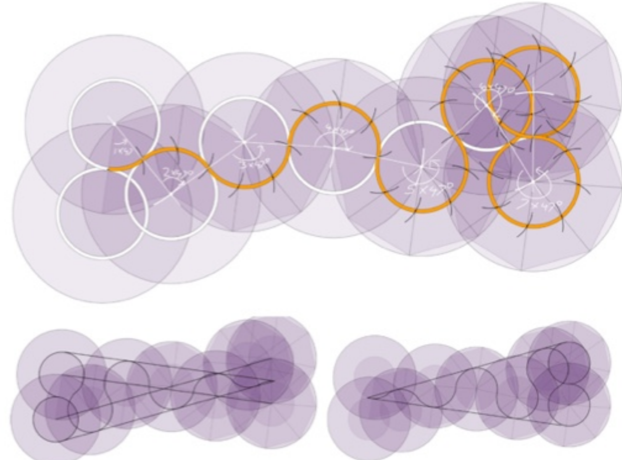


Some tatting based on an **arc squiggle** with an angle of $\frac{360^\circ}{7}$ and sequence 1, 2, 3, 4, 5, 6, 7. It could also be regarded as having units of half-knots, 70 of which go all the way around a “circle”. In this case the sequence could be thought of as being 10, 20, 30, 40, 50, 60, 70, and the angle $\frac{36^\circ}{7}$.



Some variations on the squiggle idea including crescents of rhombi and recursive squiggles.





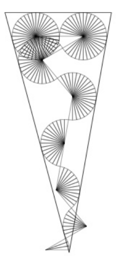
Experiments with ruler and compass and the idea of complementary squiggles.

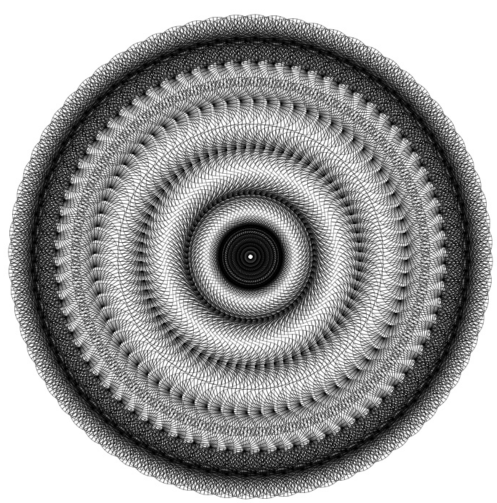
When can a rotational pattern be found for a linear squiggle?

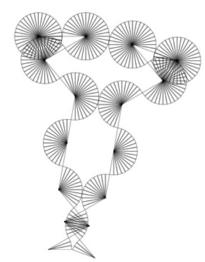
If we accept that, given a squiggle angle β and a sequence which increments by k, then the bounding triangle of the squiggle will contain an angle $\theta = \dfrac{\beta k}{2}$ around which we can rotate.

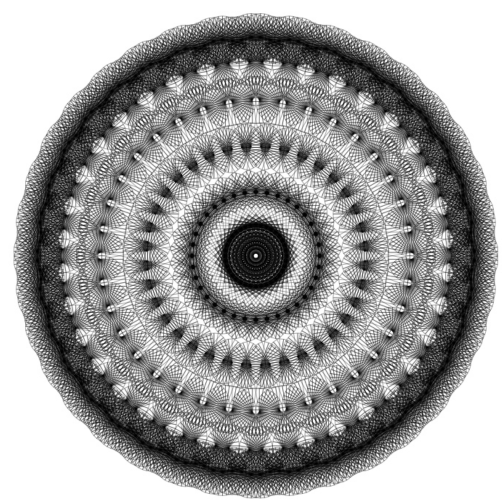
If θ divides 360°, then the squiggle will at least be able to rotate singly to form a pattern which does not have to overlap. If, in addition, 2θ divides 360˚, then we can take the squiggles in mirrored pairs so that there will be a lot of points of contact in the rotational pattern.

However, is fairly easy to construct cases which do not satisfy the constraints, for example taking a squiggle angle of $\beta = 10$˚ but incrementing by 5 units each time will give 25˚ which may result in a pretty pattern, but would have a great deal of overlap.

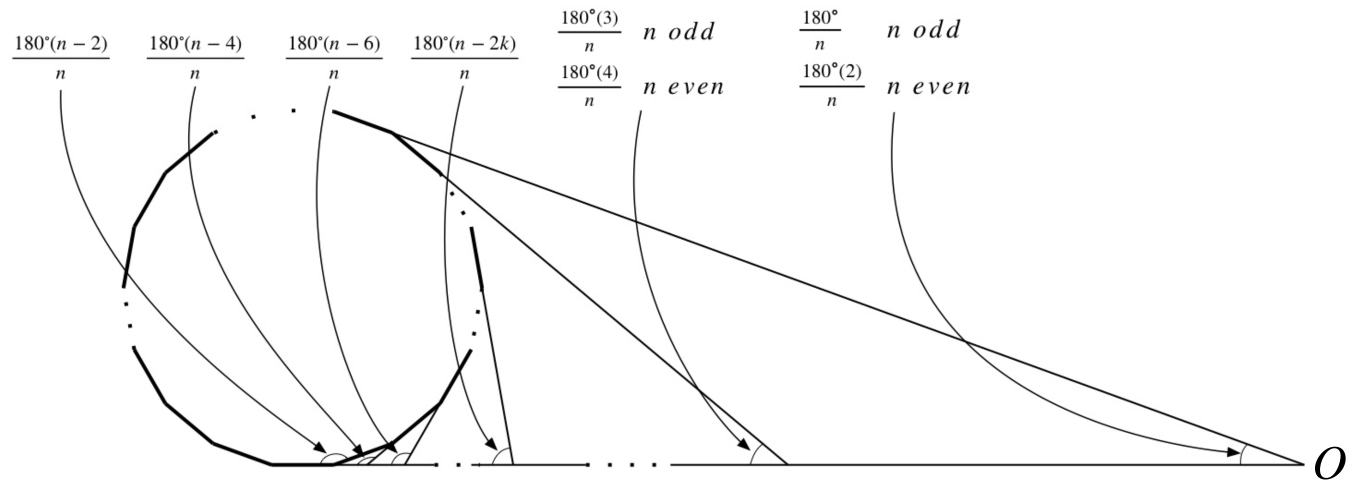








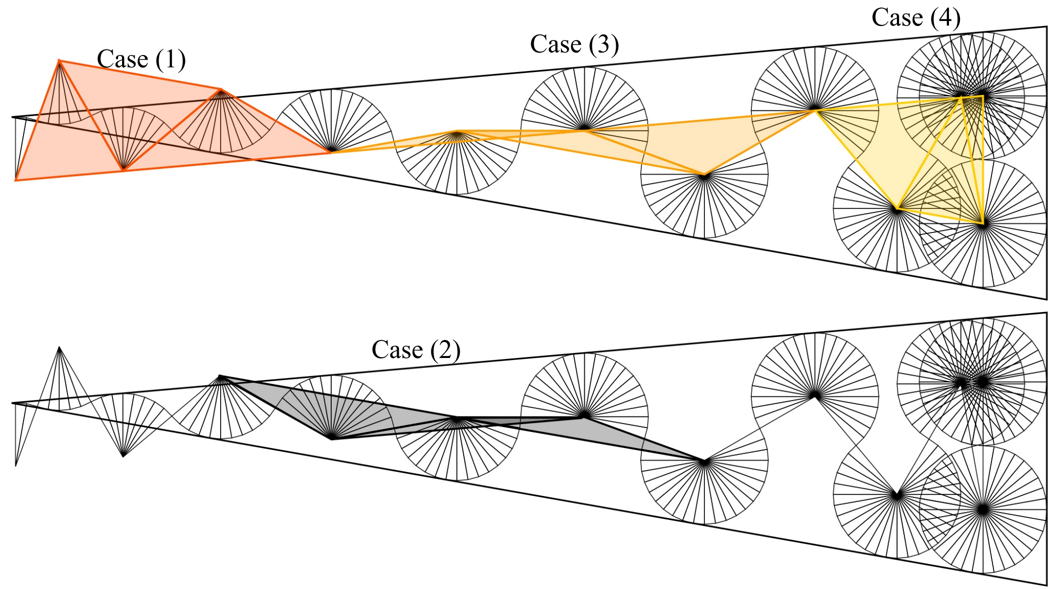
This could also be done for linear polygon squiggles, but there is more freedom in choosing the squiggle angle when using the wedge squiggles because the polygon squiggles are very restricted by the angles that can be formed when extending the sides of regular polygons (in order to get a center of rotation to form a polygon ring). Note that the restriction in the definition requiring full polygon rings to be formed in order to define a polygon squiggle may not be strictly necessary, but even if this is relaxed, the choice for a polygon squiggle angle must remain a rational multiple of 360˚.



Angles formed by the sides of a regular n-gon

Depending on which set of edges we choose to extend, for n-gons with an odd number of sides, the angle O will be one of $\frac{180^\circ(n-2)}{n}, \frac{180^\circ(n-4)}{n}, \ldots, \frac{180^\circ(n-2k)}{n}, \ldots, \frac{540^\circ}{n}, \frac{180^\circ}{n}$, and for an even number of sides, one of $\frac{180^\circ(n-2)}{n}, \frac{180^\circ(n-4)}{n}, \ldots, \frac{180^\circ(n-2k)}{n}, \ldots, \frac{720^\circ}{n}, \frac{360^\circ}{n}$. If this angle O divides 360˚, this will produce an edge-touching ring of a whole number of polygons. Otherwise there will be overlap, but eventually, the last polygon will touch the first one. We will always be able to obtain at least one non-overlapping ring of n polygons (for n odd) or $n/2$ polygons (for n even) by choosing the point of rotation obtained by joining the furthest-apart sides of the n-gon that is possible.

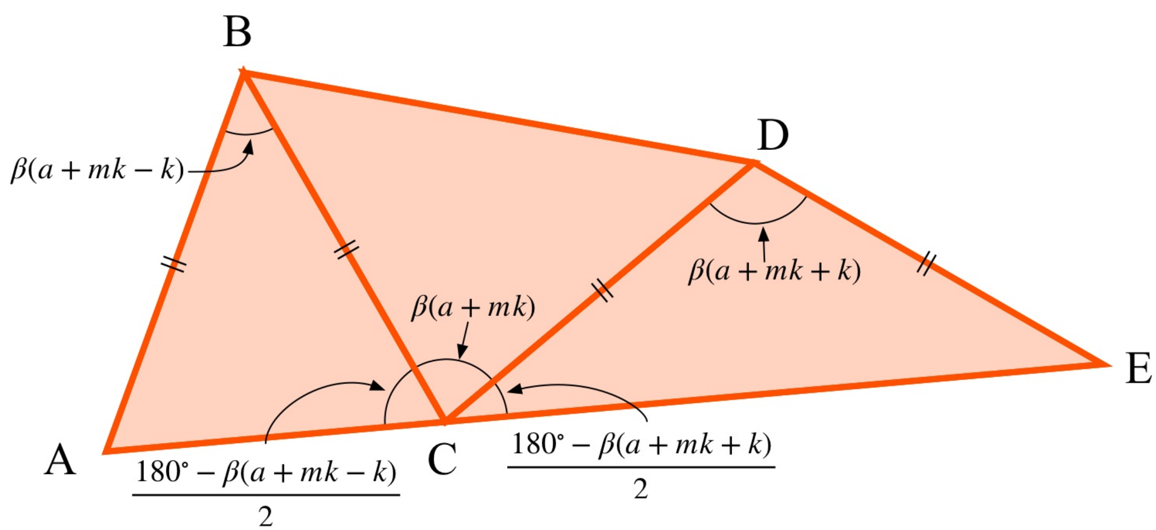
Why do straight lines go through alternating centers for linear squiggles?



Consider a linear squiggle with sequence $a, a+k, a+2k, \ldots a+nk$ and squiggle angle β. If we join any 5 adjacent centers of rotation A, B, C, D, and E to form 3 triangles $\triangle ABC$, $\triangle BCD$, and $\triangle CDE$, and can prove that line segments $\overline{AC}$ and $\overline{CD}$ meet at 180°, then we will have shown that alternate centers of rotation lie on straight lines.

Considering negative angles may simplify the proof, but would complicate illustrating it, so instead, I will break it down into cases. The group of wedges can either be towards the inner side, or completely outside of the triangles which have been drawn. Ignoring the degenerate case where k is 0, we have 4 cases:

(1) All three triangles lie on their corresponding groups of wedges.
(2) Two triangles lie on their groups of wedges, but the other lies outside.
(3) One triangle lies on its group of wedges, but the others lie outside.
(4) All three lie outside their corresponding groups of wedges.



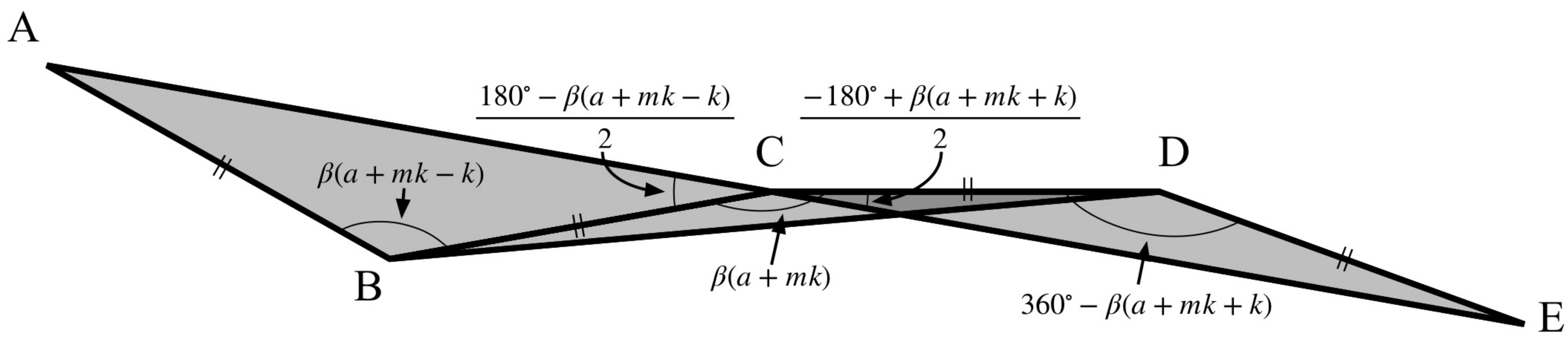
Case (1)

Because the squiggle is linear, for some integer m, we must have
$\angle ABC = \beta(a + mk - k)$
$\angle BCD = \beta(a + mk)$
$\angle CDE = \beta(a + mk + k)$

By construction, $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

So $\angle ACB = \dfrac{180^\circ - \beta(a + mk - k)}{2}$ and $\angle DCE = \dfrac{180^\circ - \beta(a + mk + k)}{2}$

$$\angle ACE = \angle ACB + \angle BCD + \angle DCE = \frac{180^\circ - \beta(a + mk - k)}{2} + \beta(a + mk) + \frac{180^\circ - \beta(a + mk + k)}{2} = 180^\circ$$



Case (2)

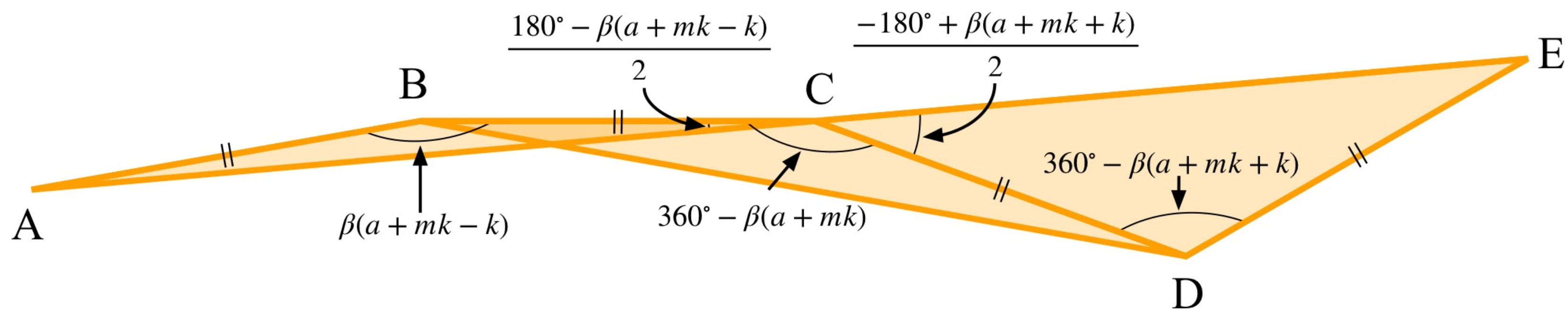
Because the squiggle is linear, for some integer m, we must have
$\angle ABC = \beta(a+mk-k)$
$\angle BCD = \beta(a+mk)$
$\angle CDE = 360^\circ - \beta(a+mk+k)$

By construction, $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

So $\angle ACB = \dfrac{180^\circ - \beta(a+mk-k)}{2}$

and $\angle DCE = \dfrac{180^\circ - (360^\circ - \beta(a+mk+k))}{2} = \dfrac{-180^\circ + \beta(a+mk+k))}{2}$

$$\angle ACE = \angle ACB + \angle BCD - \angle DCE = \frac{180^\circ - \beta(a+mk-k)}{2} + \beta(a+mk) - \frac{-180^\circ + \beta(a+mk+k)}{2} = 180^\circ$$



Case (3)

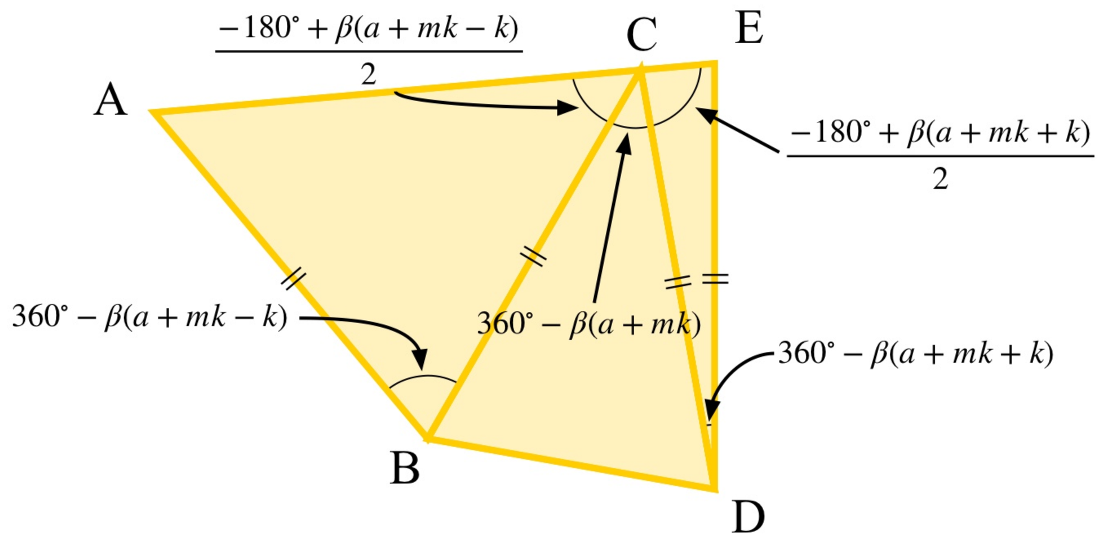
Because the squiggle is linear, for some integer m, we must have
$\angle ABC = \beta(a + mk - k)$
$\angle BCD = 360^\circ - \beta(a + mk)$
$\angle CDE = 360^\circ - \beta(a + mk + k)$

By construction, $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

So $\angle ACB = \dfrac{180^\circ - \beta(a + mk - k)}{2}$

and $\angle DCE = \dfrac{180^\circ - (360^\circ - \beta(a + mk + k))}{2} = \dfrac{-180^\circ + \beta(a + mk + k))}{2}$

$$\angle ACE = \angle BCD - \angle ACB + \angle DCE = 360^\circ - \beta(a + mk) - \frac{180^\circ - \beta(a + mk - k)}{2} + \frac{-180^\circ + \beta(a + mk + k)}{2} = 180^\circ$$



Case (4)

Because the squiggle is linear, for some integer m, we must have
$\angle ABC = 360^\circ - \beta(a + mk - k)$
$\angle BCD = 360^\circ - \beta(a + mk)$
$\angle CDE = 360^\circ - \beta(a + mk + k)$

By construction, $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

So $\angle ACB = \dfrac{180^\circ - (360^\circ - \beta(a + mk - k))}{2} = \dfrac{-180^\circ + \beta(a + mk - k))}{2}$

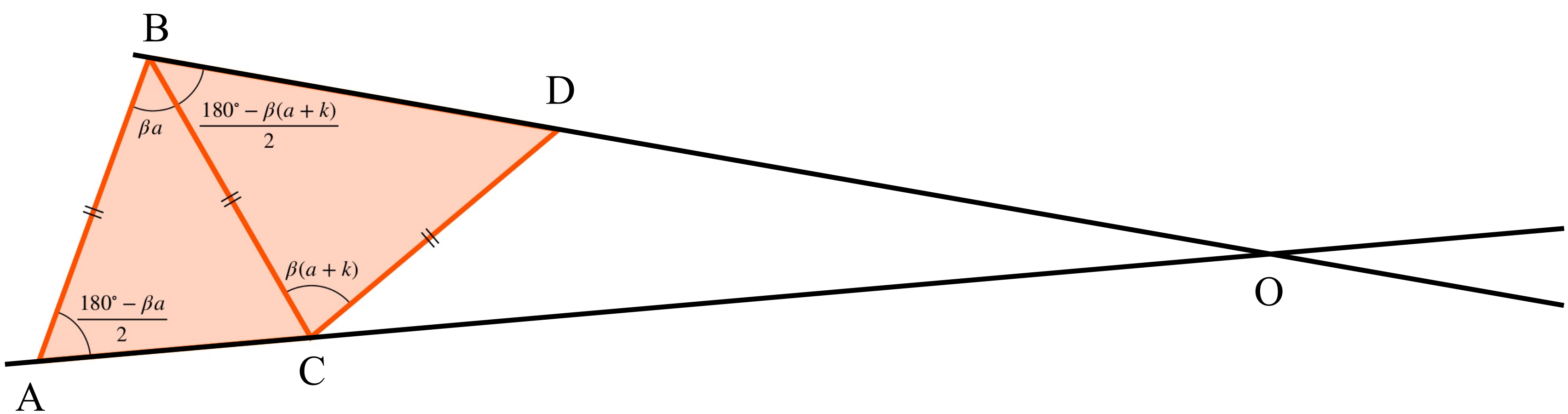
and $\angle DCE = \dfrac{180^\circ - (360^\circ - \beta(a + mk + k))}{2} = \dfrac{-180^\circ + \beta(a + mk + k))}{2}$

$$\angle ACE = \angle ACB + \angle BCD + \angle DCE = \frac{-180^\circ + \beta(a + mk - k))}{2} + 360^\circ - \beta(a + mk) + \frac{-180^\circ + \beta(a + mk + k))}{2} = 180^\circ$$

$\square$

At what angle do the lines through the centers of linear squiggles cross?

We showed above that there are indeed straight lines through alternating centers of rotation for linear squiggles, so we can simply choose any 3 centers, draw the lines through them to their point of intersection (if that intersection exists), and calculate the angle of intersection.



For convenience, let m=0,

Then by construction, $\overline{AB} = \overline{BC} = \overline{CD}$ and $\angle ABC = \beta a$ and $\angle BCD = \beta(a+k)$

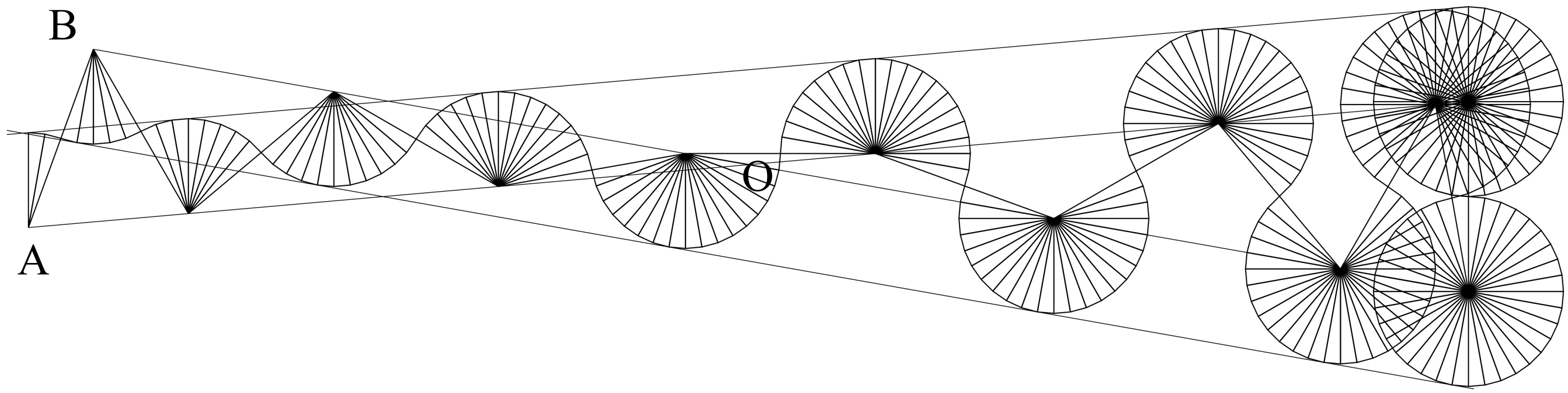
Since $\triangle ABC$ and $\triangle BCD$ are isosceles,

$\angle BAC = \dfrac{180^\circ - \beta a}{2}$ and $\angle CBD = \dfrac{180^\circ - \beta(a+k)}{2}$

So $\angle AOB = 180^\circ - \dfrac{180^\circ - \beta a}{2} - \beta a - \dfrac{180^\circ - \beta(a+k)}{2} = \dfrac{\beta k}{2}$

$\square$

Does the "bounding" triangle of a linear squiggle really have an angle of $\frac{\beta k}{2}$?



We showed above that the lines through the alternating centers crossed with an angle of $\angle AOB = \frac{\beta k}{2}$.

If the squiggle is made up of circular arcs (all with the same radius), it should not be difficult to establish that a line going through the centers of a group of them should be parallel to a line tangent to the circles in that group. This would lead to the "bounding" triangle containing the same angle as $\angle AOB$ as required.

However, I need to think a bit more about polygon squiggles and other variations to be absolutely sure that they do not introduce issues.

What about non-linear squiggles?

Initially, when I was playing around with the idea, one of my children helped me out by writing a program to display squiggles based on the sequence used. They noticed that when extending the pattern for linear squiggles beyond rotating 360°, it would end up repeating, and the result would be the same as its image when rotated 180°. They also ended up deciding to try various non-linear sequences to see what would happen. But I am not quite sure what came of these explorations.

Some of the squiggles that I have drawn or made had linear portions, but their sequences included added bits to cause the patterns to turn around, or to increase, then decrease in size. Others essentially turned back and forth to form basically polygonal shapes.

I have given a few examples below which involve non-linear or at least not-fully-linear squiggles. Most of the non-linear sequences that I have tried do not seem to result in squiggles which fit together very well. In the future, I may have to consider what will happen if the squiggle idea can be extended to sequences which allow the radius of the "circle" to vary in some way.