Maths Craft in Class

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Abstract

Maths Craft New Zealand is a non-profit initiative founded in 2016 and run by mathematicians Dr Jeanette McLeod and Dr Phil Wilson from the University of Canterbury. Together with the rest of the Maths Craft Team, we bring maths to the masses by celebrating the links between mathematics and craft. We have run numerous festivals and workshops across New Zealand, reaching over 10,000 people, and are the largest mathematics outreach programme in the country. In October 2018, we ran our first ever professional development Maths Craft workshop for primary and secondary school maths teachers. We taught them the mathematics behind a selection of crafts, facilitated their creation of lesson plans, and supported them as they introduced Maths Craft into their own maths classes.

In 2016, authors Jeanette McLeod and Phil Wilson created Maths Craft New Zealand as a way to bring maths to the masses. For the past two and a half years, Maths Craft have been running free, publicly accessible Maths Craft festivals across New Zealand where we invite the general public to experience mathematics through craft [2]. At these events, we have met many teachers, students, and parents from around New Zealand who are excited by the prospect of using craft as a gateway to mathematics, and have asked us for ways to bring Maths Craft into schools. In response to these requests, and funded by an Unlocking Curious Minds grant, in October 2018 Maths Craft New Zealand ran a free two-day professional development workshop for school teachers, dedicated to bringing Maths Craft to maths class. In this paper we describe the workshop, focusing on the mathematical thinking and affective experiences of the thirteen primary and secondary mathematics teacher participants. All of our participants agreed to participate in a research study of the project, and the results of this will be published in a forthcoming paper.

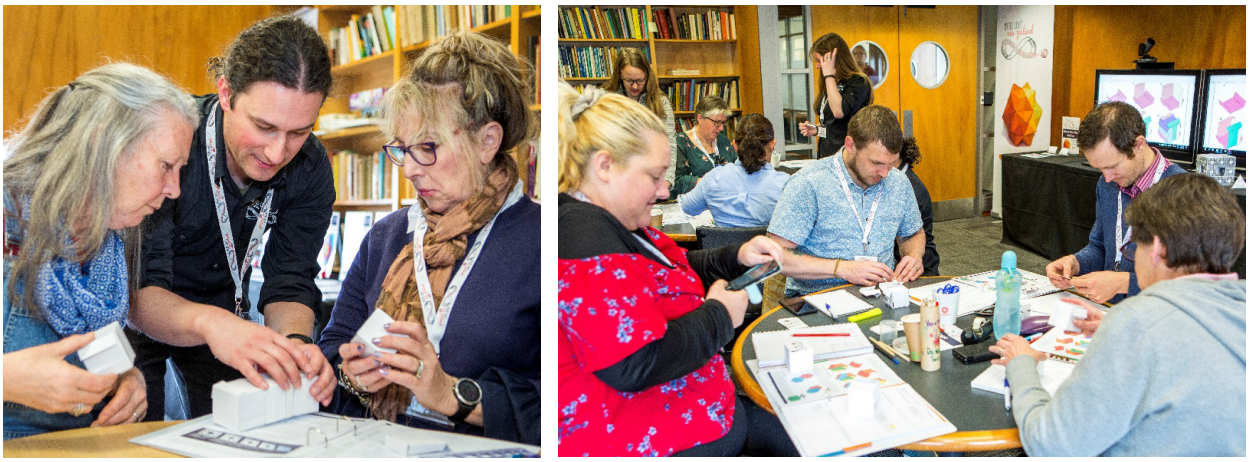

Figure 1: *Helping teachers to construct the Menger Sponge fractal sculpture during the workshop.*

The Workshop

The aim of the workshop was two-fold. Firstly, to help teachers improve their mathematical thinking and self-efficacy in mathematics, and to give them concrete techniques for engaging their students with mathematics in the classroom. Secondly, to decrease student fear of mathematics and increase mathematical

confidence, with the ultimate aim of instilling a higher appreciation of its importance. Using craft as a medium, we want to help them see that mathematics is more than just fractions and long-division, and instead focus on creativity and high-level mathematical thinking, to give them a glimpse into how 'real' mathematicians think. Over the two days, participants learned four Maths Craft activities under the guidance of mathematicians Drs Jeanette McLeod and Phil Wilson, and created draft lesson plans based on each of the activities under the supervision of mathematics education lecturer Dr David Pomeroy.

Each of the four Maths Craft lessons involved both an explanation of the underlying mathematical theory, and the construction and exploration of craft objects. We chose the first lesson to be making and manipulating paper Möbius strips, since the topology of non-orientable surfaces is surprising and confounds intuition, and the craft itself is accessible. Although many teachers had previously seen Möbius strips, our activities helped them model mathematical thinking in novel ways. For example, we asked them to predict the result of cutting a Möbius strip along its centre line *before* performing the activity. The result and their predictions were completely different, which surprised and delighted them. (This also acted as an effective ice-breaker and set the tone for the rest of the workshop.) Having them explain the result, the disparity, and why their intuition had misled them involved high-level mathematical thinking, as did the other cutting experiments which again defeated their intuition. By this point, the teachers had learned that making and examining mistakes leads to deeper understanding, something well known by professional mathematicians. To further demonstrate this, we shared brief stories of our own mistakes and subsequent discoveries.

The second lesson involved the construction of a Menger Sponge from business cards [4] and an introduction to fractal geometry. This was challenging for some teachers, especially those with weaker mathematical backgrounds, however the hands-on nature of the craft activity scaffolded their understanding of key concepts. Central to this lesson was the distinction between fractals as abstract mathematical objects and the real-world approximations we were building. This led to a discussion of how the mathematical object can be seen as the limit of an iterative process. For many teachers, their understanding of the mathematics came only after having completed their sculpture and imagining extending it through an iterative process. The teachers found this new insight exciting, and were immediately seized with ideas on how to bring it into the classroom. They also found the craft addictive and many continued to work on their sponges during lunch!

The final two lessons were all about symmetry. In our origami lesson, we built polyhedra from a simple origami sonobe unit, and explored the symmetries of polyhedra and how to use symmetry to construct more complex origami models. We also explored the proof that there are only five Platonic solids. Our most daring lesson was a cross-town field trip to the Teece Museum of Classical Antiquities [6]. Here we examined the mathematics of meander patterns in ancient Greek and Roman art. The study of meanders is not well-explored in the mathematical literature, so we developed our own simple mathematical framework and presented it as an example of how teachers might do this themselves. We emphasised how our mathematical thinking so far could be used, including generalising observed patterns and asking questions to which we may not have an answer. We also introduced the teachers to the seven frieze groups [1]. Identifying and classifying the patterns on ancient art was such a hit that the teachers continued to identify repeating patterns outside the of museum after the session ended.

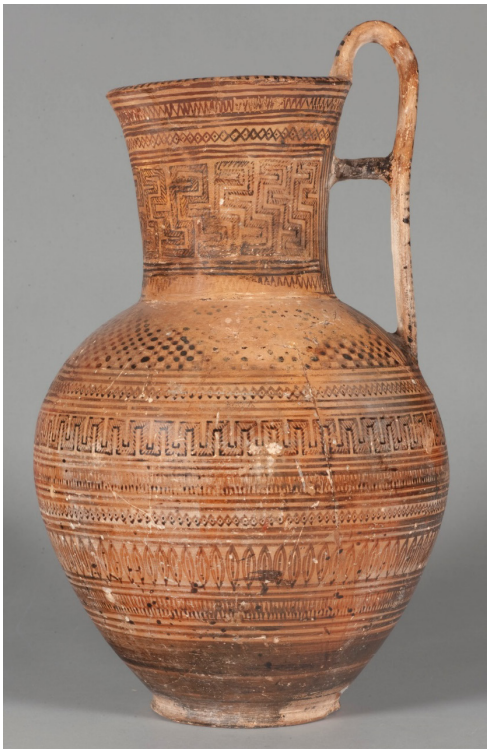

Figure 2: *Geometric Pitcher 36.56 from the Teece Museum (Copyright UC).*

Following these Maths Craft lessons, in which participants took on the role of students, we devoted an hour to debriefing and planning. Firstly, participants discussed how the Maths Craft tasks facilitated deep engagement and higher order thinking, generating principles that could support their teaching in many areas of mathematics. Secondly, participants designed lessons for their own students, adapting the activities to suit the ages of their students and the practical constraints and affordances of their classrooms.

Participants left the workshop with a portfolio of Maths Craft lesson plans with ties to the New Zealand Curriculum [5], detailed booklets containing mathematical explanations of each of the four activities, and a swag of craft material and instructional handouts to use in their own classrooms (see [3] for our free public resources). Best of all, they left buzzing with enthusiasm and full of ideas for their own Maths Craft lessons.

The Feedback

We evaluated the effectiveness of Maths Craft for teacher professional development. We collected written and oral feedback through questionnaires and group discussions, during, immediately after, and five weeks after the workshop. The feedback from the workshop was overwhelmingly positive, with one teacher saying "Best PD [professional development] in years!" Participants reported their appreciation for the way mathematical ideas and mathematical thinking were linked to hands-on craft, and for being provided with a framework in which to think while preserving the elements of discovery and surprise. For example, one teacher said: "I like the theoretical foundation and scaffolding, then leaping into the unknown."

Participant feedback shows that the workshops were helpful and directly applicable to the classroom. One participant wrote that they "wanted to reiterate how useful, educational and relevant I found the Maths Craft workshops. Thank you all for your passion and dedication!" Every participant was able to use Maths Craft in their classrooms and reported that their students were highly engaged with the activities and with the mathematics (see Figures 3 and 4). Some teachers were so inspired by our visit to the Teece Museum ("Loved the field trip!") that they went one step further and took their students on their very own field trip to the Teece Museum.

Figure 3: *Origami at Hornby High School.*

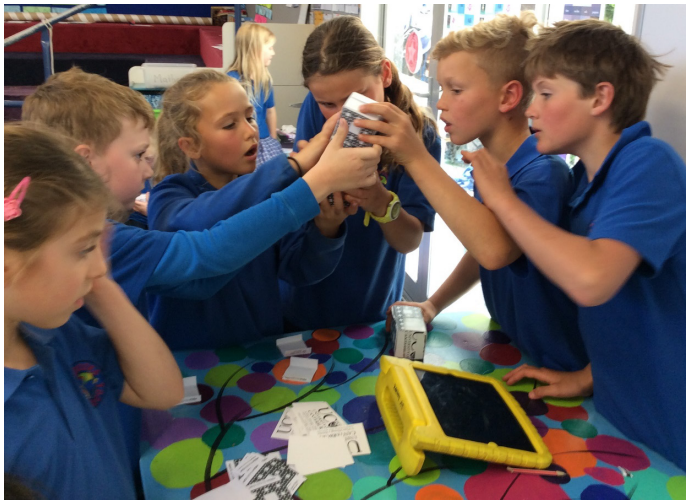

Figure 4: *A team effort at Ladbrooks School to build the Menger Sponge.*

We were especially excited to learn that some of our participants have undertaken quite ambitious projects in their schools. For instance, two high school teachers had almost 40 students ranging from Years 7-10 (ages 11-15) opt to do Maths Craft as their 'passion project'. Those students spent seven hours a week for five weeks working with the teachers on developing and extending Maths Craft projects. Students displayed their resulting work in the school and on a website they created to share their work with their families.

These two teachers also aim to run their own mini Maths Craft Day in 2019, at which the high school students will demonstrate Maths Craft to a local primary school (ages 6-11).

Further analysis of the survey data and discussion transcripts is underway to investigate the effectiveness of Maths Craft as a teaching and teacher training tool in more detail. We aim to examine how Maths Craft influences teachers' perceptions of mathematics, mathematics teaching, mathematical self-efficacy, and mathematical Pedagogical Content Knowledge. In addition, some self-reflective work of the research team (in particular the professional mathematicians on the team) is underway to document the experiences with the teachers, understand the challenges facing the primary and secondary education sector, and how future tertiary students are being prepared. What is already apparent, however, is how beneficial, inspiring, practical, and useful the participants found these workshops. One workshop participant has even created and run her own Maths Craft professional development workshop for teachers at her school based on our workshop. The workshop participants and the authors all enthusiastically decided to stay in touch and to meet up periodically throughout 2019.

Finally, we note that the participants recognised and deeply appreciated the large amount of work required to prepare and deliver these workshops. One teacher wrote "Thank you to both [Jeanette] and Phil (and team) for such a rich mathematical two days. The enormous amount of effort to develop, write and deliver maths with such a generous spirit is very impressive. I thoroughly enjoyed learning."

Conclusion

Maths Craft New Zealand is the country's largest maths outreach programme. We have run numerous festivals and workshops across New Zealand and reached over 10,000 people to date. In 2018, we created a pilot two-day professional development workshop for school teachers in which they learned four crafts, the mathematics behind them, and their links to the New Zealand Curriculum. The mathematics and craft was taught by two research mathematicians (authors McLeod and Wilson), and the curriculum links were guided by a mathematics education researcher (author Pomeroy). In collaboration with an educational researcher (author Brogt), we designed a formal qualitative study into the effectiveness of our approach, which involved seeking written and oral feedback from the workshop participants during, immediately after, and several weeks after the workshops. An initial review of the feedback shows that the workshops were overwhelmingly useful and enjoyable, with all participants using Maths Craft in class within a few weeks of the workshops, and some teachers planning their own mini Maths Craft festivals and leading their own professional workshops for colleagues. We will maintain an ongoing relationship with the participants in this pilot workshop, and will use their feedback to develop and expand this professional development programme.

Acknowledgements

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References

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