Genesis of an Interesting Zometool-related Lattice Geometry

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Abstract

The unorthodox Zometool construction described in this paper might be considered either a failure to obey the rules of Zome, or a geometrical artwork in its own right. As is often the case in a creative process, mistakes can lead to scientifically interesting outcomes, provided we analyze them rigorously. For this construction, some magical serendipity, fed by wishful thinking, led to some geometric lattices that are either mistakes or artworks.

From "*Super-short* Red Struts for Reducing the Size of the *Biggie*" to "*Bogus Struts*"

The addition of a downsized strut to the range of Zometool parts occurred at the then largest ever barn-raising event during the Bridges London 2006 conference. To give the building of a 3D shadow of the most most complex 4D convex uniform polytope, the omnitruncated 120/600-cell, a chance to survive the weight of its 14,160 struts/7,200 balls, Paul Hildebrandt advocated the tooling of a new *super-short* red strut [5]. The now *R0* was scaled down by a golden ratio factor φ against the then smallest *short red*, now *R1* (See Figure 1) and allowed the use of struts one size smaller, reducing weight by a factor φ^3. (Remarkably no collapse occurred when the *Biggie* was later built with now old regular parts ! [6])

With the urgency of the enterprise, a special temporary super-short red strut was developed: a simpler cylindrical shape replaced the usual 5-sided prism with a central 180° twisting transition to bring the next pentagonal connecting end stub aligned with corresponding ball hole assuring the Zometool law for all the balls to be aligned to the same direction, as a close look at Figure 5 shows on a model with legal struts. Balls indeed have a polyhedron shape with 12 opposite pentagonal holes turned 180 degrees away from each other, just as happens with the closely related dodecahedron 12 pentagonal faces.

Now, during the first new tool production testing batches in white and red plastics, Paul discovered that the end studs of the new struts were invalid, as not axially 180° apart. He recently attributed the mishap to a simple tool mounting error, see the *bogus strut* (as he calls it) on Figure 1 (see larger figures in the Appendix to the paper [7]).

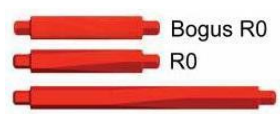

Figure 1: *Struts Compared.*

After the conference Paul asked if some Zometool enthusiasts would like to play with the bogus struts and avoid their destruction. Embracing the idea, I soon got an appetizing set of red and white odd struts !

Early Realization and Recent Thoughts

Model Building with Bogus Struts

After some fiddling, I miraculously managed to use *exactly all* the struts in two nicely symmetric models featuring regular tetrahedral substructures :

(a) large bicolor model, appears as a large tetrahedron (Figures 3–5, &7, down left in red and white)

(b) small unicolor red model, annulus made of 6 alternating up and down elementary tetrahedra identical to bottom left part of the large model in Figure 4 & Figure 7, left part, above left, in red.

The two models were originally presented united by a spare single red strut [7], and were shared in a message on an early Zometool blog. The connecting bogus strut eventually broke [7.3=3rd section of Appendix], so that I later put them on display at home, both just superposed, nicely fitting, but separated. Recent Figure 3 of the large model gives 2 upper right details discussed in the Appendix [7.5, Figure 10].

Mirage of A New vZome Functionality to Virtually Build Models With the Bogus Struts

Recently, thanks to the pandemic, I came across the models and started examining closely the structure, long forgotten, and realized there could be other ways to assemble this inventory of struts into interesting works. I am quite familiar with Scott Vorthmann's powerful vZome simulator [8] for Zometool model development and far beyond, and contemplated that if it could accept the bogus struts' criminal behavior in a new functionality that would allow not one but many ball orientations to exist in space, then each new ball attached at the end of a new strut would be obliged to fit to the end stud, whatever comes out. I expected this completely different geometry would certainly make some connections impossible.

I was unfortunately soon brought back from dreaming to reality as Scott found my suggested changes impossible to implement, as vZome is deeply based on the normal Zometool geometry [7.1.3].

Actual vZome to Build a Mock-up of our Model

Examining again our entangled model, I felt that, without knowing it, my so-called creativity may have been induced in the tentacles of the tetrahedron, an important element of Zometool's icosahedral symmetry, and here the essence of our tetrahedral lattice structure. I realized that two consecutive bogus struts end up being equivalent to a perfectly normal strut, and observed that the balls strangely pointing in two directions in fact generates 2 families of alternating ball orientations, a perfect symmetry that propagates in long lines. When embedded in our lattice, the 6 tetrahedron edges bring-in 6 bundles of parallels of such lines, crisscrossing at the vertices of the regular tetrahedral lattice (among infinite possibilities) now in 4 ball orientations at each tetrahedron (see the 4 balls on Figure 2).

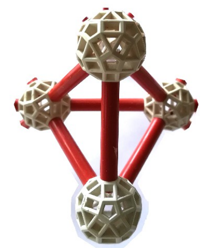

Figure 2: *Ball orientations.*

Clarck Richert and later Jean Baudoin had the idea of green struts, geometrically implemented by our late friend Fabien Vienne [2] to give Zometool access to regular tetrahedra. The actual vZome program allowed me to model exactly my large model lattice geometry, and to mimic more fully their resemblance I painted half of the green struts in red and the other half in white, see Figure 4. Note: there always exist two chiral versions of all models exclusively built with green struts, due to the non-symmetrical kinks near the green struts ends, preventing tetrahedra mirroring about a face.

Actual Zometool Models with Green Struts

With my large inventory of real Zometool short green struts I assembled the sister models, which now stayed fully green, see Figure 5. My regular Zometool white 'green struts' that fit in the fully white struts inventory of all 4 shapes that architects and artists are fond of, that could indeed have given nice green/white models of 'correct greens', showing a relevant opposition with the red/whites of bogus struts, but they aren't presently offered in this small size...

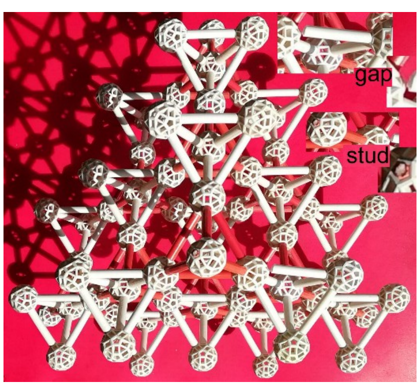

Figure 3: *large bogus model & details as broken gap and stud.*

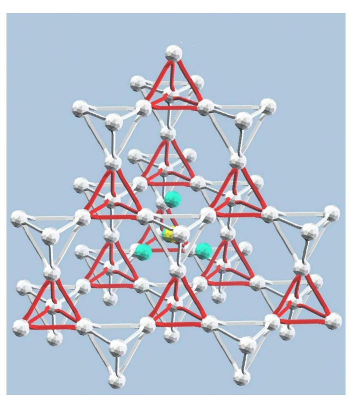

Figure 4: *large vZome model.*

Figure 5: *large Zometool model.*

From Wishful Thinking to Reality

A negative magical feat made me wrongly pave the way to an impossible corollary : "the bogus red struts can, as blue struts (main Zometool stuts), construct regular triangles, and as green struts, also construct regular tetrahedral lattices". An interesting weird reality occurred stemming from a lapse of rigorous attention on my part, despite a hint that could have alerted me : I faced a serious difficulty extracting bogus struts from my model (to show one in Figure 2) that I inadvertently attributed to tight balls in my Zometool part inventory (obtained as end-of-production balls, when the tooling gets too worn-out, yet proving useful for larger models as tightness adds strength, despite harder to assemble...). Indeed I knew perfectly well that red struts can't give a triangle, hence no lattices involving triangles [7.1].

The Bridges community may kindly display forgiveness for reasonable accidents when repaired: in this case it reminded me that adjacent pentagonal red (or green) ball hole directions aren't exactly 60° apart, but close, so when seeing such real triangles, it expects you'd need *...bending to succeed !*

The true angle α, the same as the one of a regular dodecahedron, results from two sources, $\cos \alpha = 1/\sqrt{5}$ as given by a reviewer and namely from Hart and Picciotto in their excellent Zometool reference book [1] as $\alpha = \arctan 2 = 63.4349...°$. Using the rectangular triangle 1, 2, $\sqrt{5}$ I could easily find the two to give the same result without readily finding the source. A long search identified a helpful reference [4] expliciting long trigonometric forms via dodecahedron inradius, dihedral face angle and apothem.

Using Figure 6, let's first play with a red strut sitting between two balls, these will look in a same direction, and we get 5 axial symmetry planes. If we try to insert struts at both sides to form a triangle the adjacent holes will bring the struts ends far away of each other, and only long struts will be flexible enough to get an approximate triangle at the expense of tremendous bending and torsion badly affecting the strut-ball connections, we will never create a tetrahedron ! Instead try the same with bogus struts, the system now has a symmetry plane perpendicular to the middle of the strut and the next pair of struts at both sides sit in a same plane and come together just in front of two pentagonal holes of a 3rd ball at an angle of $180° - 2\alpha$, about 6.9° too large that must be regained by forcing 60° into α with an equal and almost unnoticeable outward bending of the 3 struts in a perfect triangle working similarly in alternate positioned tetrahedra in the entire finite lattice, with apparently little change at the boundaries [7.2].

To get a tetrahedron, 3 struts fully inserted in 3 adjacent pentagonal holes of a ball, will automatically reach pentagonal holes waiting for them on a triangle to get them fully inserted, all 6 struts being equally bent outwards form a tetrahedon, and subsequently each of its 4 vertices receive 4 other tripods forming 4 lines of 3 alternating bulging struts forming together a lattice that can grow in all directions. One can imagine all resulting tetrahedra bulging and forming between them hollow truncated tetrahedral spaces.

Figure 6: *Different views of red & bogus struts before getting triangulized and Large red struts when +/- triangles.*

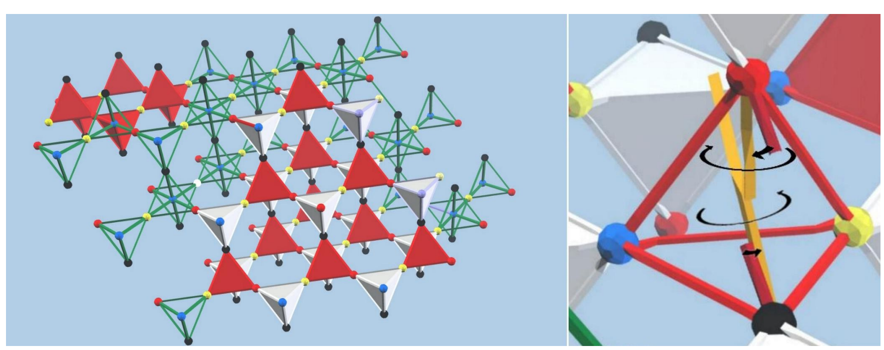

Figure 7: *Paneled models built in green lattice showing four families of balls, and detail of all balls to slightly turn along yellow axes to get best oriented for receiving the bogus struts slightly oriented for receiving the bent bogus struts.*

In the Appendix [7.4] I detail an interesting reasoning shared by Scott Vorthman [9], more involved than my phenomenological reasoning towards triangle, tetrahedron and lattice I just presented above. In Figure 7, I show part of such a regular lattice that Scott constructed with vZome from green struts (in which I embedded my two models) using 4 regularly distributed ball colors, 4 per tetrahedron (so coloring the balls of Figure 2), on 6 line families of alternative color pairs, and the detail sketches said reasoning.

The Appendix then digs into illustrated details which I consider part of the creativity adventures interested participants and the Zometool community may expect, such as why did the strut connecting the 2 models collapse, rediscovering the reasons why there does not exist a dense lattice of tetrahedra and additional scholar reading references.

Ironically, it is thanks to the present paper, that the struts bending of my models that went unnoticed for years is now starting to get almost visible with my newly educated sight.

Conclusion

I hope this paper resonates with the Bridges participants fond of unraveling visual illusions and those active in education, the Zometool and vZome specialists and hopefully the newcomers not yet aware of the tremendous power of of these systems. I believe it proposes some practical interest in the subject matter of geometrical lattices and their implementation in these architectures thanks to the play with our improbable bogus struts. It also stresses a “high scientific rigor” against easy misconceptions...

Dedication and Acknowledgments

With the sad loss of our great friend, stone and wood sculptor Jacques Beck, also a creative Zometool lover, who participated with his wife Françoise also as an artist in Bridges conferences and their late dog Pomme strolling through the aisles of Péch artworks exhibition to the delight of participants, I dedicate this paper to his memory.

Grateful thanks for interesting discussions with my dear supporting friends : Zometool's Paul Hildebrandt who shared the bogus struts; Scott Vorthmann, for using his vZome to help providing an extended proof of what the reviewers discovered; Eric Laysell, also a longtime friend of the Becks, who kindly corrected and enriched once again the English. And obviously the reviewers' team insight and helpful suggestions are deeply appreciated !

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