Concept Images of Infinity in Drawings of Pre-Service Teachers

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Abstract

We conducted a descriptive and non-experimental study that aimed to uncover the concept images of infinity as expressed in original drawings of 47 pre-service math teachers (PSTs) for primary school. The drawings were part of an assignment in an online asynchronous course that integrated art in the math curriculum. The findings indicated that about half of PSTs associated infinity with everyday life phenomena. Less than third of them offered a concept image of infinity in a numeric domain and most of these drawings visualized potential infinity. In addition, the study uncovered that almost quarter of PSTs have misconceptions of the concept. We propose that drawing mathematical concepts as has been done in this study, may offer an additional tool to explore PSTs perceptions of abstract concepts in math teacher training.

Introduction

One of the most important concepts in mathematics is infinity, which is challenging both cognitively and philosophically [1]. This is because it includes a contradiction between our existential experience of reality that is perceived as finite and our perception of infinity [2]. In other words, even though we have no experiences with infinity in everyday life, infinity is still one of the core mathematical concepts that elementary school math teachers need to assist their students in understanding. To serve this end, and to create a meaningful educational experience for their students, teachers need to first develop a deeper understanding of it themselves. That means that a teachers concept image of infinity should match its concept definition [3]. In this research, we explored the concept image of infinity as expressed in pre-service math teachers' (PSTs) drawings and artworks, as part of their assignments in an online course that integrated art and mathematics.

Literature Review

The foundation for understanding infinity begins in elementary school, where infinity is "implicitly present in many of the topics, e.g., in arithmetic, when dealing with fractions or in geometry…" [4]. However, understanding the concept of infinity is difficult and it depends on one's ability to visualize it mentally ([1, 5]). According to Tall and Vinner [3], in order to gain deep understanding, the concept definition that a learner possesses must match its 'concept image'–cognitive structures that are associated with the concept including the mental pictures, properties and processes. Concept images may be completely or partially compatible with a concept definition or even contradict it.

In the literature, a common distinction in the numeric domain can be found between potential and actual infinity: (1) *potential infinity* is a process that lasts forever, such as counting natural numbers; (2) *actual infinity* is related both to cardinal and to ordinal aspects of infinite sets. "*In a context where number means a comparison of size of sets, cardinal number gives a theoretical extension to the counting concept*" [2]. That is, infinity is conceptualized as a realized "thing"–an object, like a size of the set of all-natural numbers. Thus, actual infinity is counterintuitive and difficult to understand, whereas potential infinity is easier to grasp [4].

Understanding the concept of infinity among PSTs and in-service math teachers has been the focus of several studies. These revealed that some of them hold partial knowledge that may be expressed by quoting

rules and providing ambiguous explanations of these rules; those who comprehend infinity as a continuous and endless process, confront difficulties and hold misconceptions (e.g., [6]). Although some studies (e.g., [7]) found that the learning strategy called "learner-generated drawings" is effective for learning science concepts, no study examined how PSTs and in-service teachers "visualize infinity" [5] in drawings or artworks. To answer this gap in the literature, we designed a study that addressed the question: "What perceptions of infinity can be uncovered in PSTs' original drawings of the concept?"

Method

The research was conducted in an online asynchronous course for PSTs in their second-year teacher training, over one semester (14 weeks) of integrating fine arts with math curriculum. To uncover the PST's perceptions of the concept of infinity, we designed an assignment in the course were we asked each PST to create and post a drawing or an artwork that illustrated the mathematical meaning of the concept together with a short explanation to a collaborative Padlet gallery. Their 47 original drawings and their short explanations served as research data.

This study focused initially on PSTs who will teach math in elementary school and will have an influence on the formation of mathematical concepts of children. Therefore, it is important to explore their perceptions and knowledge of infinity, as we did in this study.

Since we were unable to find a theory that identifies concept images of infinity that PSTs possess, this study was exploratory in nature and used a descriptive and non-experimental design [8]. This method allowed us to explore the PSTs' perceptions of the concept of infinity.

We analyzed the data in three steps according to Imdahl's method [9] and qualitative content analysis [8]. First, we systematically described all structural details of the drawings. Second, we derived the contextual knowledge of the drawing from names and explanations that were stated by the PSTs. These two steps mirrored the PSTs perceptions of infinity. Third, we compared PSTs' perceptions of infinity to the known mathematical meaning of the concept.

Results

The content analysis unfolded four categories from the 47 drawings: correct concept images in a numerical domain, incorrect mathematical content images in a numerical domain, concept images with correct non-mathematical content like natural phenomena, emotions, senses and philosophical aspects and concept images with incorrect non-mathematical content.

Correct mathematical content (13 out of 47)

From thirteen drawings, nine expressed potential infinity such as: a set of nonnegative integers; a series; a reflection between two mirrors and an endless road from a starting point (zero) to infinity (Figure 1a). This result agrees with the research literature that indicates this notion of infinity as a stage towards fully understanding this concept–the actual infinity (e.g., [1]). Four drawings could be considered as the visualization of actual infinity. One demonstrated the whole number set within the infinity symbol (Figure 1e)–this can represent the Real Numbers set as an object and not as a process. Additional three drawings expressed infinity as the immeasurable distance between zero and infinity.

Incorrect mathematical content (4 out of 47)

From four drawings in this category, three expressed potential infinity incorrectly: as drawing in the snow (Figure 1b) and as pieces of information. These drawings do not represent infinity because they do not refer to any timeframe–if the timeframe is finite in both cases, different drawings in the snow or pieces of information can be elements of a finite set. However, if the timeframe is infinite, both sets should have been infinite. One drawing (Figure 1f) identified infinity as a finite set–several people. This observation is in line with previous findings that recognized PST's difficulty in understanding big finite sets (e.g., [1]).

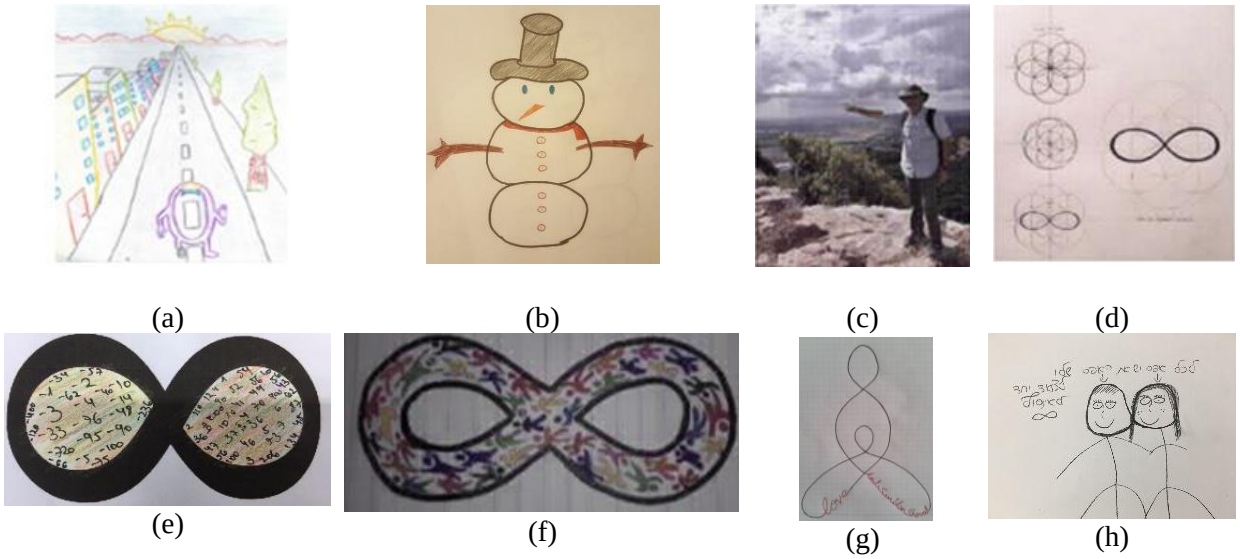

Figure 1: *Pre-service teachers' original drawings and artworks: (a) zero on its way to infinity, (b) drawing in the snow, (c) infinitely as far horizon, (d) how many infinities can we produce from zeros?, (e) infinitely many numbers), (f) infinity as many kinds of people, cultures, and connections between them, (g) infinite mother's love, zero questions, (h) every zero have a zero to walk with.*

Correct non-mathematical content (22 out of 47)

Twenty-two drawings attempt to connect infinity with everyday experiences like the universe, horizon (Figure 1c), thoughts, sight, love (Figure 1g). This may suggest that in order to overcome the complexity of the concept, the PSTs turned to metaphors. According to Lakoff and Johnson [10], metaphors help us understand abstract concepts through verbal associations to an everyday life-known phenomenon. These concept images of infinity may represent an expression of potential infinity as was found in previous studies (e.g., [12]).

Incorrect non-mathematical content (8 out of 47)

Eight drawings represented a visual connection between the symbol of infinity and zero, numerical symbol for 8 and a composition of two zeros (Figure 1b & 1h). These concept images may represent the PST's misunderstanding the concept itself or of the task requirements or their difficulty to express their knowledge artistically.

Summary and Conclusions

Usually, PSTs' conceptions about infinity are triggered when they are confronted with mathematical tasks depending on the context of the representation in which the task is presented [11]. In our study, the PSTs were asked to express their interpretation of the mathematical concept of infinity, however, they were not guided to interpret or define infinity in any specific mathematical context. Therefore, the study seems to uncover the PSTs' true perceptions by drawing the concept images of infinity, which seems to be mostly metaphors of infinity in everyday life or potential infinity represented by infinite sets or series in the numeric domain.

All the drawings that represented the mathematical notion of infinity were in the numeric domain, correct or incorrect, and there was not even one drawing which represented infinity in the geometrical domain. The PSTs who participated in the research, studied representations of infinity in geometry that are very easy to draw, such as: a line that is infinitely long and thin or a line that contains infinitely many points or an infinite number of lines that pass through a single point. Also missing was the interpretation of infinity

as a very small or limiting quantity, like the area of the Sierpinski triangle, which PSTs calculated in one of the course assignments. These two observations agree with Sacristán claim [12] that traditionally students are presented with algebraic-symbolic perspective of infinity and PSTs formal math teaching does not change their conceptions of infinity. We suggest that an effort should be made during teacher training program to represent infinity as a multifaceted concept to make it easier to link formal and intuitive knowledge and to connect infinity with everyday life.

The study results suggest that drawing mathematical concepts, regardless of a specific domain, is an effective tool for exposing students' true perceptions and knowledge of the concept. This tool is non-threatening as an assessment or a test and can help teachers to explore students' knowledge of complex concepts. In the case of PSTs, by diagnosing the knowledge of second year students, we may adapt the curriculum accordingly and therefore help them construct it properly.

The study results extend our knowledge about PSTs perceptions of infinity and the visualization of it. The method used in the study, analyzing original drawings that describe a mathematical concept, holds a great potential to uncover PST's true perceptions of abstract mathematical concepts and alongside their misconceptions. However, this research is only a first step and more extended research is needed.

References

[1] V. M. Kolar and T. H. Čadež. "Analysis of Factors Influencing the Understanding of the Concept of Infinity." *Educational Studies in Mathematics*, vol. 80, no. 3, 2012, pp. 389–412.

[2] D. Tall. "Intuitions of Infinity." *Mathematics in School*, vol. 10, no. 3, 1981, pp. 30–33.

[3] D. Tall and S. Vinner. "Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity." *Educational Studies in Mathematics*, vol. 12, no. 2, 1981, pp. 151–169.

[4] M. S. Hannula, E. Pehkonen, H. Maijala and R. Soro. "Levels of Students' Understanding on Infinity." *Teaching Mathematics and Computer Science*, vol. 4, no. 2, 2006, pp. 317–337.

[5] A. Arcavi. "The Role of Visual Representations in the Learning of Mathematics." *Educational Studies in Mathematics*, vol. 52, no. 3, 2003, pp. 215–241.

[6] V. M. Kolar and T. H.Čadež. "Analysis of Factors Influencing the Understanding of the Concept of Infinity." *Educational Studies in Mathematics*, vol. 80, no. 3, 2009, pp. 389-412.

[7] P. Van Meter, M. Aleksic, A. Schwartz and J. Garner. "Learner–Generated Drawing as a Strategy for Learning from Content Area Text." *Contemporary Educational Psychology*, vol. 31, no. 2, 2006, pp. 142–166.

[8] A. Giorgi. "The Descriptive Phenomenological Psychological Method." *Journal of Phenomenological Psychology*, vol. 43, no. 1, pp. 2012, 3–12.

[9] G. B. Christmann. "The Power of Photographs of Buildings in the Dresden Urban Discourse. Towards a Visual Discourse Analysis." *Forum: Qualitative Social Research*, vol. 9, no. 3, Art. 11, 2008.

[10] G. Lakoff and M. Johnson. *Metaphors We Live by*. Chicago: University of Chicago Press, 2003.

[11] L. Medina Ibarra, A. Romo–Vázquez and M. Sánchez Aguilar. "Using the Work of Jorge Luis Borges to Identify and Confront Students' Misconceptions about Infinity." *Journal of Mathematics and the Arts*, vol. 13, no. 1–2, 2019, pp. 48–59.

[12] A. I. Sacristán. "Students' Shifting Conceptions of the Infinite through Computer Explorations of Fractals and Other Visual Models." *Proceedings of the 25th International Conference for the Psychology of Mathematics Education*, vol. 4, 2001, pp. 129–136.