Invisible Forces: Baskets without Corners

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Abstract

An isolated fragment of basketry is more or less flat. Three-dimensional forms, typically containers, are usually made by changing the number of strands at places in the periodic pattern in order to introduce corners. Three-dimensional baskets without corners, created using hexagonal open weave, are possible within limits determined by internal forces, and models of three of the four elementary topological closed surfaces are described. Such baskets have features in common with both tensegrity structures and Leonardo grids but there are also differences. All three depend on balanced forces, but the ways those forces act are not the same.

Introduction

Baskets have been part of human culture for millennia, created in many different ways using a wide range of materials. Three-dimensional forms are a natural outcome of some techniques, such as those based on coiling, but others, in particular plaiting or weaving, produce flat fabrics unless changes known as corners are introduced at some places in the structure. In his talk at Bridges in 2019 Helmut Pottmann discussed the way in which the polygons in a mesh must be distorted when it is used to build curved surfaces in freeform architecture [8]. However in one case, which corresponds with hexagonal weave (*hex-weave*), he commented that there is no such distortion. This suggested to me the possibility of using it to create baskets without corners.

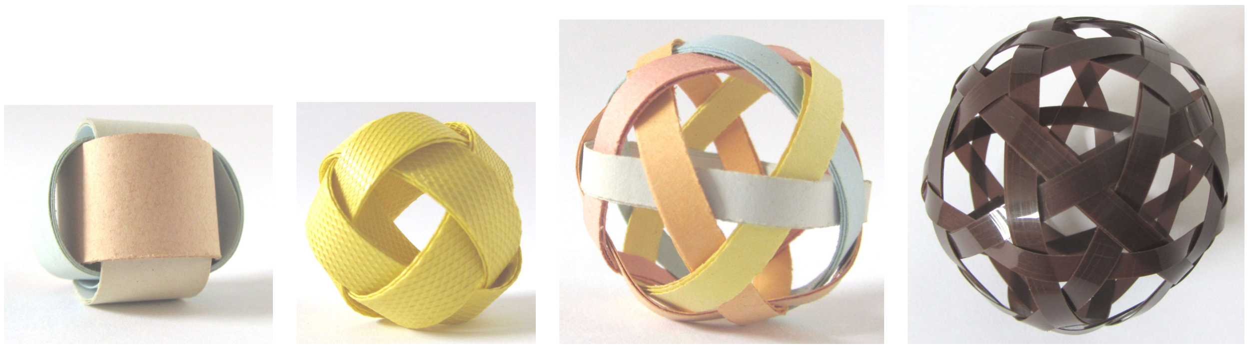
Hex-weave (Figure 2(a)), sometimes known by its Japanese name, *kagome*, is used widely throughout the East. It is quick to make and can be constructed from almost any flat, ribbon-like material. It is an example of a triaxial weave, having strands in three different directions that form alternating triangles and hexagons. Corners are created by reducing the number of strands around a hexagon, generally to five in practical applications, but four and three are also possible. This is equivalent to reducing the number of triangles at vertices in a regular tiling from six to five (four or three) to make a deltahedron. The number could be increased to make saddle shapes, which have negative curvature.

A plaited basket without corners is more or less a flat plane, so it must be constrained in some way if it is to take a three-dimensional form. Support such as a rigid boundary would do the job but another approach is to make surfaces without boundaries. Although more complicated combinations are possible, it is a standard result in the topology of surfaces that there are only four simple cases: sphere, torus, Klein bottle, and projective plane. Only the first three possibilities are considered here. The way the self-intersections interfere in a model of the projective plane without corners gets too complicated.

Details of construction methods will be outlined since they are not standard, and reference will be made to them in the subsequent discussion. Although there should be enough information for any experienced basket-maker who might be interested in using these ideas, this is not intended as a “how to” guide. Apart from the spheres, these structures are not easy to make, and I would not advise beginners to attempt them. The Basketmakers Association publishes a good introduction to hexagonal weave for those who want to learn how to make it [2].

Sphere

Spherical baskets can be constructed as interlocking circles, and symmetry allows four possible arrangements corresponding with the regular polyhedra that have opposed vertices (i.e. not the tetrahedron). The planes of the circles are perpendicular to axes constructed through the opposite vertices (Figure 1).



(a) (b) (c) (d)

Figure 1: *The four spherical baskets with axes that are diagonals of (a) octahedron, (b) cube, (c) icosahedron, (d) dodecahedron.*

The simplest (Figure 1(a)), with three perpendicular axes, is the Borromean rings configuration. In a sense it is the simplest hex-weave basket, having four triangles, and four hexagons reduced to three-strand corners. Hex-weave is Brunnian in the sense that if the strands in any direction are removed, those remaining fall apart like the Borromean rings.

Figure 1(c) shows the only type of closed basket that has been made traditionally. Rattan is used to make balls like this for a game known as *takraw* in Thailand, but which is popular throughout Southeast Asia.

Each of these baskets is closely related to some uniform polyhedra, with the circular strands matching circuits of edges. The octahedron and its faceted form, the tetrahemihexahedron with equatorial squares, relate to (a); the cuboctahedron and its faceted forms, the octahemioctahedron and the cubohemioctahedron with equatorial hexagons, to (b); the icosidodecahedron and its faceted forms, the small icosihemidodecahedron and the small dodecahemidodecahedron with equatorial decagons, and the great icosidodecahedron with its faceted forms, the great icosihemidodecahedron and great dodecahemidodecahedron with equatorial decagrams, are related to (c); and the dodecadodecahedron and its faceted form the great dodecahemicosihedron with equatorial hexagons, to (d).

Rather than having no corners most of these structures really have only corners, three-strand in (a), four-strand in (b), and five-strand in (c). The fourth example has twelve five-strand corners along with twenty standard hexagons that have not been reduced to corners.

Klein Bottle

It is not possible to make a model of the Klein bottle in three dimensions without the surface intersecting itself, but hex-weave already has holes, so self-intersections can be accommodated without any additional gaps in the structure, such as in the ones by Sequin [11]. The more familiar form would present various practical problems, such as getting the holes in the right place, but the figure-8 version is achievable. Unlike standard baskets it is not possible to build the structure in a single continuous process starting from scratch. Instead modifications to an existing piece of fabric are introduced over several stages.

The starting point is a piece of flat hex-weave constructed on longitudinal strands, six in this example (Figure 2(a)). There are many free ends, and diagonal strands can be easily lost as the fabric is manipulated, so it is important to fix them in place by turning over one end and tucking it under a strand. It is better not to turn over both ends since the strands change orientation during construction and the length needed between the turn-overs can change. It also helps to keep everything in place if the weave is made quite tight.

This version of Klein bottle is generated by extruding a figure-8 along a Möbius strip. The figure-8 shape will be made by bending the fabric so that diagonal strands pass through holes. The half-twist requires that the halves of the figure-8 match, so there must be an even number of longitudinal strands. In addition a hole and the crossed pair of diagonal strands that will fit through it must align, so each half of the figure-8 must have an odd number of longitudinal strands. The consequence is that there must be 4n + 2 longitudinal strands.

Flat fabric is easier to bend than a structure with a figure-8 shape, so the next step is to join the longitudinal strands with a half-twist making a Möbius strip (Figure 2(b)). The picture shows a coloured (red) strand that has been put into the woven structure. When there are lots of loose strands it is easy to lose track of them and it helps to leave markers in suitable places.

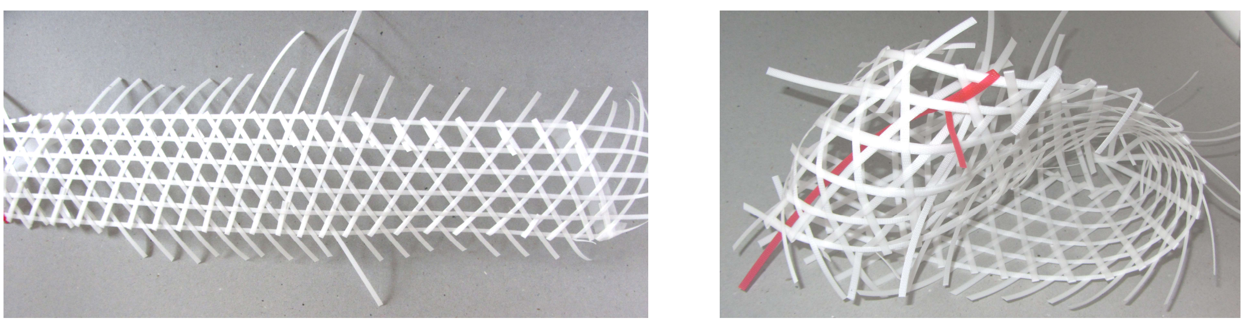


Figure 2: *(a) A piece of hex-weave, (b) joined with a half-twist.*

Rather than trying to bend the fabric into a figure-8 immediately the short diagonal strands are replaced in turn by longer ones, following a sequence of going through a hole and then on to the next short strand to be replaced. This removes the free ends so there is no longer any danger of strands falling out. Correct connections can be made without having to struggle to bend the fabric, but it is surprisingly easy to make mistakes. The usual ways of keeping track, such as knowing which strands go over and which under, no longer work, because the surface is one-sided. In fact the diagonals in both directions might form a single continuous strand, even though this is a triaxial weave. In addition, the half-turn requires the basket to be turned over from time to time, adding to the confusion. It might be necessary to correct mistakes, so it is better not to use very long replacement strands that might need to be taken out. They will be replaced again in any case.

Eventually the structure will be properly connected (Figure 3(a)) and the fabric can be bent by gradually working around it, pulling the now continuous diagonal strand a little at a time.

There is an inherent problem with this structure, because in some places the flat longitudinal strands need to bend in their own planes, which is not possible. The only solution is to replace the longitudinal strands with some material with a circular cross-section. I happened to have some 2mm trimmer line available, and found that it works very well. It is considerably stiffer than the plastic strapping used in the rest of the fabric, and pulls it into a good shape.

I have found that the process of replacing strands tends to relax the fabric, so I carried out the exercise with medium length strands once more before finally replacing them with a single length of fresh strapping (Figure 3(b)).

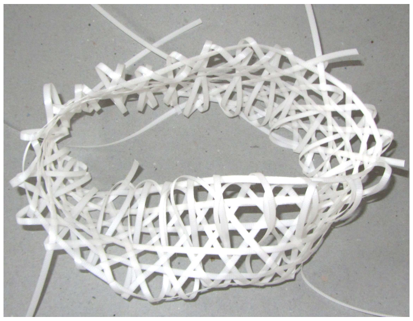
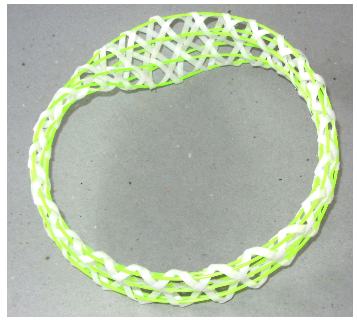



Figure 3: *(a) A properly connected structure. (b) The completed basket.*

Torus

A torus does not intersect itself so hex-weave has no particular advantage, but it can be used. One method of construction is similar to that for the Klein bottle, with longitudinal strands of circular cross-section, but since the surface does not self-intersect a different approach is possible that minimizes the problem of flat strands having to bend in the wrong plane. The idea is to make two cylinders with different radii so one can fit inside the other. The number of diagonal strands does not matter, but the number of transverse strands must match because they will be joined, and the inner cylinder must be woven more tightly than the outer one (Figure 4(a)). In addition the diagonal strands will tend to slide apart until they are joined, so it is important that as many strands as possible are fixed in place.

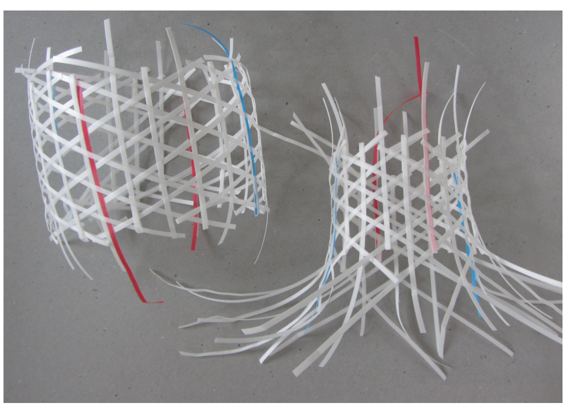
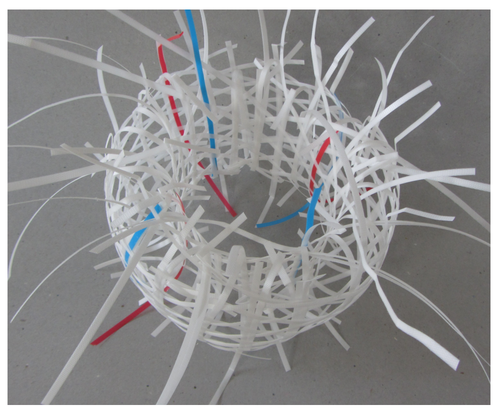



Figure 4: *(a) Two cylinders, one tightly woven, the other loose. (b) One cylinder inside the other with the transverse strands joined.*

The transverse strands could be joined to make rings but an interesting alternative is to replace them with longer strands and displace the joins by one step to form a continuous line winding around the torus (Figure 4(b)).

The next stage is to join the diagonal strands: the free ends first, leaving the turned-over ends until the structure is more stable. Again it is easy to make mistakes, and it is important to check that strands that are adjacent in one cylinder continue to be adjacent in the next. Even then changes might be needed once the circuit nears completion because things might not match up properly at the end. The turned-over ends will be too short for joining so extension strands have to be added in when the other edges of the cylinders are joined (Figure 5(a)).

Tidying up follows the same process of repeated replacement as the Klein bottle but the region at the top and bottom of the torus, where the joins have been made, is much looser than the inside and outside cylinders, so it must gradually be made tighter.

The problem with the direction of bending of the flat strands still exists, although less severely than it would be with longitudinal strands. There is no problem where a pair of strands cross, forming an X shape, but a hexagon (O shape), is widened as the strands are forced into the cylindrical region away from the top and bottom of the surface (Figure 5(b)).

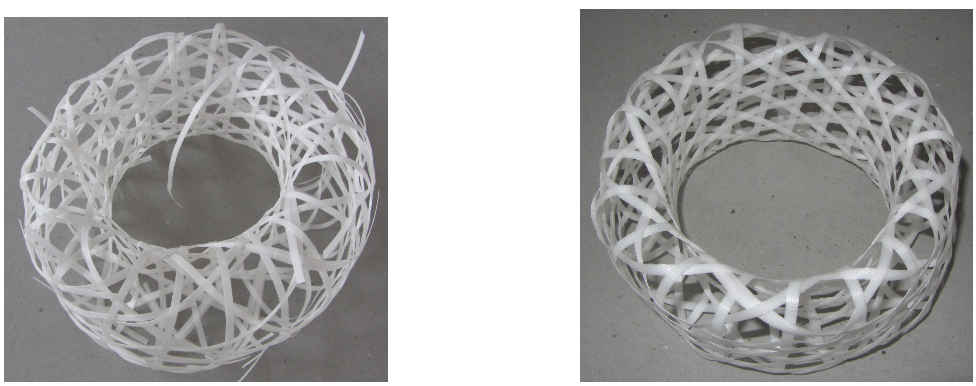


Figure 5: *(a) The diagonal strands joined. (b) The finished torus.*

An alternative method of construction is a modification of the one used to make the Klein bottle. The transverse strands in a cylinder like one of those in Figure 4(a) can be joined with loops, on either the inside (concave) or the outside (convex) of the cylinder, to form either rings or a continuous line that winds around a complete circuit. There are fewer problems if the loops are on the inside and the structure is kept as loose as possible. The free ends of the diagonal strands are plaited through the loops to continue the weave from both edges but, as before, care is needed because there is a tendency for the structure to slide apart. At some point, depending on the size of the loops, the fabric growing from each edge will meet, and can be joined. Further plaiting must maintain consistency with this first meeting.

If fabric has been created fairly uniformly all around the edges the final form looks much the same as Figure 5(b). If, however, most of the new structure is added around the place where the edges first meet, the basket tends to take up a cylindrical shape at that point, and once started it will continue in a straight line along an axis perpendicular to the transverse strands. Eventually the rest of the structure will stop its progress, and force a bend. This generally results in an irregular final form, and control is difficult, but an achievable procedure is to create a lot of new fabric in the opposite region across the torus, forming a second cylinder, parallel to the first. The final form has six cylindrical sections: two pairs in semi-circular bends, like halves of Figure 5(b) and two parallel straight columns. If a single transverse strand winds around the torus it causes the form to twist slightly (Figure 6).

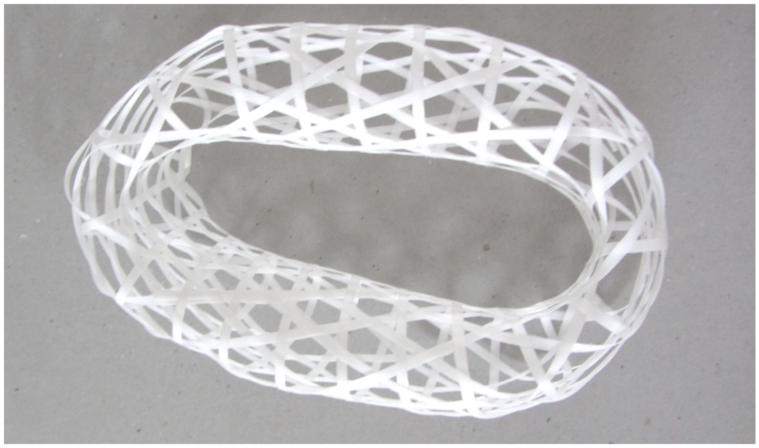


Figure 6: *A toroidal form with six cylindrical sections.*

Forces

Kenneth Snelson has called weaving the mother of tensegrity [12] based on geometric analysis, but the forces in a woven basket are not tension and compression, like those in tensegrity. A much closer parallel is with planar reciprocal frame structures [5], also known as Leonardo grids [10]. John Wallis investigated these structures in the seventeenth century, and one of his diagrams [13] looks like hex-weave (Figure 7). He explains that the lines in his Fig.253 have been shortened to make the structure clear, and they represent planks that meet underneath the one that crosses them in the gap. If they were actually joined to make continuous strips it would be hex-weave, but these grid structures were originally designed in order to use short bars in extended structures, and joining them would defeat that object.

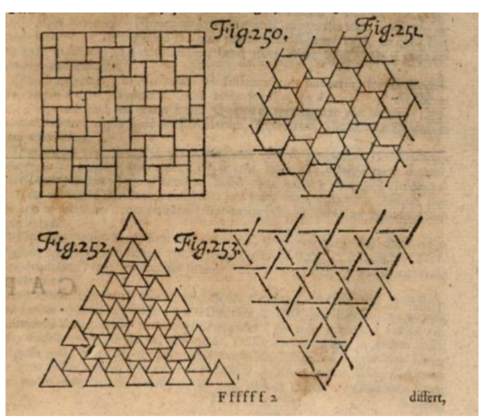


Figure 7: *Diagrams of Leonardo grids by John Wallis.*

There is no tension in flat hex-weave, so if strands were broken alternately at each crossing point to create a Leonardo grid, nothing would happen. Of course any disturbance would disrupt the structure because there would be so little friction holding the pieces in place. Wallis's grids with bars having only three points of contact have a similar problem, so grid structures are usually designed with bars that extend beyond the crossings and have four points of contact.

The difference in flexibility between the materials generally used for baskets and for grids is not fundamental. Rinus Roelofs has used flexible bars [9], and flexible sheets have been used to investigate laminar reciprocal structures [3]. The differences that are significant are that strands are continuous, and that during construction tension can be used to mould the shape, in forming the figure-8 cross section of the Klein bottle, for example. As a consequence changes to the form of a basket are generally irreversible. Removing a single bar from a reciprocal structure causes catastrophic collapse, however removing a single strand from a basket has a negligible effect. Similarly the form of a basket cannot be changed by manipulating single strands, so it is difficult to change once it is established. There is an important exception. If a bent strand is pulled tightly, the length of the bend, and so its radius, is decreased, increasing the force it exerts on its neighbours, including the friction that keeps things in place, so distortions are stabilized. In addition nearby strands are generally displaced to new positions and also get locked in. That is why in the constructions described it is advisable to use gentle tension and make only gradual changes, because once there is any distortion to the form it is virtually impossible to correct it.

Traditional methods of plaited basketry use corners to control the form, so tension can be avoided almost entirely. Recent practitioners have used corners with more than six strands to create surfaces with negative curvature (saddles). For example Richard Ahrens has made tori using mad-weave, which he derives from hex-weave [1], and Alison Grace Martin has created many sculptures that use hex-weave to model surfaces [6, 7]. Tension cannot be avoided during the construction of the baskets described here otherwise only developable surfaces with zero Gaussian curvature would be possible, but it must be kept to a minimum to avoid distortions. Details of the Klein bottle (Figure 8) show that tension is not significant in the finished forms. There are places, best seen along the lower edge of the figure, where straps are not supported, so the shape there must depend only on bending forces. Also the longitudinal strands have been joined for neatness but this had no effect on the form, and they could have been left free.

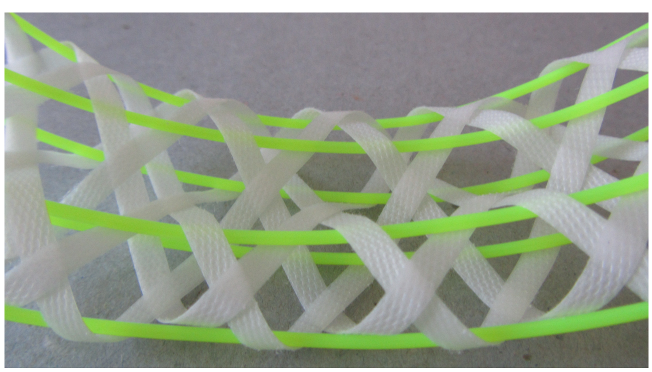


Figure 8: *Detail of Klein bottle (Figure 3(b)).*

Construction is a delicate balance between the tension, needed as strands from different parts of the structure are joined, and the bending forces within the structure that oppose it. The final form is as much determined by those structural forces as by the maker. In particular non-zero curvature, for example in the arches that join pair of cylinders (Figure 5(b)), is resisted, so that although the geometry would allow free-form hex-weave, as Pottmann commented, structural forces severely limit what can be made.

Aesthetics

"Forces made visible" is the subtitle of a book about the sculptures of Kenneth Snelson [4]. It is certainly true that, uniquely, tension and compression are explicit in tensegrity structures, but the visual impact surely follows from the impression that the normal laws of physics have been subverted. Reciprocal frame structures similarly seem to defy nature, as if they pulled themselves up by their own bootstraps, and only a little more effort is needed to understand the forces at work. In contrast, basketry is a commonplace structure made with continuous strands, and it is visually unsurprising. The interplay of internal forces (tension, bending and friction) in the baskets described here is far from obvious, even though it has a more direct influence on form than either of the other techniques that give the sculptor much more freedom. If these baskets are about "the aesthetics of structure" [4, p.30] then they demand a lot from the viewer.

Aesthetic interest might derive from the forms themselves. The figure-8 Klein bottle is not especially well-known, and I set out to model it as a hex-weave basket, which I believe has never been done before. On the other hand the shape of the torus in Figure 6 came as a surprise to me, arising from the interplay of internal forces. Paradoxically it might be more visually effective in a different medium, but pure form would only partially express its nature. Matter is more than appearance, as we all know since we live in a physical world, and the form of these baskets results from the way their matter behaves, even if they are models of abstract closed surfaces. As a result they are more interesting than, say, a 3D-printed circular torus, and their essential character results from processes of construction and internal forces that are invisible.

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