Symmetry Notation as a Multidisciplinary Method for the Design of Origami Tessellations

Laureen Mahler

School of Arts, Design and Architecture, Aalto University, Finland; laureen.mahler@aalto.fi

Abstract

As the practical and artistic applications of origami tessellations continue to expand, so too does the opportunity to pursue multidisciplinary approaches to the design of and for complex origami forms. The ongoing research introduced in this short paper demonstrates how the combined approaches of mathematics and design—in particular the use of symmetry notation to better illustrate aesthetic and kinematic characteristics of origami structures—can produce novel perspectives and methods that support experimentation and innovation.

Background and Context

Origami is the art of folding—typically, though not exclusively paper—to create representational or abstract structures, and it represents a unique art form. With both its Eastern and Western origins dating back centuries [10], origami has received particularly heightened recognition in recent years as its practical applications become more prevalent [4]. In the 1980s, origami practitioners began to systematically document their techniques, providing the foundation for myriad innovations in the medium [7], including the development of technical folding, or *origami sekkei*. Origami tessellations in the form of corrugations, which constitute the focus of this paper, are one manifestation of technical folding.

In the wake of the increased interest in origami, research on the generation of novel folded structures has proliferated, with much of this work positioning itself in the fields of structural engineering and mathematics [2, 4, 8]. While these approaches are integral to the expansion of the practice, there exists an overlooked opportunity to consider technical folding from a multidisciplinary perspective that includes the expertise of art and design. This short paper aims to document ongoing work that combines artistic research and mathematics to provide a novel approach to the design of corrugated tessellations. While it takes as inspiration existent generative approaches, including models proposed by Lang, Bateman, Palmer, Barreto, and Hudson [9], the objective of this research is not to provide algorithmic data or mathematical proofs, but rather to illustrate how design and mathematical approaches can be combined to foster alternative perspectives on the design of and for origami structures.

Definition of Terms

Origami tessellations are folded from grids (square, triangle, or 45-degree/diamond), upon which unit cells are periodically repeated via translation, reflection, and/or rotation [7]. The resulting tessellations are often categorized into three sub-groups: 1) standard tessellations without gaps made by overlapping layers of paper; 2) self-similar patterns that utilize scaled iterations; and 3) corrugations that have no overlaps and are frequently flat-folding [5]. The work presented here focuses on origami corrugations and the role that symmetries play in their creation.

Corrugations include the recognizable Miura, Yoshimura, and waterbomb folds as well as Resch and Kresling structures [4], and their kinematic properties have been well documented in both mathematics and engineering [4, 8]. The tessellations and corresponding dynamic motion produced by these structures provide a unique opportunity to examine how symmetries shape not only their crease patterns, but also the characteristics of the resulting folded models. The examination of symmetries in this research utilizes the Thurston-Conway orbifold notation system, as it provides accessible descriptors that indicate the symmetry actions inherent to patterns in the Euclidean plane [3]. In that context the symmetry notation of both two-dimensional crease patterns and their three-dimensional folded outcomes are depicted in terms of the 17 wallpaper tilings [6].

Symmetry analysis using orbifold notation includes the identification of reflection lines and rotation points as well as instances of translations and non-kaleidoscopic reflection, wherein the symmetry signatures are categorized into rotations, reflections, and mixed hybrids [3]. As depicted in Figure 1, the crease pattern of the Miura tessellation (a well-known corrugation) has a symmetry signature of 22*, indicating that the symmetry is composed of two points of rotation order 2 as well as one type of kaleidoscopic reflection at one repeated vertex. The model folded from this crease pattern has an identical symmetry signature, and in both instances the shape traced in blue is defined as the fundamental domain, or the smallest crystal of the pattern required to create the tessellation [3].

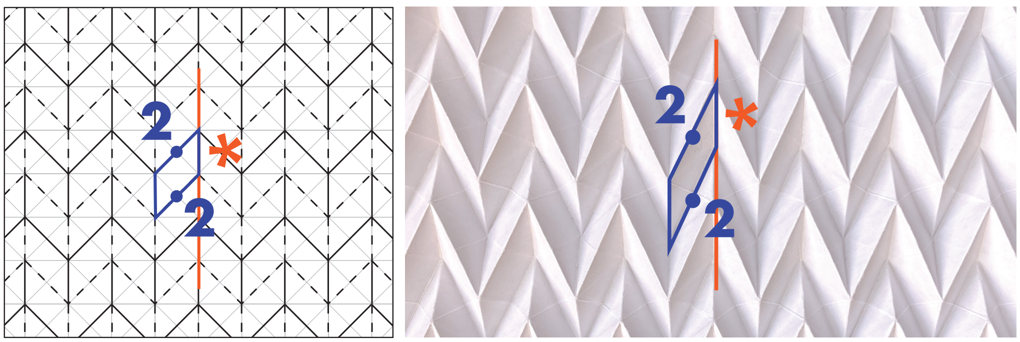


Figure 1: *Orbifold notation for the crease pattern and resulting origami model of a Miura tessellation.*

Approach and Process

The basis of this research lies in the concept that symmetry embodies motion and action—whether in planar patterns, surfaces in three-dimensional space, or objects in the hyperbolic plane. Symmetry notation is an effective method of translating that motion into representative visuals as well as a signature that summarizes the action (rotations, reflections, etc). These movements manifest isometric motion, or movement in space that preserves the distance between points—applicable in both two- and three-dimensional space.

This research focuses on origami tessellations in the form of corrugations as previously described, including standard and hybrid versions of the Miura, Yoshimura, and Resch folds, as well as iterations of the waterbomb base. Corrugations were selected because they reliably demonstrate a direct correlation of symmetries between their crease patterns and the resulting folded origami models. Preference was given to flat-foldable and/or rigid structures that could be assessed in the context of Euclidean plane tilings, with spherical structures reserved for a future stage of research. As a reflection of the ongoing research, this paper firstly analyzes the direct correlation of symmetry patterns in origami corrugations, then develops the concept of symmetry motion and tiling formations as tools that designers can use in the creation and visual design of corrugated structures.

Results and Discussion

In a background examination of origami structures for the present research, it was established that tessellations of the standard and self-iterative types—in which layers of paper are typically folded over each other to achieve a repeated pattern—do not present a consistent correlation between crease patterns and folded models. However, the corrugation type of tessellation is unique in that it does not use hidden layers of paper in the periodic repetition of the unit cell. As a result, when orbifold notation is applied to the crease pattern and its resulting corrugation model, a direct correlation is consistently demonstrated. Figure 2 represents an excerpt from the visual data collection phase of the research, which allowed for comparison of crease patterns and their generated forms. In these comparisons, the digital crease patterns were first analyzed for their symmetry behaviors and signatures (Figure 2, left), followed by the folding and analysis of the corresponding tessellated structures (Figure 2, right). Subsequently, several notable characteristics of the direct correlation between corrugation crease patterns and folded models were observed.

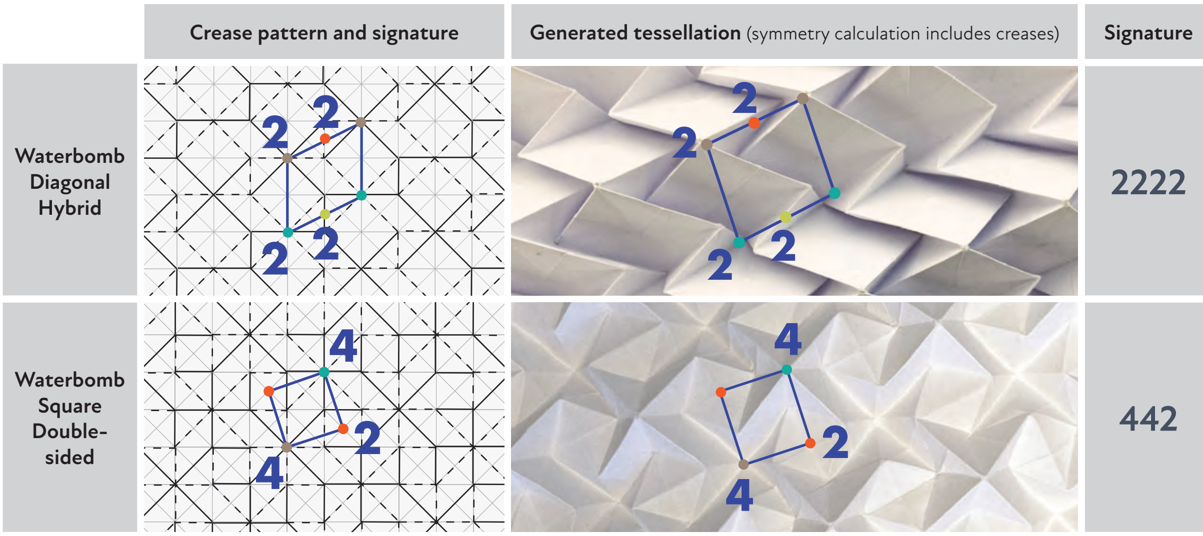
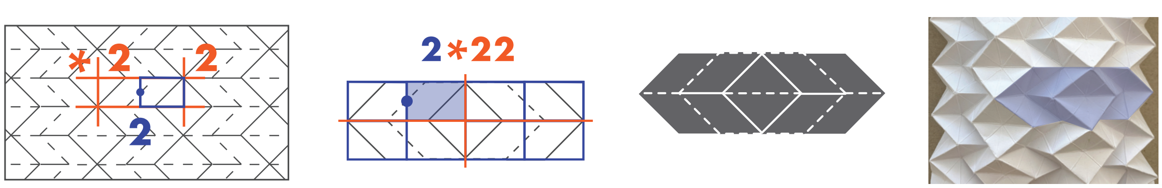


Figure 2: *Excerpt from the rotations category of the visual data collection; colored points and numbers indicate different points and orders of rotation, and blue lines specify the fundamental domain.*

In each crease pattern/folded model pair, the fundamental domains are identical (Figure 2, shapes outlined in blue). This correspondence demonstrates a key characteristic of origami corrugations: the symmetry movements indicated by the crease pattern and performed in the folding process are visually preserved in the model. Thus, two significant behaviors can be generalized: 1) the physical iteration of the fundamental domain directly corresponds to its positioning in the original crease pattern, and 2) the resulting structure can be moved dynamically in three dimensions, in a way that manifests each step of the folding process. This allows the designer to essentially "preview" the three-dimensional qualities of the physical model by examining the two-dimensional crease pattern, as well as approximate its kinematic movement based on the shape of the fundamental domain and the motion described by the symmetry signature. In other words, the dimensionality and kinematics implicit in the crease pattern become explicit in the resulting model.

When the physical model preserves the symmetry of the crease pattern, the designer can also identify primary shapes that make up the periodic repetition of the tessellation. This is the structure's "tiling formation", created by isolating the fundamental domain and reconstructing its symmetry motion. With the Yoshimura-Miura arch hybrid as an example (Figure 3), the tiling formation is created by determining the fundamental domain (Figure 3a), reconstructing the symmetry motion as indicated by the signature (Figure 3b), and identifying the formation of shapes that make up the tiling (Figure 3c). In the resulting model, this tiling formation represents the dominant visual motif repeated in the structure (Figure 3d).



(a) (b) (c) (d)

Figure 3: *Tiling formation: (a) crease pattern notation, (b) reconstruction of the symmetry motion from the fundamental domain, (c) identifying the tiling formation, (e) the resulting model.*

Tiling formations provide the designer with a complete picture of precisely how the unit cells of a tessellated corrugation move relative to each other as well as the overall structure. From a design perspective, this correspondence aligns with the Gestalt doctrine of isomorphism, in which objective and subjective visual perceptions share the same pattern form [1]. This not only supports the design of novel origami structures, it also establishes an important foundation for a method of designing *for* origami corrugations: including the design of imagery and/or typography intended to visually enhance the structure. Thus, the tiling formation serves as a building block for interactive visual design in both two and three dimensions, as it emphasizes the embodiment of the symmetry action—and translates it into a practical tool that designers can use to create more effective aesthetic systems.

Summary and Conclusion

Although various software programs and algorithms exist to translate vector-based crease patterns into animated digital previews, the ability for a designer to predict and understand the symmetry behaviors of corrugations not only facilitates the creation of novel crease patterns and structures, it also enhances the visual design process. Additionally, the understanding of symmetries as manifestations of motion lays the developmental groundwork for a design methodology unique to origami corrugations. While CAD software can effectively map two-dimensional designs onto virtual structures, it does not offer a viable method for the ideation, visual design, and execution of dynamic three-dimensional origami structures.

When the principles of symmetry notation become an extension of the designer's visual and spatial toolkit, the result is an ability to create fully integrated visual designs in three dimensions of dynamic space—a skillset equally as vital as the design of novel origami structures themselves. The research introduced in this short paper is ongoing, and future work will include additional corrugation models as well as spherical symmetries for curved folding, with the objective of developing a design methodology for dynamic origami tessellations.

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References

[1] V. Bruce, P.R. Green, and M.A. Georgeson. *Visual Perception: Physiology, Psychology, and Ecology*. Psychology Press, 2003.

[2] S. Callens and A.A. Zadpoor. "From Flat Sheets to Curved Geometries: Origami and Kirigami Approaches." *Materials Today*, vol. 21, no. 3, 2018, pp. 241–264.

[3] J.H. Conway, H. Burgiel, and C. Goodman-Strauss. *The Symmetries of Things*. AK Peters, 2008.

[4] L.M. Fonseca, G. Rodrigues, and M.A. Savi. "Overview of the Mechanical Description of Origami-Inspired Systems and Structures." *Intl. Journal of Mechanical Sciences*, vol. 223, 2022, pp. 107316.

[5] E. Gjerde. *Origami Tessellations: Awe-inspiring Geometric Designs*. AK Peters, 2009.

[6] M. Hann. *Symbol, Pattern and Symmetry: The Cultural Significance of Structure*. Bloomsbury, 2013.

[7] R.J. Lang. *Twists, Tilings, and Tessellations.* AK Peters/CRC Press, 2017.

[8] L. Lu, S. Leanza, and R.R. Zhao. "Origami with Rotational Symmetry: A Review on Their Mechanics and Design." *Applied Mechanics Reviews*, vol. 75, 2023, pp. 1-27.

[9] K. Miura, T. Kawasaki, T. Tachi, R. Uehara, R.J. Lang, and P. Wang-Iverson. (Eds.). *Origami 6* (Section IV. Mathematics of Origami: Design Algorithms). American Mathematical Society, 2015.

[10] P. Wang-Iverson, R.J. Lang, and M. Yim. (Eds.) *Origami 5*. CRC Press, 2011.