Quadrilateral Lace

Susan Happersett

Beacon, NY, USA; fibonaccisusan@icloud.com

Abstract

This paper will describe the exact procedures I use to draw algorithmically generated mapping patterns in the framework of quadrilaterals. I began with a square format, then rhomboid and trapezoidal. Layering concentric self-similar shapes creates the illusion of 3-dimensional space. Shifting the self-similar shapes out of the concentric order creates the sense of movement across the plane.

The development of my lace drawings has been a multi-year process. By a lace pattern, or point-to-point mapping pattern, I mean carefully laying out a collection of points in the plane according to a pattern, and then joining certain pairs of points with line segments, where the pairs to be joined are specified by a definite procedure. The first lace drawings I algorithmically generated were restricted to points on the Cartesian Coordinate system: the *x*-axis, the *y*-axis and the lines $y = x$ and $y = -x$ [1]. Expanding on the types of point-to-point mappings I could produce, I decided to use the perimeters of quadrilaterals to assign my points. I began by working with a square with 7 equally distant points on each side. I picked an odd number of points for aesthetic reasons. Four sides on the square, 7 points per side, produces 24 points in total. I wanted a point on the center of each side. Once the parts were defined, I needed to write a rule to define the mapping process to be carried out for each point. I decided each point should be connected to the point 8 points (the number of points per side plus one) away in the clockwise direction. This mapping was completed for all 24 points as in Figure 1. The next quadrilateral I used for a structure was a rhombus – Figure 2 – with 120-degree obtuse angles and 60-degree acute angles. Again, I used 7 equally distance points on each side. The third quadrilateral I explored was a trapezoid with three congruent sides (I call it a "Trisosceles" trapezoid), Figure 3. The fourth side measures half the length of the other three sides. Again 7 points on each side, but on the shorter side the points are half the distance apart than on the longer sides.

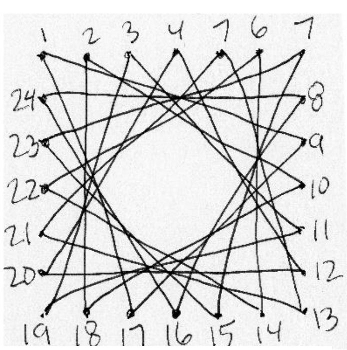

Figure 1: *N=7 Square*

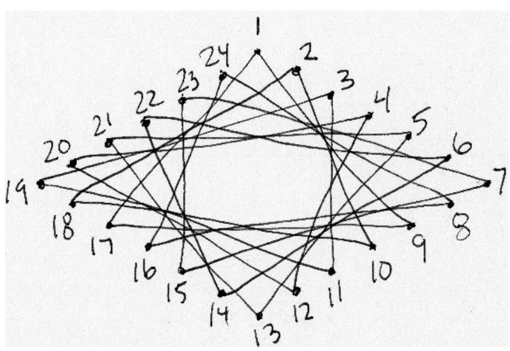

Figure 2: *N=7 Rhombus*

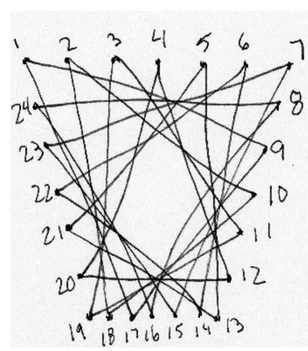

Figure 3: *N=7 Trapezoid*

To change the number of points on each side of the quadrilaterals from 7 to 5, 9, 11, ... I needed to create some general rules; *N*=the number of points on each side, *T*=the total of points, then $T=4(N-1)$. For the mappings, point *x* maps to point *y* but *y* must be (less than or equal to) *T*. In formulas:

$$(1)\ \text{if } x \leq 3N-5 \text{ then } y=x+(N+1)$$

$$(2)\ \text{if } x>3N-5 \text{ then } y=x+(N+1)-T$$

Using these formulas, I can draw lace patterns of different sizes, but of the same shape with the same distance between points. This layering technique creates some interesting perspectives in the drawings.

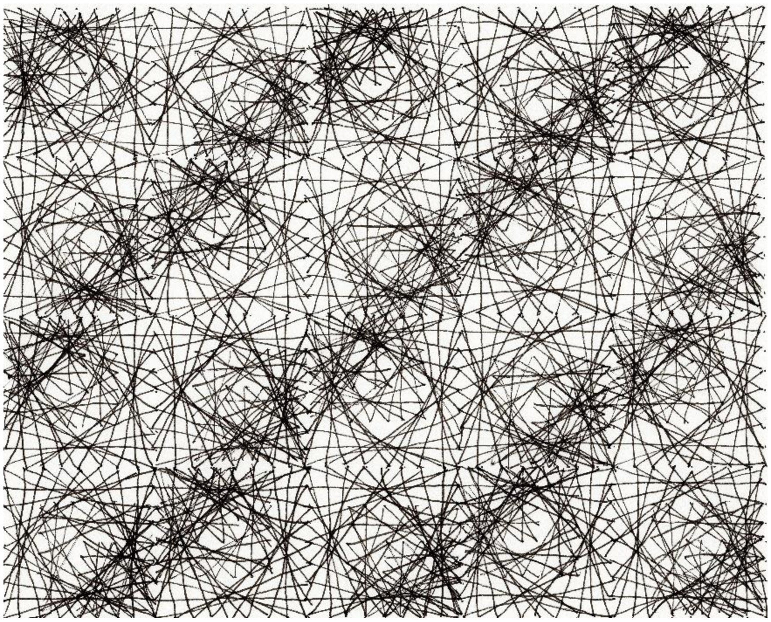

Figure 4: *Spiraling Squares, Ink on Paper, 2020*

"Spiraling Squares" (Figure 4) uses a grid of squares with 9 points on each side. Within each is an N=7 square and an N=5 square. The placement of these additional squares is alternatingly lining up with the perimeter points on the top left corner and the bottom right corner. This creates the sense of a spiraling motion.

"Cubic Lace One" (Figure 5) consists of an arrangement of three N=9 rhombi each with a N=7 and a N=5 rhombus positioned in its center. The altered perspective of this drawing offers a 2-D rendering of a cubes within a cube.

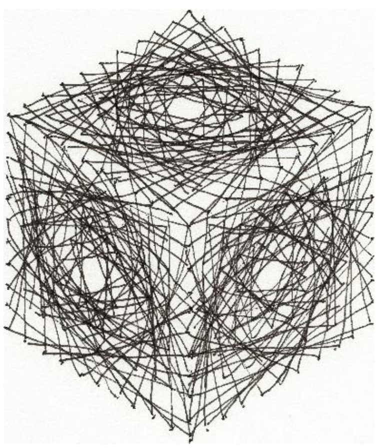

Figure 5: *Cubic Lace One, Ink on Paper, 2019*

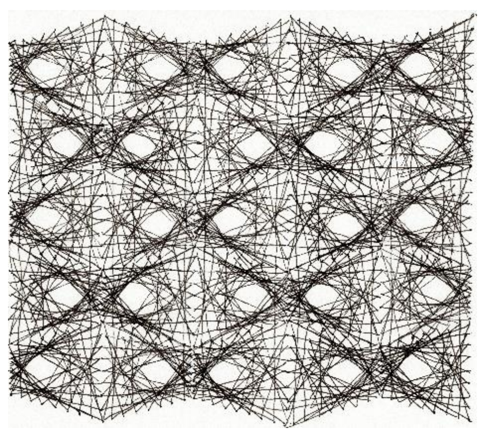

Figure 6: *Trapezoid See-Saw, Ink on paper, 2020*

"Trapezoid See-Saw" (Figure 6) is a tiling of N=9 trisosceles trapezoids with N=7 trisosceles trapezoids situated so that the points on the smaller side of the N=*7* trapezoids are centered on the points on the smaller side of the N=9 trapezoids. My exploration into point-to-point mapping patterns has allowed me to explore numerous possibilities in altering the perception of space and movement while still adhering to the restrictions of hand drawing on a flat piece of paper.

[1] S. Happersett. "Cartesian Lace Drawings." Bridges Conference Proceedings, Waterloo, Canada, Jul. 27–31, 2017, pp. 391–394. http://archive.bridgesmathart.org/2017/bridges2017-391.html