Arts and Magic Square Symmetries

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Abstract

The *arts* have great potential in education. They enhance reasoning abilities, intuition, perception, inventiveness, creativity, problem-solving skills, and expression, especially in primary school. Here we report connections between *arts* and *magic square symmetries,* particularly the unifying structures of 3x3 magic squares. First, we explore the application of different symmetries visualisation in education. Then, we extend the potential of the visualisation of magic square symmetries into the creation of multiplication tables and using different media. It appears that the visualisation using different media and manipulative games is a particularly efficient way for representing magic square symmetries to primary school pupils.

Introduction

The *arts* have great potential in education. They enhance reasoning abilities, intuition, perception, inventiveness, creativity, problem-solving skills, and expression [9], especially in primary school. At that age, pupils' brain is more dynamic and flexible than adults' one for elaborating on the information. Plus, their freedom and curiosity favour exploring all aspects of artistic creativity without inhibition. Indeed, the *arts* are mostly connected with multiple ways of looking at the world. Thanks to such artistic perspectives, children experience infinite possibilities to integrate knowledge and art with the environment and artifacts. To John Hiigli [7], art is an activity lying at the nexus of learning where learners' pathways intersect as mathematics, writings, and drawings.

Teaching pupils to view their surrounding world through artistic and mathematical approaches is a key to improving their math skills. Several studies [11] show a positive association of pupils' art training with their test scores and later academic achievement. The shift in seeing mathematical concepts in an artistic light is critical for the interdisciplinary educational outcome of teaching/learning. Every topic in mathematics is more attractive if it is explored artistically. The wonders in the visualisation with colours, lines and shapes are the key to mathematical beauty and a portal to the understanding of mathematical structure.

As for the potential of mathematics ideas in building bridges with art, we focus on *magic square symmetries*, particularly the unifying structures of 3x3 magic squares. First, we explore the implementation of different visualisations in education to reveal the magic square symmetries more clearly and intuitively. The description of manipulative games involved comes from an ongoing experimentation at a primary school in Montegranaro (Italy). Then, we extend the potential of artistic visualisation of magic square symmetries into the creation of multiplication tables. Finally, we shortly present a work in progress concerning the magic square symmetries' visualisation using a computer-based audiovisual media at the intersection of mathematics and art.

Magic Square Symmetries

A normal magic square is a series of natural numbers arranged in a square matrix whose sum of each row, column, and diagonal is always the same. This number is an arithmetic invariant for the magic square, defined as the magic constant. The number *n* of integers along a side determines the order of magic squares. The third-order magic square is the smallest, not trivial normal magic square. It has 9 cells and a magic constant of 15, as expected by the formula $C_n = n(n^2+1)/2$ with $n=3$, easily explained by the well-known sum of the first n consecutive integers 1 through n^2 divided by the order of the magic square.

6	7	2
1	5	9
8	3	4

8	1	6
3	5	7
4	9	2

4	3	8
9	5	1
2	7	6

2	9	4
7	5	3
6	1	8

2	7	6
9	5	1
4	3	8

8	3	4
1	5	9
6	7	2

6	1	8
7	5	3
2	9	4

4	9	2
3	5	7
8	1	6

Figure 1: *All 8 equivalent magic squares of order 3, related by symmetry.*

Upon closer look at all solutions of 3x3 magic squares, shown in Figure 1, geometric structures, number bonds and symmetries are revealed. There are 8 distinct magic squares obtained through the symmetry operations of the square. The operations are squares' rotations and reflections forming the closed group D_4, the dihedral group of order 4.

Art and Magic Square Symmetries in Education

Exploring symmetries and number bonds shows connections between geometric properties and mathematical patterns. Although the superior aesthetic appeal of magic squares is questionable [5][6], it is undeniable that they have captivated artists and mathematicians since ancient times. Magic squares have appeared in works of art for many centuries and continue to do so [8]. The production of artworks using new and creative visual methods is yet ongoing [4].

We made use of this connection between art and mathematics in the investigation of magic square symmetries in an educational experiment carried out at the primary school level in Montegranaro (Italy). The experiment consisted of 6 activities developed over three weeks to encourage pupils to work, discuss and reflect on the solutions of the magic square in Figure 1. A sample of 101 pupils (8-year-olds) assimilated reflections and rotations associated with this magic square of the third order using visual and manipulative methods [1]. The symmetries are often hidden from learners simply observing the numbers in the grids. Pupils discovered the symmetry more intuitively through drawing artistic magic lines or through using coloured visualisation [4]. The magic line is designed with a straight line from 1 to 9, connecting each integer in a numerical sequence. Considering the pupils' age, we preferred to use coloured designs to identify the 8 equivalent squares. Thus, we allowed the children to reproduce their magic square with colors on transparent paper, shown in Figure 2.

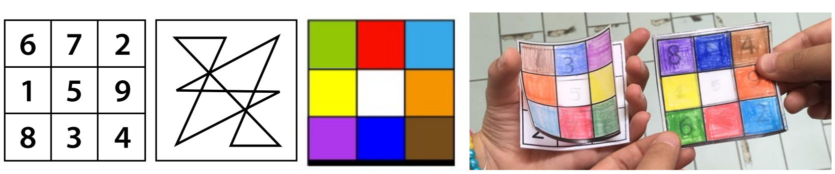

Figure 2: *Creative visualisation of order 3 magic squares.*

Pupils had the opportunity to become familiar with the symmetry groups of the square. They grasped the language of transformations intuitively using the concept of invariance through a joyful and concrete situation. We assessed the effectiveness of the experimentation by analysing the learning changes in the level of pupils' knowledge and skills in performing the symmetry operation using magic squares.

The findings indicate that most children completed the tasks correctly. The flips along the diagonal were more difficult to apply than rotations and reflection in the vertical and horizontal axis. The identity operation was very simple to detect for all pupils. Some students discovered transformation compositions by specifying the odd and even numbers for rotations followed by reflections in different directions. All combinations of rotative and flipping actions can indeed be replaced by a single symmetry operation.

Based on the geometry intuition of pupils, Figure 3 presents two artistic multiplication tables of coloured magic square symmetries. Such tables were not part of our teaching experiment but were designed for a potential use in high school. The manipulation game experiment requires intense spatial reasoning and offers the starting point for reflections and geometric abstractions. Following Skemp's theoretical framework [10], manipulation tasks with visually engaging art have the potential to facilitate the reorganisation of the cognitive processes involved in building abstract and formal knowledge on symmetry operations synthesised by the multiplication table.

The multiplication table, called the *Cayley Table*, sums up all possible results of composing two symmetries operations in sequence. We applied the superposition of the two creative visualisations of the magic square used by pupils in a multiplication table for the dihedral group D_4. The visual representations in Figure 3 show 8 unique magic squares and 2 unique magic line patterns. In addition, the colour pattern reveals the structure of a smaller group within these tables. Indeed, the tables are divided into four quadrants. The top-left and bottom-right quadrants consist of rotations alone, while the remaining two quadrants are pure flips.

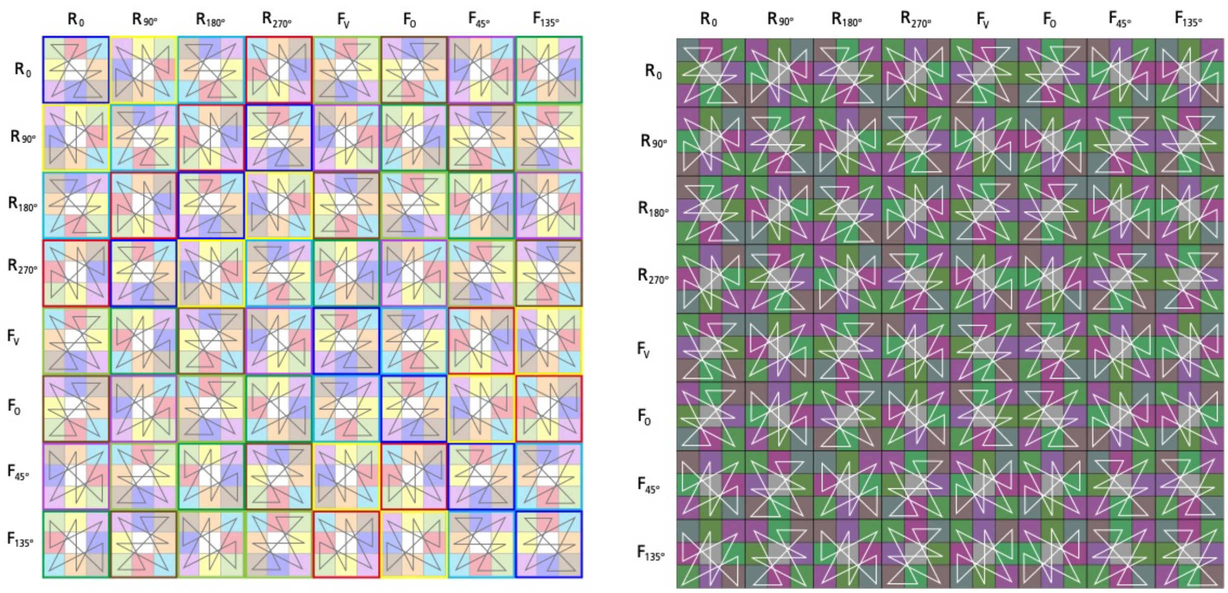

Figure 3: *Multiplication tables of coloured magic square symmetries with magic lines.*

In line with our research project a software is being developed: Magic Square Study 1 (MSS1). In MSS1 a 3x3 magic square is used to investigate audiovisual technologies that could also be applied to magic squares of higher order. In MSS1 a magic constant colour is used to which the colours in the rows, columns, and diagonals of the magic square sum up when mixed. Figure 4 shows the mixing of magic constant colours with magic square colours by rotation of magic square cells in 3-dimensional space. A more detailed description of the calculation of the RGB values of the magic square colours used in MSS1, as shown in Figure 5, will be presented in a future publication.

Figure 4: *Calculation of magic square colours.*

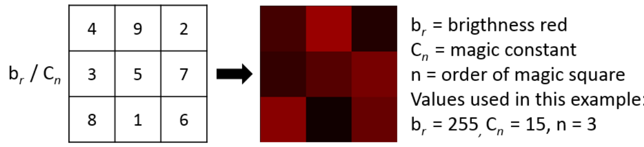

Figure 5: *Example of the calculation of red magic square colours in MSS1.*

Audification of the x- and y-coordinates of the magic line rotated in 3-dimensional space is used to synchronously complement the colour mixing symmetries of the rotating magic squares in the visuals of MSS1. The z-coordinates of the rotating magic line are used for calculations only. Audiovisualisation of the same geometric shape that forms a curve or polygonal chain can be achieved with different types of waveforms [2]. The following approach is used in MSS1: the ratio of audio samples to 3-dimensional Euclidean distances between the vertices is the same for all rotations. Like this, the waveforms retain their fundamental frequencies, only change in timbre, and produce a drone-like sound [3].

Conclusion

We have found that magic squares and their symmetries can be translated into different media, like audio, visual and tangible artifacts. It appears that the visualisation using different media and manipulative games is a particularly efficient way for representing magic square symmetries to primary school pupils. Multiplication tables and audiovisual media can be complementary and support each other like pieces of a puzzle to form a greater picture.

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