Mathematics and Maypole Dancing

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Abstract

Our workshop will allow participants to explore some of the mathematics of maypole dancing. In groups of 6-10 they will dance the maypole dance, find representations of the generated patterns on the pole, and discover properties of the generated ribbon patterns. The inverse problem will also be explored: given a pattern, can we find a ribbon constellation that generates the pattern? The question becomes even more interesting if we change the traditional dance pattern.

Workshop Goals

Figure 1: *Students Dancing Maypole Dance*

Exploring the patterns on a maypole gives rise to a surprising amount of mathematical ideas. Educators at the K-12 and college level will be able to use the workshop activities to bring authentic mathematical explorations which involve a physical and esthetical motivation into their mathematics classrooms. Besides the authentic exploration and the generation of theorems and proofs, students in their classes can learn about different symmetries, fundamental domains, mapping a cylinder onto the plane, combinatorical techniques when counting the patterns, and thinking about equivalences in general. (Maypole) dancers can use the workshop ideas to make choices about ribbon orders and colors that will result in the most pleasing pattern. They can also use the idea of creating different maypole dances for their future dances.

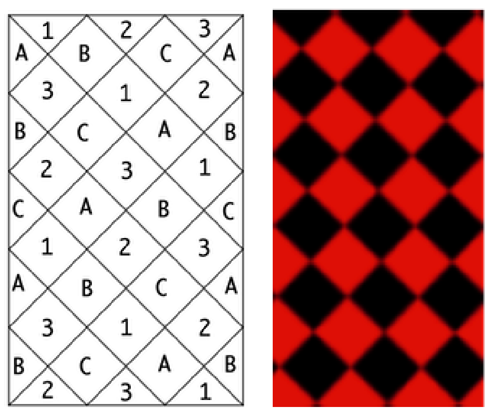
The workshop is run in an inquiry-based way, which allows participants to also learn about pedagogical tools by observing the facilitators. They may notice how we ask questions (instead of providing answers), guide the participants while leaving a lot of room for new ideas, and facilitate whole class discussions that engage all participants. The participants also learn about the curriculum and teaching resources at www.artofmathematics.org.

Dancing the Maypole Dance and Representing the Pattern

In the beginning of the workshop, groups of participants do the maypole dance to experience the movement, and to see how the pattern on the pole is generated. To mimic the wooden pole, we will bring shorter plastic tubes as in Figure 1. The dancers hold ribbons that are attached to the top of the pole and dance around the pole in particular patterns. The ribbons get woven around the pole in interesting patterns.

Maypole dancing has clear connections to braids and group theory, see for example [1], [3], and [7]. The focus for this workshop is different. Instead of analyzing the moves to create the ribbon pattern or braid, we are interested in the geometric ribbon patterns that are being generated. Understanding the connection between the dance pattern (including the number and color of ribbons chosen) and the resulting ribbon pattern is our goal.

A first problem participants have to wrestle with is how to represent the patterns. Figures 2a and 2b show an example of a ribbon pattern with three couples. Assume that there are three black ribbons (called 1, 2, and 3) and three red ribbons (called A, B, and C). Each black ribbon is paired with a red ribbon. All of the people holding black ribbons move to the right (i.e. mathematically positive when looking at the pole from above) and go first over the red ribbons. At the same time, the people holding red ribbons move left (i.e. mathematically negative from above) and go first under/inside the black ribbons. After this first step, the black ribbons would go under while the red ribbons go over. This pattern keeps alternating. While the dancers move around the pole, the ribbons will cross over and under to generate the ribbon pattern.



(a) **(b)**

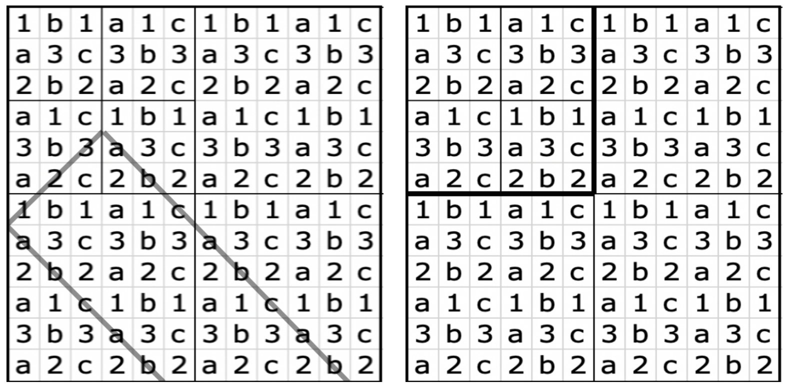
Figure 2: *A General Maypole Ribbon Pattern (a) and Black-Red Maypole Ribbon Pattern (b) in Tree Representation*

We call the representation of a ribbon pattern using diagonal ribbons as in Figure 2 the **tree representation**. The ribbon pattern on the maypole is 'cut vertically' so that the cylinder can be unwrapped to see the full pattern. Leaders are represented by numbers and followers are represented by letters. **Leaders** are defined as ribbons which first 'go over' in the dance. Accordingly, **followers** go under first in the dance. In the image, 1, 2, 3 are leaders and A, B, C are followers; we can also see that the 1-ribbon first passes over the A-ribbon, the 2-ribbon passes over the B-ribbon, etc.

We can also represent a ribbon pattern by the shorter number-letter representation $1A\ 2B\ 3C$. We call this in short the **letter representation**. $1A\ 2B\ 3C$ is the basic unit of all dances for 3 couples. If we want to talk about a specific pattern, we can, for instance, write $BWBWBW$ for the patterns resulting from 3 black ribbons (leaders) and 3 white ribbons (followers).

To automate the process, the patterns can also be generated with Excel. The ribbons are now horizontal and vertical instead of diagonal. We call this the **screen representation** of a ribbon pattern. Figure 3a shows how the tree representation relates to the screen representation. The leader ribbons (numbers) are now horizontal while the follower ribbons (letters) are vertical.

In both, the tree and the screen representation, we can extend the pattern horizontally to see the larger pattern. A **fundamental domain** of the ribbon pattern is defined as a smallest region that repeats itself inside the whole pattern. See Figure 3b to see a fundamental domain of $1A$ $2B$ $3C$ in screen representation. Notice that a fundamental domain has to exist since the dance repeats itself, but that it is not unique.



(a) **(b)**

Figure 3: *Comparing Tree and Screen-Representations (a), and the Fundamental Domain of* $1A$ $2B$ $3C$ *in Screen Representation (b).*

First we notice that in order to dance the traditional dance pattern we need an even number of dancers, since the roles of leaders and followers are different. But the n umbers of pairs (pair=leader and follow) can be even or odd.

Activity 1: Compare a dance with 3 pairs to a dance with 4 pairs. How are the ribbon patterns the same, how are they different?

When comparing the fundamental domains the participants notice that for n pairs the fundamental domain has dimensions nxn if n is even. If n is odd the dimensions are $2nx2n$. The reason is that for n even you meet your partner in the same over/under position when you meet again. If n is odd the over/under position is reversed when you meet your partner again and so the fundamental domain is not complete yet. You can see this in Figure 3b: The fundamental domain for 3 pairs is the same size as the fundamental domain for 6 pairs. When you look closely at the smaller regions inside the fundamental domain you can see how the diagonally positioned regions are the same. This happens because the over/under pattern is reversed when all dancers (from the odd number of couples) dance with their original partner again.

Pattern Puzzles

Activity 2: Given a retangular pattern of 2 or 3 different colors, can we decide which maypole dance would generate this pattern? Does the dance have to be unique, or could there be several dances generating the same pattern? How can we determine if the traditional maypole dance can not generate this pattern?

When presented with the pattern in Figure 4 the participants try to decide if this could correspond to a traditional maypole dance. Notice that all ribbons follow a consistent over/under pattern. From the first

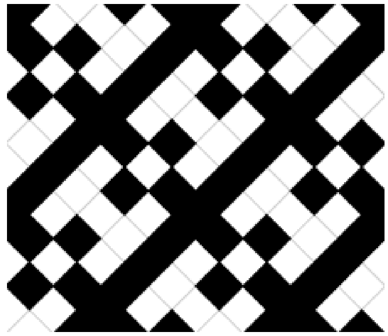


Figure 4: *Puzzle Image 1: Find the Dance!*

row, we can see that the ribbons going from top left to bottom right are B, W, B, B, which means these are the leader's colors. In the next row we see the ribbons going down from top right to lower left, these are the followers: B, W, W, W. Therefore the letter representation of the dance that generates this pattern is $BB\ WW\ BW\ BW$.

The pattern in Figure 5 is generated by 4 different colors. The leaders are green, white, and black, while the followers are black, white and red. Therefore the letter representation of the dance is $GB\ WW\ BR$.

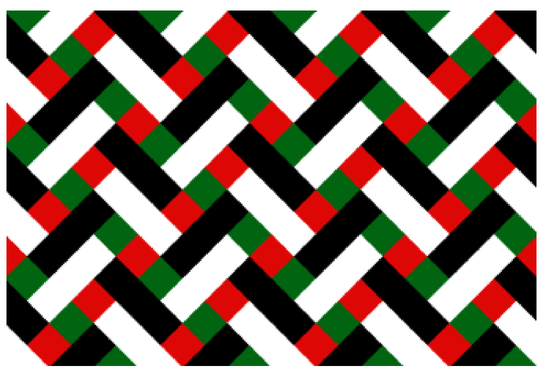


Figure 5: *Puzzle Image 2: Find the Dance!*

Notice that the letter representation of the ribbon pattern is not unique. The pattern could also have been generated with 6 pairs: $GB\ WW\ BR\ GB\ WW\ BR$. Moreover, if you move the fundamental domain over, you can also generate it starting with the next leader: $WW\ BR\ GB$ or $BR\ GB\ WW$. In fact, there are more ways, how equivalent ribbon patterns can be generated, see our paper [5].

But the pattern in Figure 6 can not be generated by a traditional maypole dance. The over/under pattern is not present even though there is a lot of regularity.

Non-Traditional Maypole Dances

Looking at patterns that can not be generated by the traditional maypole dance, calls for the next question:

Activity 3 Could we modify the dance so that we can generate other ribbon patterns? For example the pattern in Figure 6?

Unfortunately, the pattern in Figure 6 can not be generated by any maypole dance. The leader ribbons are green, red, and black, while the follower ribbons are red, black, and green. Notice that the first leader

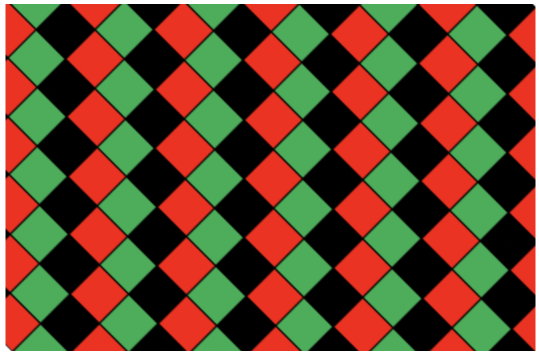
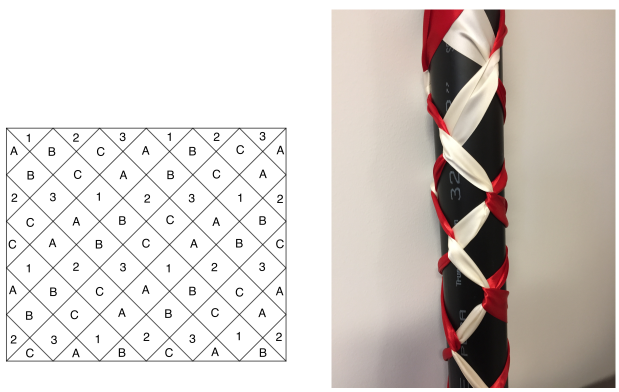


Figure 6: *Puzzle Image 3: Find the Dance!*

ribbon (green) and the third follower ribbon (green) meet in a red square which is impossible.

But there are other consistent dance patterns that generate new ribbon patterns, for example when the leaders go “over under under” while the followers go “under over over,” see Figure 7a. Notice that the generated ribbon patterns always contain at least two adjacent squares of the same color for each ribbon. It is an open question, how many different patterns we can generate with this new dance pattern.



(a) **(b)**

Figure 7: *(a) Non-traditional Maypole Dance in Tree Representation, (b) Non-traditional Maypole Dance with Turns*

Another traditional version of the maypole dance is the “Spider’s Web” dance, or “The Gypsies’ Tent,” in which the ribbons form a web away from the pole. You can see a video of this dance at [2] in maypole dance number 3. This spider web doesn’t touch the pole anymore though. Using this idea, we are wondering about other dances that create patterns/braids that lie on the pole. Figure 7b shows a simple example of this idea. Here each dancer walks around one neighbor, then moves back into the original position and walks around the other neighbor. The ribbons on the pole can not lie smooth on the pole anymore since they get pushed together during the turn. Therefore the pole is visible and part of the geometric pattern.

We will end the workshop with an open exploration of such new maypole dances. Which dances are most fun and which patterns are the most aesthetically pleasing?

Students Engaged in Inquiry

Most of the work in this paper has been created with students in mathematics for liberal arts classes at Westfield State University. [4] describes in detail how the class is run and gives access to the book "Discovering the Art of Mathematic: Dance." None of the students were mathematics majors and most, if not all of them, didn't want to take a mathematics course. One of these students was co-author Julianna Campbell who describes her transformative experience below (see also [6] and [5]).

"Traditional mathematics K-12 education had failed me. I struggled with low confidence, believing that I could not succeed in math the way my peers did. Instead of fostering curiosity and creativity, my math teachers resorted to 'drill and kill' worksheets and timed multiplication quizzes. As a naturally inquisitive learner I found success in reading and writing, these classes allowed me to explore worlds and ideas that opened my mind to new possibilities. However, when I walked into the honors center at Westfield State University and heard about this strange new class that combined English and math and art in a way that gave people a new perspective on the disciplines, there was something about the challenge of this class, and the thought of giving math a second chance that made me sign up.

The English and math interdisciplinary class made me realize that these two disciplines are not all that different. In both classes we had opportunities to write, reflect and question. Professor von Renesse's class relied heavily on the writing of mathematical proofs for each assignment. I found success in this type of mathematical exploration and inquiry. Writing about math, and proving complex mathematical ideas was interesting and rewarding for me. After handing in four, five, six page proofs I felt a sense of accomplishment that I had never felt before with math. Months later I decided to embark on an independent study with Professor von Renesse, engaging in mathematical inquiry and practices regarding the math of maypole dancing. We thought about the intersection between math and dance, and I was given opportunities to develop my own proof, the pair swap, which was perhaps the most rewarding aspect of the entire project. I had the courage to voice my questions and ideas in a mathematical setting, which was incredibly freeing for me. Discovering that English and the arts was not completely separate from mathematics, not only made mathematics more accessible to me, but also more interesting."

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