Exploring Symmetry Through Portuguese Tiles in Historical Monuments

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Abstract

Portugal has a rich and long cultural heritage which can be used to promote the teaching of Mathematics in schools. Geometry can be explored through the analysis of decorative art found in Portuguese historical buildings, in particular through the analysis of ceramic tile panels and facades used extensively throughout several centuries. Through the analysis of decorative art found using ceramic tiles in two Portuguese monuments, we propose several teaching activities to explore the topics of isometry and symmetry in schools. We give some examples of rosettes, friezes and wallpapers found in the monuments, classified according to their symmetry groups. We also give an example of an applied activity using GeoGebra and exploring Islamic patterns.

Introduction

Portugal was founded in 1139 and many of the national monuments are several centuries old. Due to maintenance or expansion works, performed along the centuries, certain monuments frequently exhibit many different art styles since architects and builders used the style of their own epoch in their interventions. As a result, a Romanesque church, for instance, may exhibit Gothic, Islamic and Romantic elements. Between the 11th and the 15th centuries the Iberian Peninsula was progressively conquered by Christians to defeat the Islamic Moorish Kingdoms who dominated most of the territory [1]. Many Arabic communities remained in the peninsula during the Christian domain, and it is natural to find Islamic influence in several aspects of the Iberian culture (language, art, farming techniques, etc.). In the arts and, particularly, in architecture, the Islamic influence had its greatest prominence in the 15th and 16th centuries, and because it was incorporated into the dominant European styles of the time, it is usually called Mudejar art, thus referring to a reinterpretation of European styles by Islamic culture.

In this paper, we present photos of ceramic tiles found in two Portuguese historical monuments that inspired mathematics activities to be developed by students (grades 7 to 12) [3]. The first monument, Sé Velha de Coimbra (Old Cathedral of Coimbra), whose construction dates back to the foundation of the kingdom in the 12th century, is a Romanesque Catholic cathedral. At the turn of the 15th century, a major decorative campaign was sponsored through which the columns and walls of the aisles were covered with tiles from Seville, bearing multi-colored geometric motifs of Mudejar art [7]. The second monument, Palácio Nacional de Sintra (Sintra National Palace), was a medieval royal residence built over a Moorish castle and was inhabited by the royal family from the early 15th century to the late 19th century. Classified as a National Monument in 1910, it is part of the Cultural Landscape of Sintra, classified by UNESCO as a World Heritage Site since 1995 [4] [8]. The palace blends Gothic, Manueline and Mudejar styles, mostly as the result of building campaigns in the 15th and early 16th centuries. The Manueline style, is a sumptuous, composite Portuguese style of architectural ornamentation of the first decades of the 16th century, incorporating maritime elements and representations of the discoveries brought from the voyages of Vasco da Gama and Pedro Álvares Cabral.

Rosettes, Friezes and Wallpaper Patterns

Symmetry can be used to organize and classify figures in decorative art. A plane figure $\mathcal{F}$ is defined by a set of points on the same plane. A symmetry of $\mathcal{F}$ is any isometry i which leaves $\mathcal{F}$ invariant, $i(\mathcal{F}) = \mathcal{F}$. The set of symmetries of a figure $\mathcal{F}$ together with the operation composition forms a group which is known as the symmetry group of $\mathcal{F}$. In the plane there are only three categories of discrete symmetry groups: rosette groups (with a finite number of symmetries, that can only be rotations or reflections); frieze groups (with translational symmetry in a single direction); and wallpaper groups (with translational symmetry in two directions). There are two types of rosette groups (cyclic groups, C_n, and dihedral groups, D_n), seven types of frieze groups (using the crystallographic notation: p111, p112, p1a1, pmm1, p1m1, pma2, pmm2) and 17 types of wallpaper groups (using the crystallographic notation: p1, p2, pm, pg, pmm, pmg, pgg, cm, cmm, p4, p4m, p4g, p3, p3m1, p31m, p6, p6m) [2] [5]. The classification of friezes and wallpaper patterns can be done using the flowcharts proposed by Washburn and Crowe [6].

Rosettes

Figures 1 and 2 show examples of rosettes found in the tiles of the two monuments mentioned above.

Figure 1: *Cyclic rosettes found in tiles. Left to right: C_1 , C_2 (both from Palácio Nacional de Sintra) C_4 (3x) (from Sé Velha de Coimbra).*

Figure 2: *Dihedral rosettes found in tiles of Palácio Nacional de Sintra. First column: symmetry group D_1 above and D_2 below. Second column: D_4 rosettes. Third column: D8 rosettes.*

Friezes

Figure 3 shows examples of friezes found in the tiles of Palácio Nacional de Sintra.

Wallpapers

Of the 17 existing types of patterns, only six of them were found in the two monuments. Figure 4 shows two examples found in Palácio Nacional de Sintra. The most frequent symmetry group is p4m and Figure **5**: 5 shows two examples.

Figure 3: *Frieze patterns found in tile panels of Palácio Nacional de Sintra (symmetry groups top to bottom: p111, p112, p1m1, pmm2).*

Figure 4: *Wallpaper patterns found in tile panels of Palácio Nacional de Sintra (symmetry group cmm on the left and p2 on the right).*

Figure 5: *Wallpaper patterns (symmetry group p4m) found in tile panels of two Portuguese monuments (Palácio Nacional de Sintra on the left and Sé Velha de Coimbra on the right).*

As a final example we mention a school activity, inspired by the patterns found in the tile panels of Palácio Nacional de Sintra. Students are challenged to construct, given detailed guidelines and using GeoGebra, certain patterns of Islamic influence [3]. Some possible results are given in Figure 6.

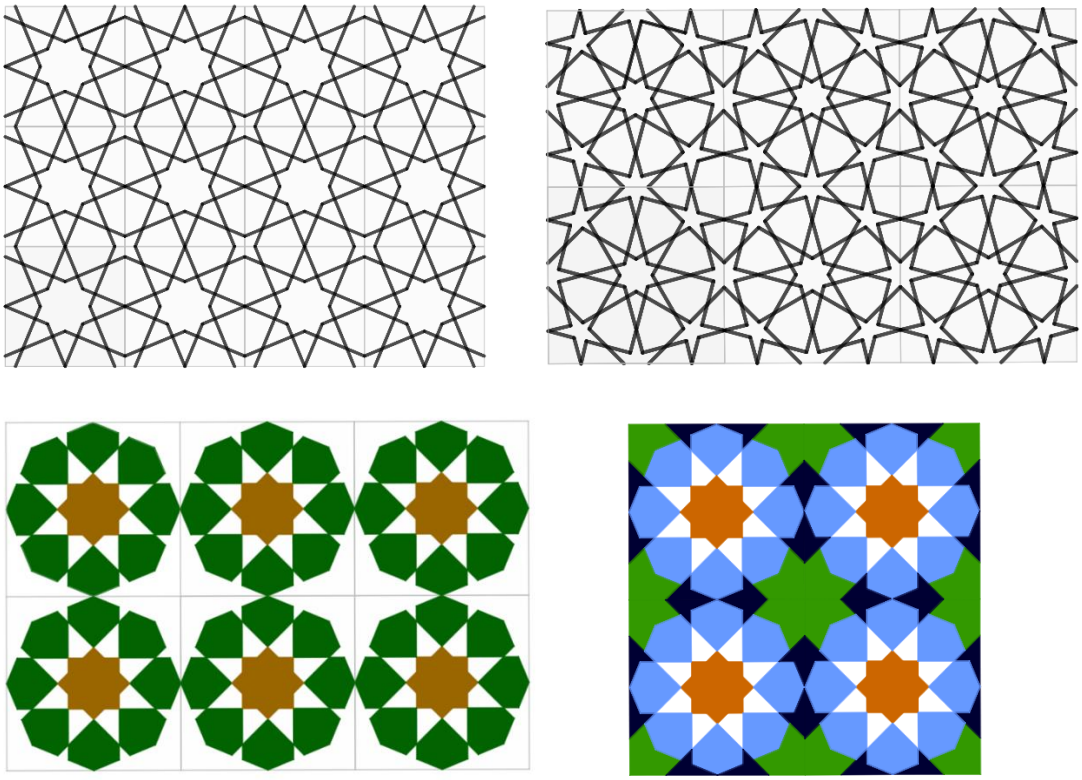

Figure 6: *Results of pattern constructions based on Islamic motives from Palácio Nacional de Sintra.*

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