Adapter Tiles Evolves the Girih Tile Set

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Abstract

This document presents an evolved Girih tile set beyond the five Girih Tiles popularized by Lu and Steinhardt. Additional tiles have been presented in multiple papers before, but here, a wider set is officially defined. I refer to this as the evolved tile set or evolved Girih tile set. It includes the adapter tiles, which also have been presented before, but not defined as a group or part of a tile set. Adapter tiles differ from the five Girih Tiles as the latter only have equilateral sides. This paper presents one category of adapters containing tiles having at least one side with the length of Φ. The aim is to establish non-equilateral sides as a necessary concept to enable tiling of a greater number of existing Islamic geometric patterns. This opens up for future introductions of other categories of non-equilateral tiles.

Introduction

Hankin's polygons in contact (PIC) [7], Bonner's polygonal technique [1, 2], and Cromwell's modular design system [5], all describe a way to tessellate a plane with tiles where the motifs follow certain edge rules for how the motifs are to interact with each other. I refer to this concept as tiling. The most notable edge rule is the angle by which the lines cross the edge (sometimes referred to as the contact angle).

A paper by Lu and Steinhardt [9], about the quasicrystalline nature of Islamic geometric patterns, also argues that tiling could have been a key method used when these historical patterns was made. To support their claim, they defined five tiles from the 5-fold pattern in panel 28 of the Topkapı scroll [10] as the tile set example, see Figure 1. These tiles have equilateral sides. They named the set Girih Tiles [6].

Analysis of existing 5-fold Islamic geometric patterns shows that these five tiles only enable tessellation of a small portion of the patterns, so the Girih Tiles tile set ought to include more tiles. Hereafter, I refer to the existing five tiles as the *core 5* tiles.

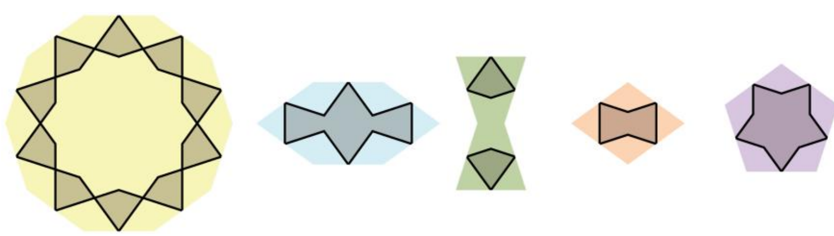


Figure 1: *The core 5 – the five Girih Tiles defined by Lu and Steinhardt. Here with an edge rule angle of 72°.*

The need for additional tiles isn't a new revelation, as new tiles have been presented in earlier papers. Jay Bonner published the tiles of the *core 5* as well as additional tiles in his paper [1] from 2003 (four years before Lu and Steinhardt), see Figure 2a. In 2016, Jean-Marc Castera mentioned the same tiles in his paper [4]. In his paper [5] from 2010, Peter R. Cromwell also provided an evolved Girih tile set. His key tile set contribution was to specify two versions of the line pattern depth for the tile he refers to as the Barrel tile, see Figure 2c.

What these papers have in common is that they don't focus on the tile set. The tiles were used as parts to communicate other topics.

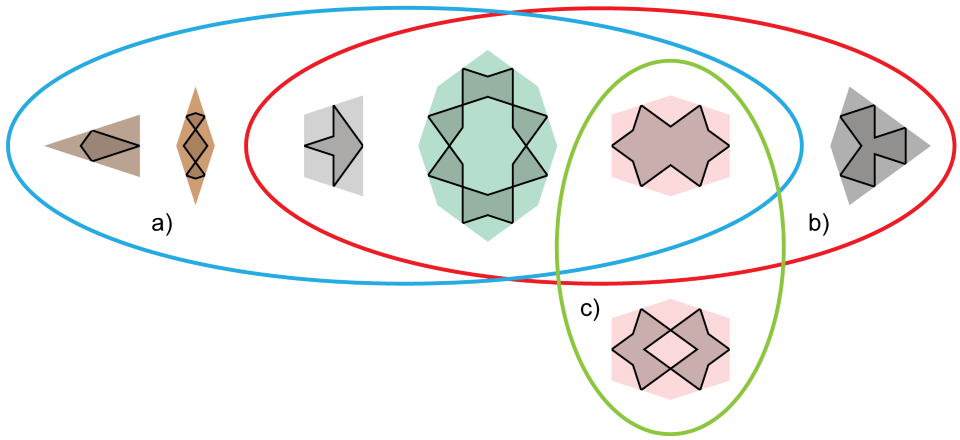


Figure 2: *Additional tiles previously published, Bonner (a), the Topkapı scroll (b), Cromwell (c).*

Figure 3 shows five additional tiles from Sir Roger Penrose' *P1*, *P2*, and *P3* tile sets from the 70's [11]. Not all of them pass the cut into the evolved tile set due to *acute angles* of the tile shape. Mathematically they should, but from an aesthetic perspective, an artist would argue that acute angles provide limitations in the way they carry the pattern lines (the motifs). The motif won't get enough space to develop in the narrow area between the edges. The omittance of these tiles is supported by their scarce occurrence in historical Islamic geometric patterns. The shape of the **Kite** tile doesn't have acute angles, so it manages to elevate into the set; but even though the **Dia** (thin rhombus) is very much acute it still passes as it does provide satisfactorily solutions for some edge rule angles.

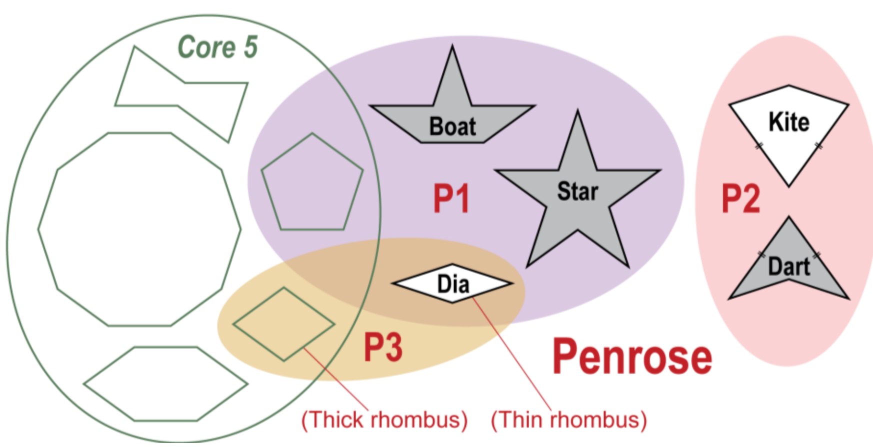


Figure 3: *Non-core 5 tiles from the Penrose P1, P2, and P3 tile set.*

The Topkapı scroll [10] is a collection of instructional patterns from the Timurid era. In some patterns the tile edges are visible. They disclose, and thus confirm, several of the additional tiles in the evolved tile set, see Figure 2b, including the Kite from the Penrose P2 tile set. These patterns, and many Islamic geometric patterns, cannot be tiled only with the *core 5* tiles.

As these additional six tiles (in Figure 2) have been acknowledged by multiple sources, stretching from the Timurid era and forward, their relevance for replicating historical 5-fold patterns cannot be overlooked. It's time to give them the elevated status they deserve. It's time to evolve the Girih tile set.

Non-Equilaterality

With its five tiles, the *core 5* tile set is easy to keep track on. With more tiles added, the evolved tile set would benefit to be more structured. The next step is to organize the tiles based on side length. This will form the base for future development.

All the *core 5* tiles have equilateral sides (unit length of 1), but many Islamic geometric patterns require non-equilateral tiles. The focus of this paper is the Φ-category, see Figure 4, which is the most common non-equilateral category. Tiles in this category have at least one side that is 1.618 times longer than the unit side, that is, the golden section, or phi (Φ). Three of the six additional tiles belong to this category, see Figure 7.

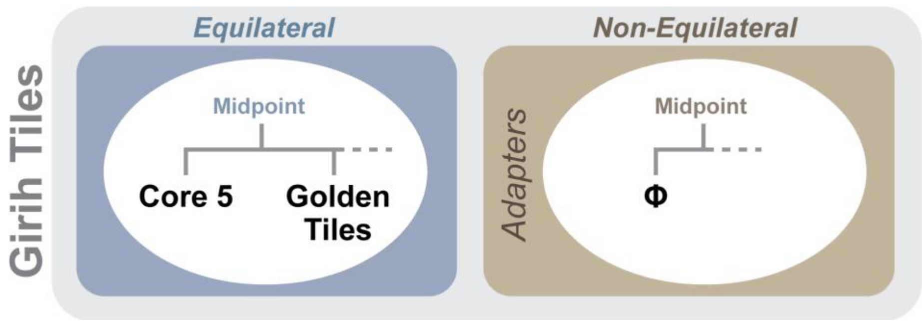


Figure 4: *Equilateral and non-equilateral tile categories chart. The non-midpoint categories are omitted as they are not relevant for this paper.*

Examples of these Φ-sided tiles can be seen in the pattern that Jules Bourgoin described in his publication [3] from 1879. In Figure 5, the design has been tiled with tiles from the evolved tile set, including two of the tiles with non-equilateral sides, the **Cone** and the **Pyra** tile, see Figure 7.

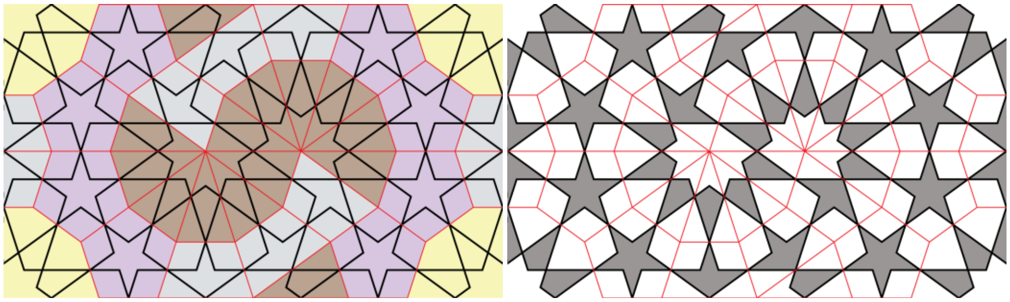


Figure 5: *Jules Bourgoin, panel 188b with the non-equilateral tiles in gray and brown.*

In addition, Bourgoin also depicted a tile from the *core 5* Girih tile set – the **Bow-tie** – where all sides have the length of Φ, see Figure 6.

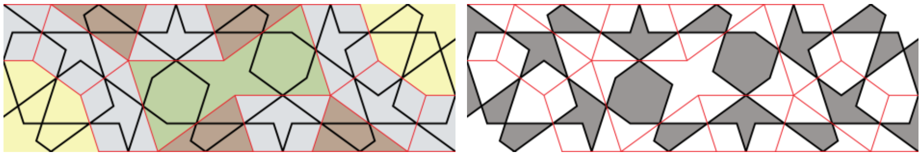


Figure 6: *Jules Bourgoin, panel 187b with the ΦBow-tie tile in green.*

Bonner acknowledged this in his book [2] from 2017. He specifies three all-Φ-sided tiles from the *core 5* set; what I refer to as the **ΦPenta**, the **ΦRhomb**, and the **ΦBow-tie** tiles. I call these Φ-sized tiles the **Golden Tiles**. As equilateral midpoint tiles they fit into the chart in Figure 4 next to the *core 5* category.

The two Bourgoin patterns are examples of historical Islamic geometric patterns that cannot be tiled with the *core 5* tiles. They require tiles from a non-equilateral category. The Φ-category is one of two categories that can tile them.

Adapters

The non-equilateral tiles make up a large part of the additional tiles. As they are crucial for recreating many Islamic geometric patterns, they are interesting enough to be defined as their own class and discussed in detail. I call these adapter tiles (see examples in Figure 7), or **adapters** for short, as they can adapt from one edge rule angle to another. Figure 7 shows the most common adapters with both unit and Φ-sides. They belong to the Φ-category.

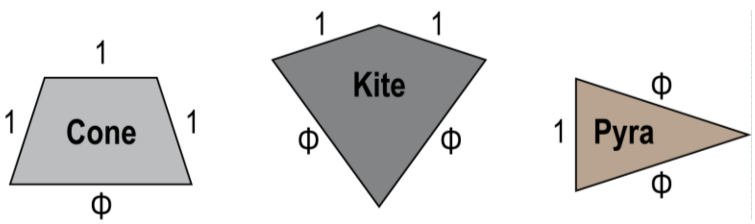


Figure 7: *The most common adapter tiles from the Φ-category.*

In his publication from 1925 [7], E. H. Hankin describes a way in which the tile pattern interacts with each other. For sides with equal length, lines cross over to the other tile seamlessly, that is, the rule is that they have to cross at the same point(s), and with the same angle. This is the "polygons-in-contact" method or PIC for short.

In Islamic geometric art, the main edge rule has two lines crossing at midpoint. For 5-fold patterns the angle between the lines mostly is 36 ° or 72 °, see Figure 8.

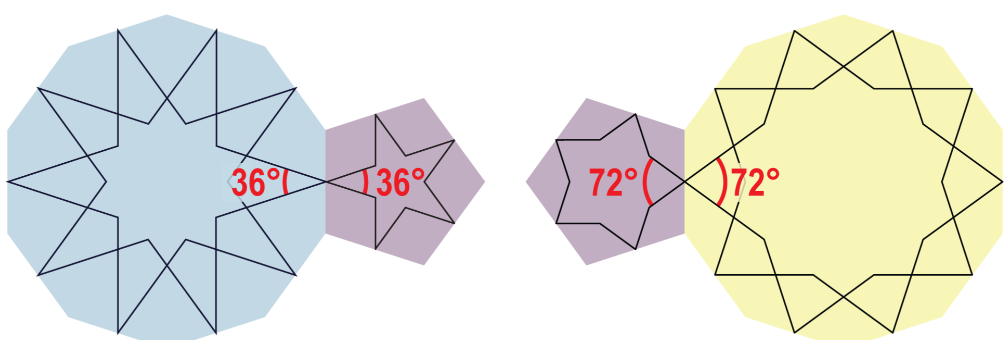


Figure 8: *The 36 ° and 72 ° edge rule for 5-fold patterns.*

In his book from 2017 [2], Bonner talks about different *pattern families*. This means that tiles with a different edge rule angle belong to a different family. He defines four pattern families that dominate the existing patterns in Islamic geometric art. Three families have a midpoint crossing, **Acute** (36 °), **Median** (72 °), **Obtuse** (108 °), and one has two non-midpoint crossings, **Two-point** (72 °). The angles are for 5-fold patterns. Other symmetries (e.g. 6-, 8-fold) have their own characteristic contact angles for each family.

As adapters have more than one edge rule angle, they belong to more than one pattern family. Adapters allow a pattern to have a more dynamic design. Figure 9 shows a pattern that is populated with tiles from two pattern families; the main part of the pattern (outer area) has tiles with median angles (referenced with black and blue lines). The ring of adapters (brown tiles), surrounding the acute star tiling in the center, acts like a “mediator” when these two pattern families meet – the pattern **adapts** to the center star. The tiles in the inner circle have a bigger scale, Φ/(3-Φ) = 1.171.

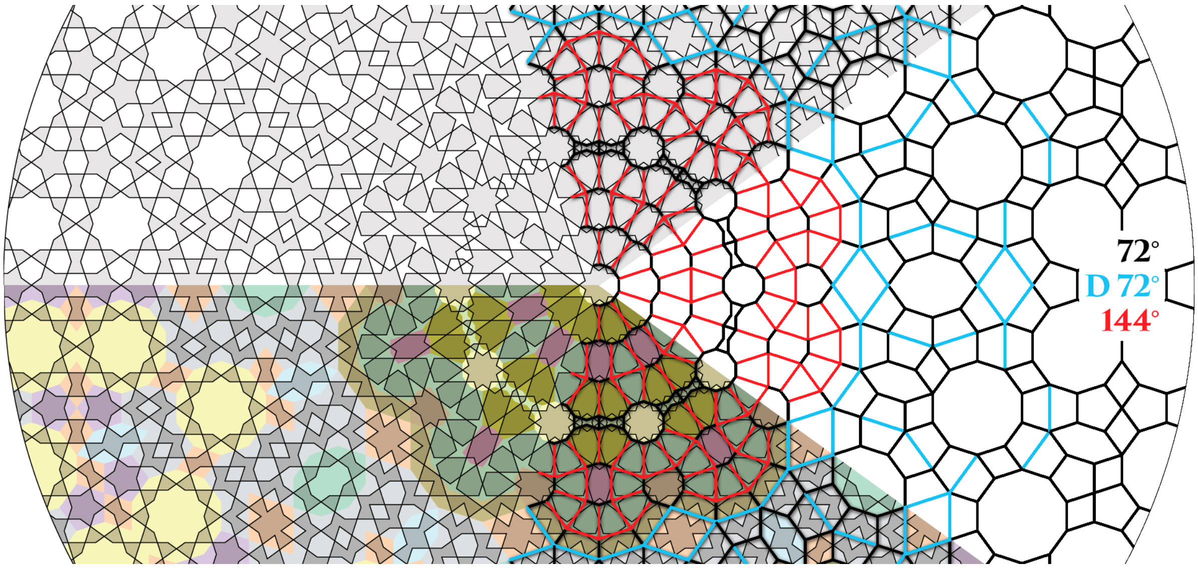
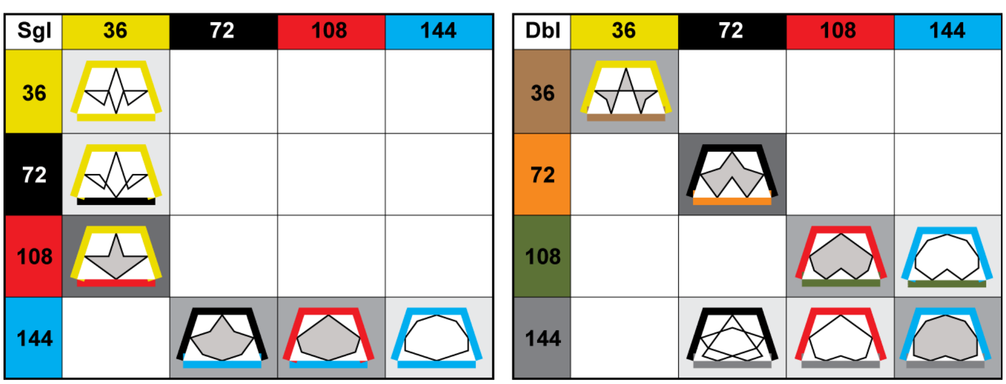


Figure 9: *This pattern adapts from a 72 °edge rule angle (black) and a two-point 72 °edge rule (blue), to 144 °edge rule angle (red) in the center. The D in the legend stands for “double”, indicating the two-pointed side.*

Using the Cone adapter tile as an example, the possible combinations of the edge rule angles results in six tile motifs for the single crossing of the Φ-side, see Figure 10a, and seven for the double crossing, see Figure 10b. The 36/108 Cone and the 72/D72 Cone can replicate many of the historical Islamic patterns in the acute and median pattern family respectively. Note that here, I’ve separated the obtuse pattern family into a 108 ° and a 144 ° column, and I provide a second table for the tiles with a double crossing of the Φ-side.



a) b)

Figure 10: *The possible edge rule angles for the Cone tile. The darker the background, the more useful the tile pattern is. The tiles with edge rule angle combinations that cannot be constructed when following the criteria of symmetry for each side of the line’s midpoint are shown as empty areas.*

For creating new patterns, any angle can be used, and the number of crossings can vary. The edge rule of the tiles in Figure 11a have five symmetrical line crossings, with eight lines crossing as pairs or single lines, with 144°, 72° and 0° angles. To optimize the tiling options, the lines at the edge have to be symmetrical around the mid-point, that is, each side of the mid-point has to be a reflection of the other. For Penrose tilings, where the matching rules are enforced to obtain quasicrystalline patterns, the lines are not symmetrical from mid-point.

The edge rule doesn't have to specify lines, it can be curves. In Figure 11b, the edge rule angle is 72°, but the lines are curved, giving the final pattern a more organic look. Curved lines are not so common in traditional Islamic geometric patterns. To avoid sharp turns, the edge rule has to contain a value for how sharp the curve is allowed to be (easily defined by the length of the Bezier curve handle). As long as the edge rule angle is the same for each respective side length, the tile pattern can be very creative. Figure 11c shows an example in which the motifs are more elaborated.

The three golden tiles Bonner showed can be found historically, but for creating new interesting patterns, the other tiles can also be enlarged to become *golden tiles*. Figure 11d shows examples of the tiles in the *core 5* category as golden tiles, with several motif variations of each tile. Note that most of these variants would render the final pattern too complex, but a few of the more simple ones could work.

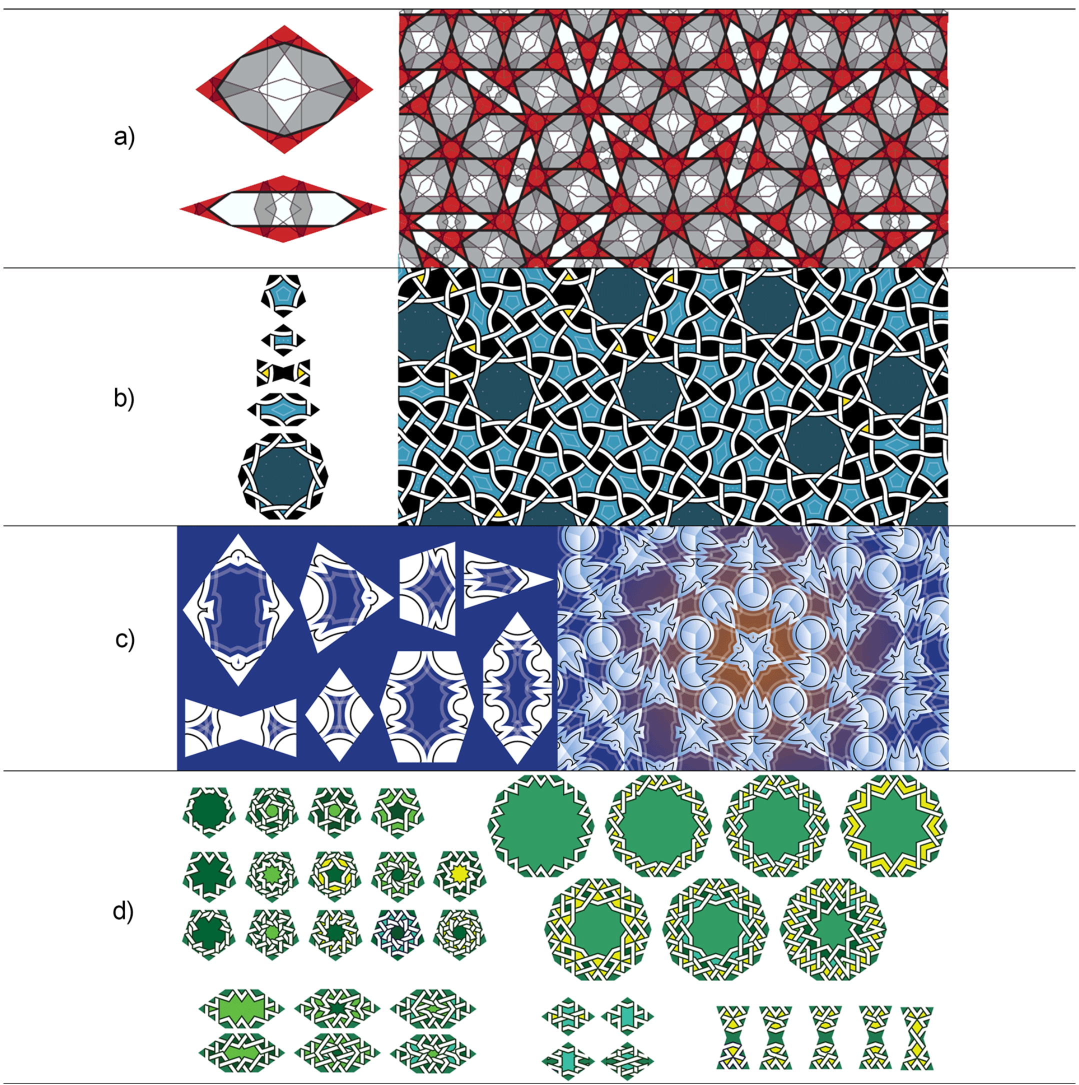


Figure 11: *Edge rule variants – multiple crossings and angles, curved lines, and golden tiles.*

The Evolved Tile Set

Lu and Steinhart's *core-5* tile set aimed to support the idea of quasicrystallinity in Islamic patterns, and for that, it was sufficient. In this paper the purpose of a tile set differs as it includes a broader scope. The objective is to create a set that can provide the ability to replicate a *wider range* of existing historical patterns, as well as, create new interesting patterns. This requires an evolved Girih tile set.

The six new tiles qualify due to either their ability to meet this objective, or their adherence to their historical mathematical relevance (like the Penrose tiles). The range of possible 5-fold tiles is more than can be covered in this paper, so tiles have to meet other criteria too, like their ability to provide diversity in the pattern. For example, the acute angles of tile shapes create narrow areas for the rays to develop within, and thus disqualify them from the set. Tiles with more than ten sides (like Kepler's monster tiles [8]) will have to be defined in a separate tile set.

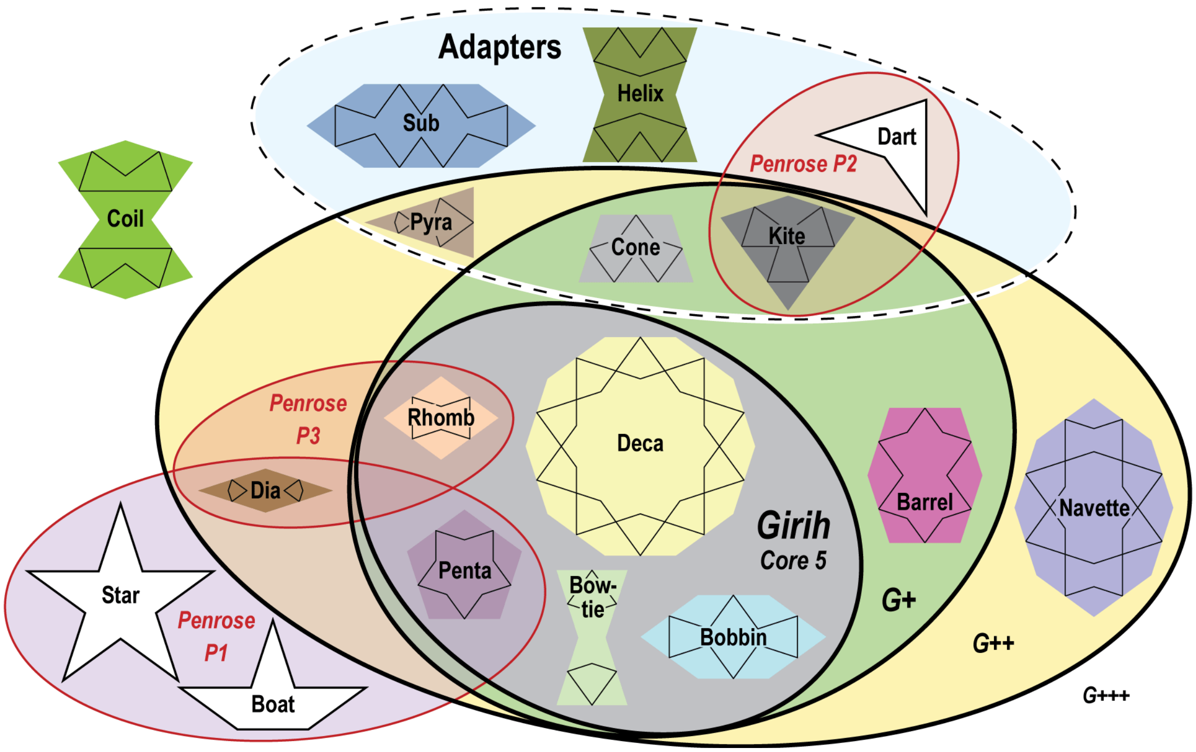


Figure 12: *The evolved Girih tile set.*

Besides limiting the number of tiles included, one way to provide clarity is to organize the tiles in groups based on *usefulness*. We already have the first group, the *core 5*, as the main Girih tile group. I've added three levels of usefulness. I call them **G+**, **G++**, and **G+++**. The closer to the *core 5* set, the higher level of usefulness. The further away from it, the classification becomes more difficult and more arbitrary. Level G+++ isn't as fixed as the other levels. It contains three tiles that I've found to be useful to create compelling patterns; the **Coil** tile and two adapters, the **Sub** and the **Helix** tile. The Coil and Sub carries motifs with shapes that isn't common in historical patterns, but without them tessellating new 5-fold patterns will be limiting.

I also see the need for a criterion that covers how useful a tile is *for certain pattern families*. Some tiles are more useful for one pattern family but don't work at all with others – for example, the Bow-tie is crucial for median patterns, but gives an odd appearance for acute patterns, while for the Pyra it is the opposite. This makes it hard to define a single set that will fit all eventualities. Therefore, the definition in this paper can only be a generalization based on an average estimate of the tiles' usefulness.

Adapters belong to a huge group, containing several different categories. This paper covers the most common category, the Φ category, including the key tiles, the **Kite**, the **Pyra**, and especially the **Cone** (as it is the "*king of adapters*"). Note that the golden tiles aren't included in the set, as all tiles can just be scaled to be used as golden tiles.

Summary and Conclusions

This new tile set has now evolved into 14 tiles; from a small fixed set with one purpose in mind, to a set with a multipurpose function and a more complex and generalized composition. This will make the definition of a set hard, and it comes down to the criteria by which one use to sort out the tiles eligible to be included. With the tiles in levels G+ and G++ one is able to replicate more historical Islamic patterns than the *core 5* tile set allows. This new tile set will update Lu and Steinhardt's Girih Tiles to version 2.0, which will be a good starting point for everybody to build/create/puzzle their own patterns, experimenting with new designs, and verifying existing historical patterns. The tile palette is set. Start puzzling!

The big question for the future is what else will non-equilaterility bring? Are there other side lengths that fit into the 5-fold context? The usefulness of tiles, and different tile sets for each pattern family, would be other fields of interest to investigate. Perhaps it is possible to classify tiles so they can be used to calculate how "pure" a pattern is? Who knows, we might even have pattern top lists? The initial evolved tile set is set. Start investigating!

Acknowledgement

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