A Trio of Beaded Surfaces

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Abstract

This article presents several items of jewelry which feature mathematical surfaces rendered with beads and wire.

Introduction

In recent years mathematical enthusiasts have been using 3D printing, ceramics, knitting, origami and other means to render mathematical surfaces. Regardless of the initial motivation, the physical objects that come out of this work often lead to a deeper level of understanding of the mathematical objects for both the creator and the viewer. In this paper we present some mathematical surfaces we rendered out of beads and wire for the purpose of jewelry making. Our process starts with thick (14- or 16-gauge) silver wire bent into the shape of the boundary of a surface. (Figure 1 illustrates the boundary of our first example -- the saddle surface.) We then model the interior of the surface using very thin (26-gauge copper-coated) wire, threaded with seed beads. Our modeling approach varies, depending on the structure of the surface at hand and the qualities of the surface we wish to highlight. In each case, the threaded wire generates the surface by traversing strategically-chosen pathways on the surface. Our technique is different from that of bead weaving, which has also been used to render surfaces (see for example [1,2]).

Figure 1: *Saddle Boundary.*

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Hyperbolic Paraboloid Earrings: After bending the boundary wire into the shape of the saddle's boundary (where we restrict the saddle surface to a circular domain), we hammered the wire to achieve a nice, finished texture and fused the ends together. (Again, see Figure 1.) This closed loop then served as the "frame" for our surface. In this particular example, we chose to render the surface using its contours. So the threaded wire in this case traversed the surface by following the contours of the saddle, which consist of hyperbolas that open in one direction at points with height greater than the height of the saddle point and in an orthogonal direction at points with height lower than the height of the saddle point. Of course, the contour through the saddle point consists of two intersecting lines, and we highlighted these lines by using larger-sized beads. We attached the beaded wire to the boundary frame by wrapping the ends of the wire around the frame several times, and we pulled these ends tight with pliers. Finally, we employed a color gradient (moving from red violet to blue, cyan, then yellow) to represent the heights of the contours on the surface.

Figure 2: *Hyperbolic Paraboloids.*

Figure 3: *Boundary of the ruled helicoid surface.*

Figure 4: *The beaded half helicoid.*

Helicoid Pendant: Next we created a half-helicoid, which is a ruled surface generated by rotating a line segment around a fixed axis at a constant angular rate. We started with a wire boundary loop consisting of a helical spiral with a straight segment down the center. (See Figure 3.) We threaded beads along eye pins that served as the rulings of the surface. By varying the size of the beads on each pin, with the smallest beads towards the center, the beaded segments filled out the surface well. We chose a color scheme to provide maximum contrast near the boundary (see Figure 4).

Soap Film Pendant: Our third surface, shown in Figure 5, started with a boundary wire that is "geometrically knotted" but not closed (such curves were shown in [3] to bound soap films). We constructed the beaded surface to approximate the least area surface filling the wire (as in the case of a soap film). We started by threading silver beads along the segment where the surface intersects itself – as we observe in soap films, the three sheets meet at a 120° angle. We then filled in with colored beads in several sets of curved arcs inspired by the patterns observed in actual soap films. We chose colors to gradually shift from dark to light so that at the intersection the three sheets are all different shades from each other.

Figure 5: *Soap film surface; the seven silver beads indicate where the film intersects itself.*

Next Steps

The surfaces featured here are just our first attempts at rendering surfaces with this bead and wire technique. While we tried to capture the salient features of the three surfaces, mathematical accuracy was not our primary goal. We view these initial steps as a proof of concept, and we have learned much about the process. In future work we plan to re-examine both the boundaries of our surfaces and the paths that our threaded wires traverse with an eye toward mathematical accuracy and aesthetics. Perhaps we can render various parameterizations of surfaces with the beaded wire. Future directions include creating different surfaces (e.g., soap film surfaces, ruled surfaces, and other surfaces) and sequences of surfaces depicting transformations.

References

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