Cosmatesque Mosaic and Conformal Mapping

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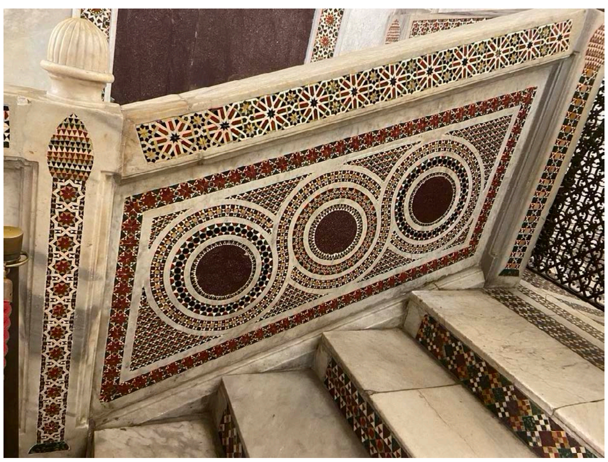
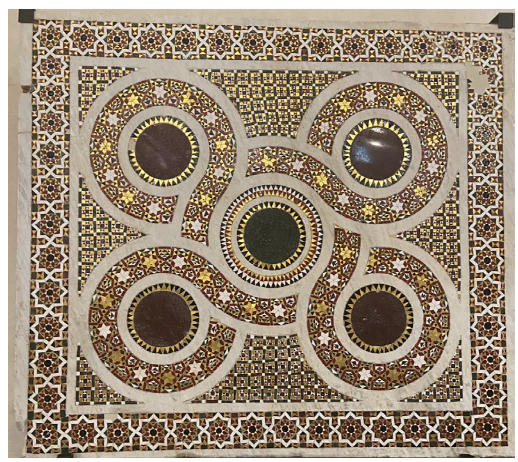
Abstract

This article highlights an application of conformal mapping as a means of generating traditional and modern Cosmatesque patterns. Many traditional Cosmatesque pattern can be modelled using a conformal exponential map of one complex variable. Alternative conformal maps lead to more modern versions.

Cosmatesque, or Cosmati, refer to mosaic patterns that were prevalent in 12^{th} century Rome, that appear throughout Italy [2] and the United Kingdom [3]. The name originates from several generations of craftsmen in the Cosmati family. While this style of ornament is almost a thousand years old, craftsmen today still manufacture and restore these designs. These patterns appear mainly on church floors. The most characteristic patterns are referred to as guilloche, a serpentine arrangement of alternating braids and the quincunx, an arrangement of four circles around a central circle again connected by a braid-like design as illustrated in Figure 1.



(a)



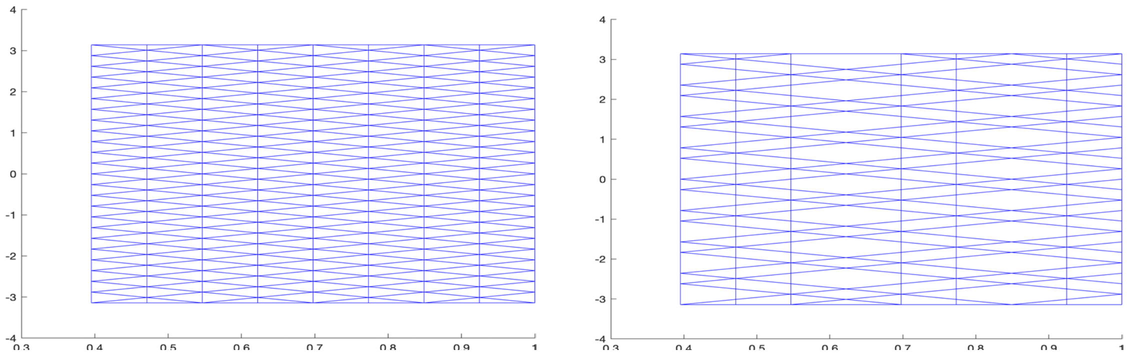
(b)

Figure 1: *Typical (a) Guilloche and (b) Quincunx patterns from Capella Palatina in Palermo .*

Within the braids, as pictured above, is a tiling pattern often recognized as a regular or uniform tiling, albeit transformed to fit inside an annular region. Many of these transformed tilings can be constructed as the result of a conformal mapping of a corresponding planar uniform tiling. Discussed below will be a particular tiling of triangles, hexagons and parallelograms.

In particular the types of patterns demonstrated below are based on a common underlying grid. On the left in Figure 2 is illustrated a design within a rectangle of arbitrary width w and height 2π that is uniquely determined by several factors, namely w_{min} , w_{max} , arbitrarily chosen to differ by w, an angle α, which measures the slope of the diagonal lines and an integer n that determines the number of vertical partitions.

As α and n vary, the shape and complexity of the annular patterns will vary. While the graph on the left illustrates a collection of diagonal and vertical lines, the alternative shown in Figure 2b selectively omits every third diagonal and vertical line and thus reveals a tiling of parallelograms, triangles and hexagons.



(a) (b)

Figure 2: *Tessellation of a strip before transformation with (a) all lines and (b) selective removals.*

The conformal mapping $(x,y) \rightarrow (e^x\cos(y), e^x\sin(y))$ transforms the grid from the rectangular domain to the annular range [1]. A further constraint on the design will force the image of y=0 to be the same as that of $y = 2\pi$. Applying the exponential conformal map to the grids above produces the following patterns which are illustrated below in Figure 3 and 4 in graphic and mosaic form.

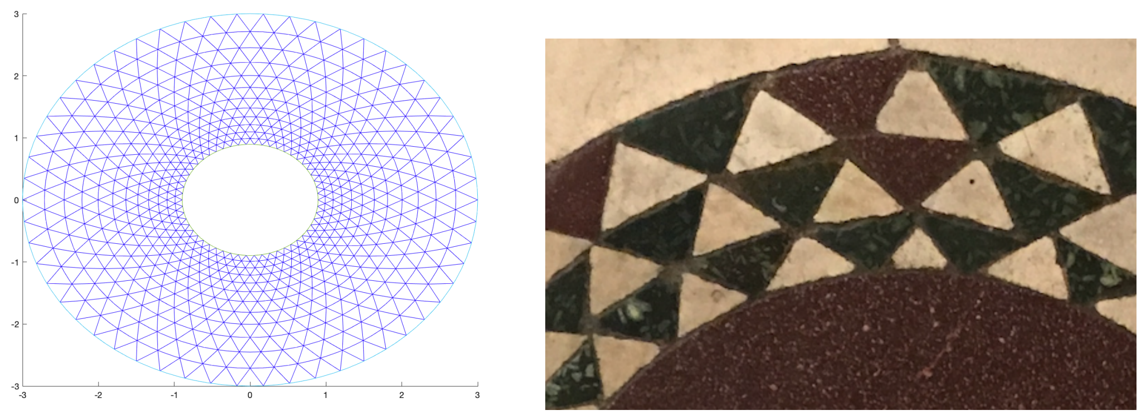


Figure 3:*Conformal result of Figure 2(a).*

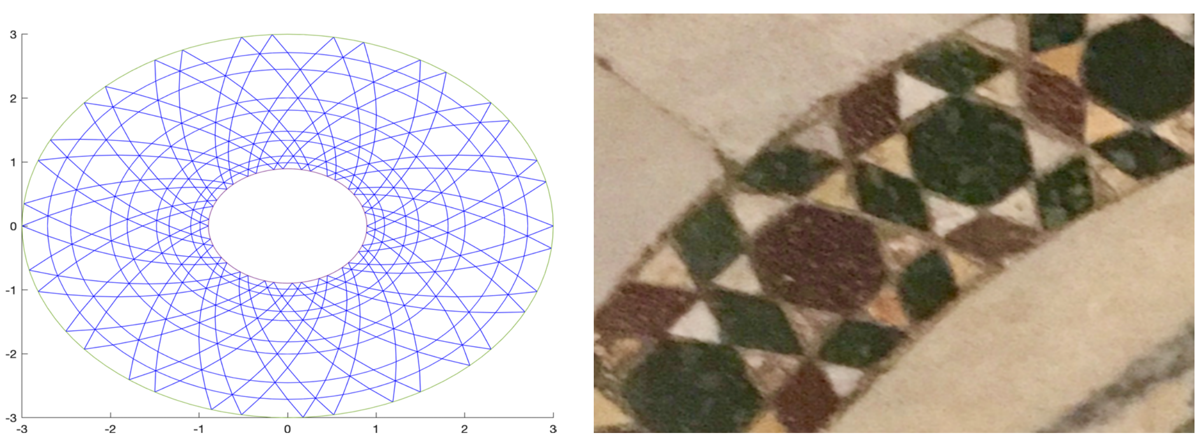
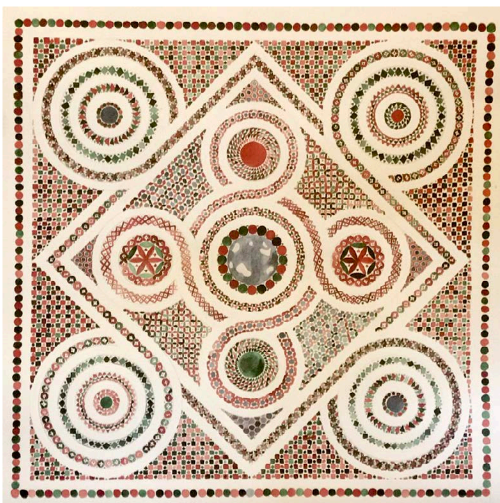


Figure 4: *Conformal result of Figure 2(b).*

More complicated designs can be constructed based on alternative planar tilings that include octagons and circular shapes, all subject to the exponential map.

(a)



(b)

(c)

Figure 5: *A Collection of watercolor paintings by the author inspired by (a) the Westminster Abbey in London and (b) the Sistine Chapel in Rome. A collection of designs by the author is in (c).*

As a modern alternative, other conformal maps of $z = x+iy$ such as z^a or (a+bz) / (c+dz) produce patterns as illustrated below in Figure 6 followed by the authors watercolor rendition.

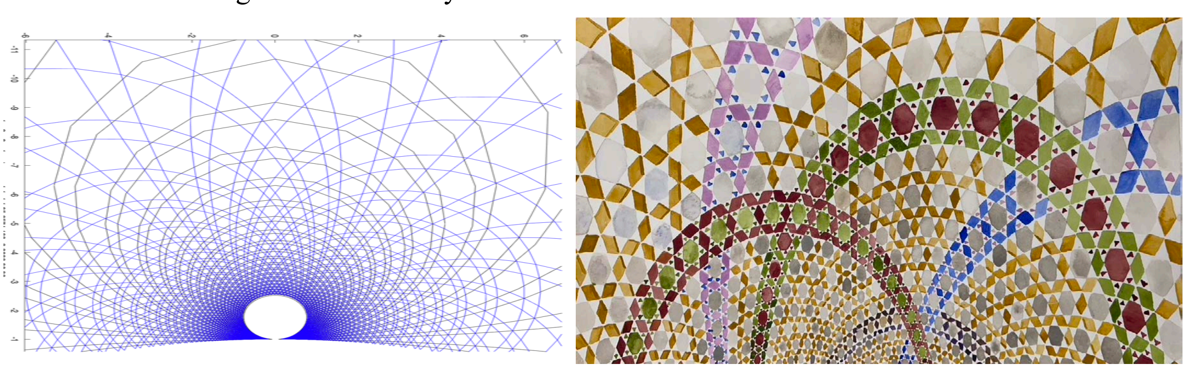
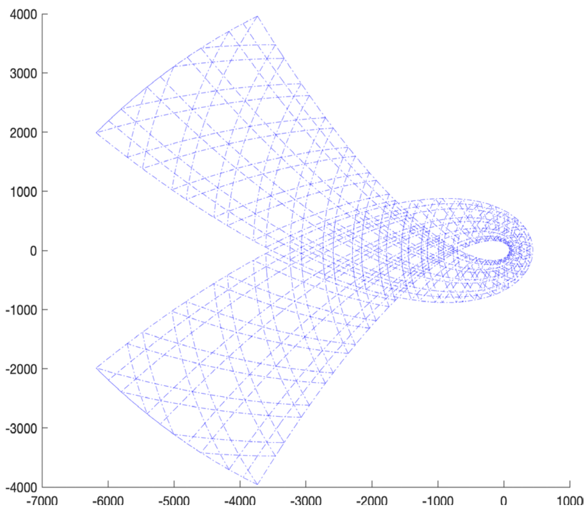


Figure 6a: *Transformation of Figure 2b by the Möbius transformation (1-z) / (1+z)*



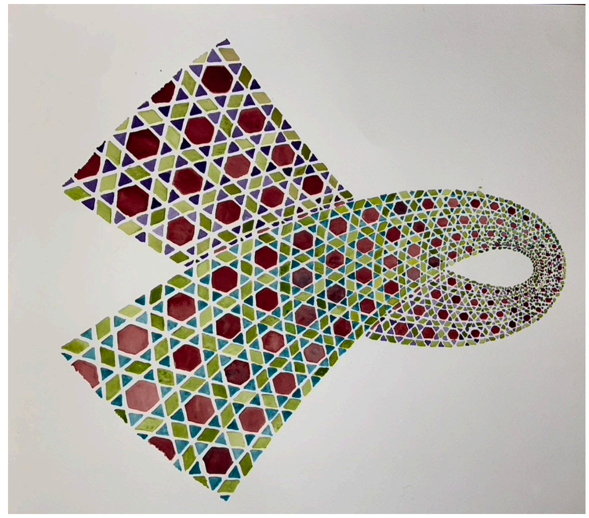
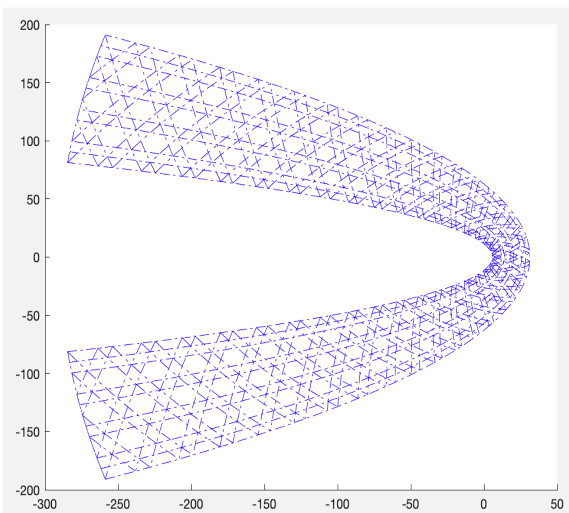


Figure 6b: *Transformation of Figure 2b by* z^3



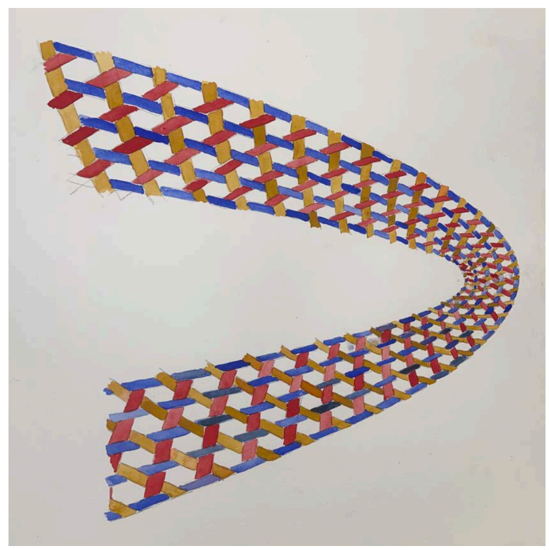


Figure 6c: *Transformation of Figure 2b by* z^2

Realizing these patterns as physical mosaics would be more complex owing to the absence of identical tile pieces but a computer-driven process could overcome this difficulty.

References

[1] T. Needham, "Visual Complex Analysis", Oxford Press, 1997

[2] P. Pajres-Ayuela, "Cosmatesque Ornament: Flat Polychrome Geometric Patterns in Architecture", W.W. Norton & Co. 2001

[3] W. Rodwell, D. Neal, "The Cosmatesque Mosaics of Westminster Abbey", Oxbow Books, 2019.