Geodesic Cities

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Abstract

This paper explores a mental experiment: what if we take an existing city grid and straighten all the roads, and make all the intersections at right angles allowing the roads to pop out of their Euclidean embedding. We perform some experiments on simplified common city grid patterns, we make some forays into a physics based computer simulation, and we show and discuss a sculpture of the author's home town road network.

Introduction

If I were to give directions to my brother's house, I might say something like, "on Bridge Street going north, turn left onto Northfield, then right onto Bearinger and then right onto Laurelwood," or I might draw a map like Figure 1a. In reality, the route would actually look like Figure 1b, because the city of Waterloo, Ontario, Canada is not very grid like. The streets are not straight, they are not well-aligned with each other, nor do they always meet at 90° angles.

(a) *A mental model*

(b) *A geographically accurate map. Map data ©2020 Google.*

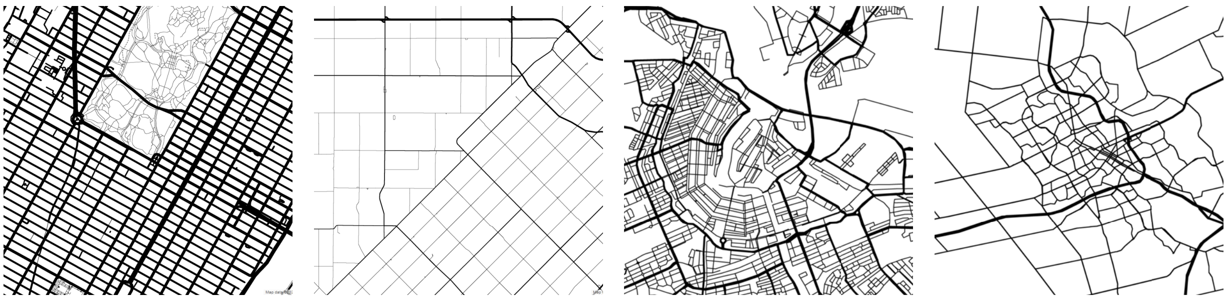
Figure 1: *The way to my brother's house.*

In a TEDxDublin talk *Making Sense of Maps* [7], Aris Venetikidis talks about cognitive maps — a simplified mental model of a city where roads are straight and intersections are at 90° angles. Could I realize such a mental model as a physical sculpture? In mathematical terms, could there be a map on a (not-necessarily-Euclidean) surface where roads are traced out by a geodesic curve? Or put even more directly, what shape will I get if I reconstruct my city's roads with straight paper strips attached at right angles? This paper explores this idea using physical paper-strip constructions of several street-grids.

The disciplines of fine art and map-making frequently borrow from each other. For instance, Mondrian's *Broadway Boogie Woogie* renders the Manhattan city grid in his own unique style [4]. More recently, Tarbell's generative art project *Substrate* has complex city-like grids emerging from simple algorithms [6]. This paper culminates with a description of the author's own full-city sculpture *Geodesic City*.

City Grids

Aris Venetikidis' description [7] of a grid-like mental model closely matches my own cognitive map. Of course this grid model works famously well for cities such as New York (Fig.2a). For the town where I grew up — Strathroy, Ontario — there are only a few weird intersections to think about where two different grid systems meet (Fig.2b). But the grid model has more difficulty with the concentric layout of Amsterdam in the Netherlands (Fig.2c). And from personal experience, my mental model regularly fails me when navigating my own home town of Kitchener-Waterloo (K-W), Ontario, Canada (Fig.2d). In the time I have lived there, I still have not gotten used to the decidedly ungrid-like triangles, bends, and "parallel" streets which cross each other up to three times.



(a) *Manhattan* **(b)** *Strathroy* **(c)** *Amsterdam* **(d)** *Kitchener-Waterloo*

Figure 2: *Four styles of street layouts. Map data ©2020 Google.*

Paper Strips

One medium for creating surfaces from its geodesic curves is with paper strip constructions. Because paper is a developable surface and is straight, a sufficiently thin paper strip glued along a surface will follow a geodesic curve. And any surface tangent to a set of paper strips will have those paper strips as geodesic curves. To illustrate, we can create an equilateral triangle, a square, and a pentagon with paper strips that meet at 90°. The resulting forms suggest a sphere, a plane, and a hyperbolic surface respectively with the paper strips indicating geodesic curves (see Figure 3) .

Figure 3: *Right angle triangle, quadrilateral, and pentagon made with paper-strips.*

I encourage the reader to try their own experiments. I have found that I get consistently even and straight strips of paper by shredding the heaviest card-stock that my paper shredder can handle and attaching them with small rectangles of invisible tape.

Sculpting with paper strips is not a novel technique. Alison Grace Martin is a designer and artist who has made many beautiful sculptures of mathematical surfaces [2]. A similar project by Matsumoto et al. [3] recreates target surfaces with paper strips by ascertaining developable patches and adding curvature to the seams. Harris [1] discusses the direct mathematical relationship between the Gauss-Bonnet theorem and the sculptural possibilities of paper strips.

One key difference in this paper is that rather than designing a paper-strip sculpture to recreate a target surface, we want to look at paper-strip configurations from "naturally" occurring networks in the real world to see what surfaces emerge.

Figure 4 shows the results of three experiments which correspond to idealized versions of Manhattan, Amsterdam, and Strathroy. As expected, the Manhattan grid remained flat. We see a bend emerge from the Strathroy grid, and the concentric Amsterdam grid resulted in something very close to a cone.

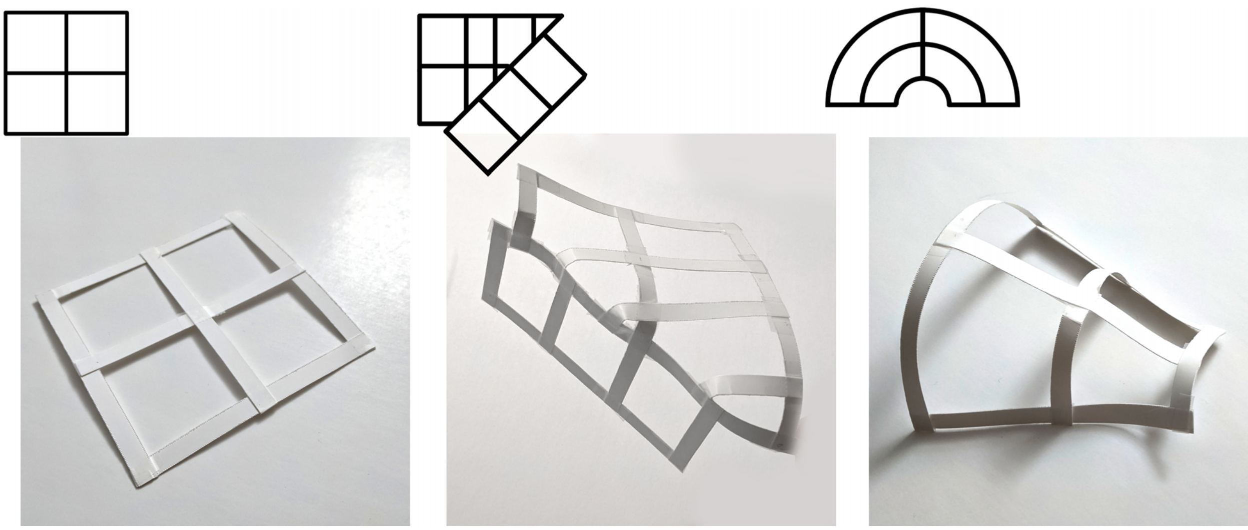


Figure 4: *Paper-strip experiments on idealized city grids of Manhattan, Strathroy, and Amsterdam.*

Geodesic City

For Kitchener-Waterloo, I wanted to be true to my own cognitive model, so I chose sixteen streets that are important to my own navigation experiences. I used online mapping software to determine the distances between intersections and cut the paper strips to length. After attaching each intersection, I had a form that was about 1m long in the longest dimension. I was pleased with the organic and brain-like shape that emerged. I spray painted the roads black and added yellow lane markers using a metallic gold marker with a chisel tip. See Figure 5.

The final result, named *Geodesic City*, was shown on June 20, 2019 at the Art Meets Tech event, part of the True North festival in Kitchener. The sculpture is my commentary on the disorderly street plan of my city. It calls out the differences we see between the design of an object and the form it takes in users' minds. If pressed, I may admit this whole thing is my excuse for why I still get lost after living here for 28 years.

Future Work

The mathematics of the projects in this paper are entirely accomplished with manipulatives. However, I have begun work on a physics based simulation of these paper strip sculptures. Some early results are shown in Figure 6. Future work could include an explanation of the physics involved and a link to the source-code. My

Figure 5: *Geodesic City.*

long term goal is to input city grids of the world from available online datasets such as OpenStreetMap [5] and create virtual Geodesic Cities on demand.

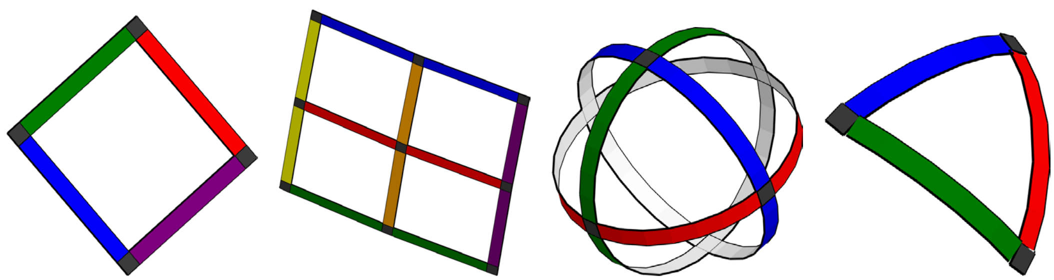


Figure 6: *Early results of physics based computer simulations.*

References

[1] Edmund Harris. “Gauss Bonnet Sculpting.”, *Bridges Conference Proceedings*, 2020. http://archive.bridgesmathart.org/2020.

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[4] Piet Mondrian. *Broadway Boogie Woogie*. https://www.moma.org/collection/works/78682.

[5] Open Street Map https://www.openstreetmap.org/.

[6] Jared Tarbell. *Substrate*. http://www.complexification.net/gallery/machines/substrate/.

[7] Aris Venetikidis. *Making Sense of Maps*. https://www.ted.com/talks/aris_venetikidis_making_sense_of_maps.