Experiencing Mathematics through Astonishment

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Abstract

In this paper, I briefly explore the notion of astonishment as a way to experience mathematics as work of art. I engage with the concept through some scholarly work, and the discuss episodes from my mathematics education researches in which aspects can be observed, and questions can be raised.

A feeling of wonder

In the last pages of his ethnological study, Ingold [1] asks: “Which of them is wiser, … the one who knows the name of every kind of bird but has them ready sorted in his head [or] the other who knows no names but looks with wonder, astonishment and perplexity on everything he sees?” (p. 134). Ingold offers this rhetorical question and draw on the work of Masschelein [2], to calls for an education aimed at developing attentiveness rather than awareness of the world. This, he argues, implies learning is done through “exposure to the world, and exposition, he explains, is about being out-of-position, pulled away from where we were, which then requires an open form of attention toward whatever is coming.

This reminded me of Heidegger’s [3] (1935/2006) questions on the origin of the work of art. Noting that “what occurs to us as natural is perhaps only the habitual of a long habit, that has forgotten the extraordinary from which it sprung. Yet that extraordinary once assailed man as estranging and brought thinking to astonishment” (p.9), he then goes on analysing how active works of art create such a feeling of estrangement. The shock of an unexpected, curious presence (that might be pleasing or unsettling, etc.) disrupt the quiet unfolding of our world to reveal something otherwise invisible, unattended to. Then the revelation process follows, in which what that presence brings forth becomes part of conscious awareness.

I this presentation, I would like to explore the notion of astonishment as a way to experience mathematics as work of art in relation with mathematics education. Can we imagine that at least a part of how we do mathematics in school could be about developing a sense of wonder? In the following sections, I briefly unpack the notion of astonishment, and discuss observations I made with some of my students. My goal is to initiate a conversation on a potentially rich topic that have not be explored *per se*.

Astonishment in Question(s)

The word astonishment comes from the Latin *ex-*, “out of” + *tonare*, “to thunder”. First, there is a strong form of externality at play. This is what Heidegger associates with work of art. The expression of astonishment is the sign of the unexpected effect of something foreign, the unanticipated sensation of, for example, idea that impose itself to the mind (rather than being constructed). Astonishment is brutally finding oneself in the process of becoming aware of something heretofore foreign. But at the same time, being struck evokes something internal, an “affect”, and the fact that what touched us is somehow already (becoming) part of oneself. That something already stops being foreign, and become part of one’s own awareness. *Might mathematics education provide occasions for students to experience unexpected ideas, to discover in themselves developing ideas?*

Few scholars explicitly engage with the concept of astonishment. Sherman [4] uses it to examine Lecoq’ pedagogical approach to theater. According to Sherman, Lecoq highly valued being surprised by his students, promoting a view in which teaching is about developing people’s ability to create, as opposed to show them models to reproduce. Sherman also presents Barbaras [5] analysis of Merleau-Ponty’s writings to stress the importance of astonishment in relation with learning: “The mystery is not solely my opening before the presence of the world, it is that I can astonish myself, which is to say, disrupt my familiarity with myself and begin to think […] an astonishment that gathers the incontestable and strange possibility" (p.10 quoted by Sherman). *Could mathematics education provide opportunities for students to surprise us, and develop their creativity by disrupting familiarities with oneself? Could mathematics education be in part oriented toward the birth of strange possibilities?*

The notion of astonishment is explored in some details by Kant [6] in his famous *Critique of pure reason*. Kant describes astonishment as an affect, an occurrence taking place when we encounter something that bypasses our expectations (e.g. A622/B6S0, p. 579). Astonishment pushes us out of our current frame of mind. It even sometimes makes us doubt, Kant argues, whether we still correctly judge what stands before us. Being astonished, then, is precisely about experiencing something in an unexpected way. The feeling is both pleasing and unsettling at various degrees. Weigler [7] explore this idea through a series of analysis of theatrical scenes known to produce that feeling of awe, showing how they hinge on our tendency to pigeonhole our experiences based on our expectations. With these scenes, he says, "spectators found themselves unexpectedly engaging with what they saw on stage as if it were the first time they had ever encountered it" (p.3). *Is it possible to present mathematical ideas and processes in a way that surprises students' expectations, and makes learning mathematics like a first encounter?*

In film theory, Gunning [8] also explores the notion of astonishment, here in regard with spectators' first experiences of moving pictures. Considering how the effect was set up by early filmmakers, he notes: "What is displayed before the audience is less the impending speed of the train than the force of the cinematic apparatus […] the one demonstrates the other. The astonishment derives from the magical metamorphosis rather than the seamless reproduction of reality" (p.35). When we think about astonishment, we generally have in mind something more than simply being (pleasingly) disturbed by the unexpected: astonishment suggests a form of understanding, or the possibility thereof. When early spectators saw the still picture projected on the screen becoming alive, Gunning explains, what must have astonished them is not much the movement of the train, but the realisation that this was produced by the projector. *How should we examine mathematics so its truly powerful machinery comes to light as in itself an object of wonder?*

A more extensive examination and deeper analysis could certainly allow the formulation of something like "operational definition" or a substantive deconstruction of the concept astonishment in the arts, and in relation to education. More modestly however, I have simply, through this review of a sort, highlighted some *questions* following the idea of turning to astonishment (as discussed in the arts) to think about mathematics education. In the next section, I continue highlighting what seems to me a possibility worth exploring by offering some illustration of what mathematics as astonishment could mean.

Mathematics as Astonishment: challenges and opportunities

For famous mathematicians T.S. Hardy [9] the : "'purely aesthetic' qualities can we distinguish in such theorems as Euclid's or Pythagoras's [are] a very high degree of *unexpectedness*, combined with *inevitability* and *economy*" (p.29, original emphasis). However, like any work of art, not everybody readily appreciates the aesthetic of a given proof. Papert [10] reports something that looks like astonishment when people work on the demonstration of why the square-root of two cannot be written as a fraction. But it is clear that appreciating such proof requires at least a bit of mathematical background. First one needs to be able to read, decipher, interpret what is at hand; and second it is also necessary to situate it on a certain scene: Is the theorem important, or difficult? What is in that proof that makes it "unexpected", or any other characteristic one might appreciate (cleverness, subtlety, uniqueness, etc.)? And then, of course, the production needs to be attended to in a particular way: encountered (as if it were) for first time. For sure, when all that happens, it is very possible to look at a mathematical result and get this feeling of surprise, displacement, and so on. *Experiencing mathematics through astonishment seems to require some familiarity with the ideas, including some kind of expectation regarding what would count as surprising.* But what kind of situations might be able to bring all that together? And does it mean that not everybody can experience, through astonishment, mathematics as work of art?

It is worth mentioning at this point that many of the questions I have raised here relate to questions mathematics educators already tackle. How to present students with new ideas (e.g. algebra)? How is it possible for them to make sense of these (when all they know is arithmetic)? To what extend can situation let (or lead) student (to) "discover" new mathematical ideas? Astonishment is different from discovery, but insights are surely to be found in the abundant literature around inquiry based mathematics education for example. It could be fruitful to reviewing such literature with astonishment in mind. I also think some more empirically grounded exploration of the what astonishment can mean in mathematics education would be worthwhile. To exemplify this, lets examine the case of Jee: a first-grade student sitting at her table with a bunch of Polydron shapes. Jee experience something reminiscent of what Heidegger or Hardy talk about. Connecting shapes together and moving her construction in space, Jee suddenly expresses satisfaction and excitement when the shapes "magically" come together in her hands to form a triangular prism.

Figure 1: *Jee during her first utterance*

Jee encounters the prism (and the pyramid) with signs of astonishment. She first simply moves the shapes around, from inside a world of familiarity, until suddenly something happens. A break in the ordinary moves her out of previous way of attending to the shapes. It is now a solid, a three-dimensional figure. As such, it seems to instantly make sense to her: she *recognises* the appearance of something unexpected, or perhaps simply the fact that she is pushed out of seeing the shapes as a set of meaningless, more of less articulated bits. The description she then offers, rapidly going over aspects of the objects, also illustrates how the shapes keep revealing themselves. The objects are at the same time strange and familiar. They can be described with common words, but how exactly those usual ideas (square, triangles, having more, having less) work together in this special case needs to be articulated. And this *can* be done! As Jee points out differences, the surprise is followed by an attending to what is now possible. Moreover, the sort of work she is displaying here is exactly what many mathematics educators wish to see happening in the classroom: a student actively engaging with mathematical ideas, a "meaning making" activity connected with mathematical ideas that are even part of school curriculum (a rich literature exist on that topic).

The notion of recognizing something unexpected or that of engaging with unforeseen potential are keys to conceptualize astonishment in mathematics education, and ask if it can be seen as the same "kind" of astonishment we talk about in relation to art. However, capturing something as fleeting as astonishment is in fact not an easy feat. Methodological considerations are needed, to include perhaps first-person perspective approaches to examine the experience of astonishment. The work carried on the ah-ah experience (a moment of sudden insight or discovery) in mathematics education (e.g. Liljedahl [11]) offer some cues, and can help compare these two kind of experiences. Some might also be tempted to contrast accounts of astonishment when encountering a mathematical or an artistic production.

Art is very good at remind us that astonishment cannot be encapsulated. While mathematics education still have a tendency to look for predictable, uniform, clearly identifiable results, a pedagogy of astonishment would certainly go against that view (what Ingold is looking for). The everyday consequence of this, however, can also shed some light on the matter. About a year ago, as part of a project in which high school students are exposed to what I called the "imperfect" aspect of mathematics [12], a students were asked if the "method" of "crossing out" digits to find equivalent fractions (e.g. "1~~6~~/~~6~~4=1/4") works and why. I remember the puzzling surprise of *my* first encounter with the technique as something beautiful and strange, disturbing and fascinating at once. That group of students, however, did not strongly react. One quickly offered to "multiply by six" both terms, corrected himself ("by 16"), and then the group continued exploring various cases. I did not get from that lesson the feeling that students experienced something like Jee with the prism. I also had the opportunity to present the method to a group of university mathematics undergraduates. Astonishment surely took place (at least for some), but the surprise sounded more like a scandalous shock than unexpected delight. So while some mathematical ideas and teaching situations bear the potential to experience astonishment, the disposition it requires needs investigation. And while astonishment can be associated with positive feelings, it can also provoke negative impressions: looking at how this is conceived of in the arts might help us move forward.

Conclusion

With this short study, I offer a way to think of the relationship between mathematics and art in a somehow unusual way. Instead of looking at how mathematics is or can be used in art, I investigate a way in which *mathematics could be experienced as art*, through the notion of astonishment. The study exposes possible ways to think of this idea, the existence of some practical prospects, and some challenges. Exploring in more detail similarities and differences between astonishment in mathematics and in the arts might also lead us to further dwell on the different kind of (artistic) experiences mathematics afford. Could we distinguish the appreciation of "styles", going from the bold, sketchy observations of people like Fermat, Euler or Ramanujan, to the patient, skilful craft of Euclid or Russel, for example? What about the gothic feel of Archimedes' writings, the minimalist touch of Peano, the grandiose effect of Wiles' proof of Fermat's theorem, or the impenetrability of Mochizuki's work? It all goes to the idea of presenting mathematics as an art, and maybe even consider how solving a mathematical problem or finding a proof could be similar to creating a work of art. And vice versa. To what extend do this suggestion adds to the ongoing conversations? What is the exact nature of the relation between astonishment in mathematics and an astonishment about a work of art? These are questions I hope we can keep on discussing.

Acknowledgements

Data collection for this study was supported by research grants from the Social Sciences and Humanities Research Council of Canada. Part of the ideas presented here emerged from a conversation with Wolff-Michael Roth, who also developed them [13].

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