Exploring Raaga Improvisations of Carnatic Music with Mathematical Proof Writing

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Abstract

This paper draws a parallel between the creative melodic expansion of a raaga in Carnatic music (South Indian classical music) and mathematical proof writing, grounded in the enactivist theory of emergence. I describe the creative processes involved in the melodic improvisation of a raaga and correlate it with mathematical proof writing activity. This work has been informed by reflections on my own experience both as a Carnatic performing artist and a mathematics graduate.

Introduction

Understanding the mental mechanisms behind creative processes has been of much interest in the field of psychology for the past few decades. Though creativity research seems to have received far less attention than other areas of inquiry into human thinking, various theories and definitions of creativity have been proposed by many researchers [7]. Since creativity is a complex construct with many interacting dimensions, addressing it with a single definition is a difficult and demanding task [2]. However, there is a general consensus among creativity researchers that it involves the generation of novel and useful or appropriate ideas ([8], [9]). While this specification is useful, I would like to add the notion that creative processes might also involve the emergence of unpredictable and unexpected ideas, even to the creator. I see it as an imperative addition, as the unpredictable ideas may or may not be completely novel to the creator, but the unpredictability of the sequence of ideas might itself be considered a novel creative experience. In this paper, I identify some similarities between creative melodic improvisations in Carnatic music (South Indian classical music) and mathematical proof writing, using enactivist theory, and explore how the successive emergence of unpredictable actions might be a key to understand creativity.

Enactivism, a framework developed from the ideas of Maturana and Varela in 1987, mainly emphasizes the inseparability of the individual and the world [3]. It strongly dissents with the view that cognition represents 'knowing' of a world that is outside of the observer. Rather, enactivism posits that 'knowing' neither represents grasping knowledge from the outside world nor constructing knowledge inside our minds, but is the result of interactions of an individual with the environment and how both are reshaped by this interaction. It further claims that since each individual's personal history and experiences are different, the outcomes of their interactions with the environment will be different. In this way, enactivism accounts for many different behaviours exhibited by different individuals in any given situation, as the outcomes that emerge is dependent both on the environment and on the unique individual interacting with it. With this frame of reference, I view the inner workings of creative melodic improvisations in Carnatic music as successive emergence of unexpected and unpredictable musical phrases and compare it with the workings of mathematical proof writing.

Emergence of Ideas in Raaga Alapana - Carnatic Music

One of the most striking and distinguishing features of Carnatic music is the raaga system [5]. The idea of a raaga could nearly be equated with the concept of mode in the western musical system. However, a raaga is much more than just the musical notes put together as a scale. Each raaga, taking in specific combinations of musical notes, has its own colour, mood, and is much like a musical organism, with its own set of unique emotional and aesthetic characteristics. Moreover, there is also a constraint that improvisations in a raaga cannot include any other musical notes that are not a part of the raaga. For

example, improvisations in raaga ‘Shankarabharanam’, (equivalent to the major scale of western music), cannot include any musical notes not pertaining to this scale (unlike western music, that permits ‘accidentals’). Thus, improvisations in Carnatic music are confined to the ‘grammatical’ rules and clearly defined boundaries of each raaga. Carnatic music is different from other improvisational genres like Jazz, in that an equal emphasis is given also to the learning and performance of compositions, as intended by the original composers. In fact, musical compositions in different raagas could be considered as tools and capsulated versions of the respective raagas, containing many of their characteristic key phrases. So, as one learns a greater number of compositions in a specific raaga, one gets fluent in handling the raaga’s key phrases and their possible arrangements. Thus, the extrapolation of these internalized key phrases and the emergence of their unpredictable arrangements during raaga improvisations results in what we might call creativity.

‘Alapana’ is a Sanskrit word that means, ‘to speak/discourse’. The purely melodic improvisation aspect of carnatic music, unrestricted by rhythm and lyrics, is called ‘Raaga Alapana’, and is technically the expansion of a raaga in all the grandeur by expressing successive musical phrases that bring out its essence and emotion of the raaga. Since each raaga evokes a singular mood and feeling, the main aim of a musician during raaga alapana is to create such an atmosphere where the particular emotions associated with that raaga are brought out. Thus, learning the key phrases and the basics of a raaga may be thought to be analogous to the learning of alphabets, words and grammatical rules of a language. However, the actual performance of a raaga alapana is more like writing a novel or poetry, which not only requires the knowledge of language rules and meanings, but also the ability to bring ideas together through intellectual, emotional and aesthetic considerations. The expansion of a raaga through alapana is hence considered as one of the most challenging and creative outlets of Carnatic music, where the artist’s ability to portray the spirit of a raaga is put to maximum test.

Carnatic music pedagogy highly relies on learning by ear, from a Guru or a teacher. Raagas or compositions are not taught from notations visually, but aurally through sound. And so, there is no particular way to teach or learn raaga alapana, as even when a Guru expands a raaga, the unpredictable succession of phrases that emerge during one raaga alapana session might not be the same, even during the next day’s session. Thus, the musician does not know what particular sequence of phrases might emerge at any particular session of a raaga alapana. This element of suspense and unpredictability is what makes a raaga alapana such a creative enterprise. Looking at this through the enactivist lens, we could situate raaga alapana as the interaction between an artist and his environment, where at every given moment both the artist and his/her environment are being constantly reshaped by their mutual interaction. For example, when an artist performs a raaga alapana for an audience, the audience might be moved by the artist’s rendition and in turn, the artist would feel the audience’s response through their nodding and clapping and is thus motivated to be more creative.

There are many factors that come into play during raaga alapana, both from the artist’s end and the environment that he/she is situated in. From the former’s end, the artist’s knowledge base of a certain raaga and tonal flexibility/limitations are directly correlated to the quality of raaga alapana that can arise from his/her improvisation. This could be related to Maturana and Varela’s idea of ‘structural determinism’, where they say that it is the structure of an organism that allows for changes to occur, both to the organism and the environment that the organism is situated in [4]. Moreover, in our case, since we are dealing with human subjects and generation of ideas, factors other than physiological make-up matters. The psychological aspects of the artist such as mood, personality traits and aesthetic connection to a particular feeling/raaga etc., also have a huge impact on his/her musical explorations. On the other hand, the environment that an artist is situated also plays an important role in shaping up a raaga alapana. For example, exploring a raaga during a practice session at home is a completely different experience from singing at a concert stage in front of a huge audience to singing with knowledgeable peers and Gurus, who

might add to the on-going emergence of musical phrases during teaching or collaborating situations. Thus, both the artist and the environment affect each other during the performance of a raaga alapana and the creativity in generating successive musical phrases during the alapana lies in the emergence of unpredictable, (not necessarily unknown) ideas that is dependent on various factors discussed above.

Similarities with Mathematical Proof Writing

Similar to raaga alapana in Carnatic music, proof writing could be considered as one of the most challenging ventures that unwraps the imaginative abilities of a mathematician. A mathematical proof entails a systematic series of successive mathematical statements, each of which follow logically from what has gone before [1]. Similar to how the same raaga could be explored by many different artists in multiple ways based on their knowledge level and aesthetic sense, writing a proof from the same mathematical assumptions by different mathematicians highly depends on their knowledge base and aesthetic/logical decisions that they make during proof writing. Thus, the art of proof writing also seems to lie in the idea of emergence as suggested by enactivist theory. Though certain key ideas and general proof strategies may be learnt or taught beforehand, one does not get into the activity of proof writing with some preconceived strategies during every session. Also, the ideas that one uses in the successive logical argument during proof writing at one time, may or may not be the same during another session of working with the same proof.

Analogous to raaga alapana, proof writing not only depends on the mathematician, but also on the environment that they are situated in. Frequently, we find mathematicians saying that they exhibit more creativity when talking about mathematical ideas with peers. Thus, writing a proof in a setting that is completely devoid of other people might be different from writing a proof while collaborating with other mathematicians. Hence, we could possibly say that creativity in mathematical proof writing also lies in the emergence of successive logical statements, that highly depends on both the intellectual and aesthetic sensibilities of a mathematician and the specific context that they are situated in [6].

My Experience with Raaga Alapana and Proof Writing

As an insider in both Carnatic music and mathematics, I would like to shed some light on my own experiences with raaga alapana and proof writing activity. While improvising, what encourages me to start the alapana with a certain phrase of a raaga is highly dependent on my knowledge base in the raaga and what I am feeling at that given moment, in that specific context. Once the first phrase has been sung, either the aesthetics, emotionality or logical patterning of this phrase informs what the second phrase would be in the sequence. This phrase now affects the third phrase similarly, and so on. The sequencing of these phrases is emergent on the spot. While the individual phrases may or may not be completely novel to me, the emergence of these phrases in a particular sequence at a given time and a space is a completely novel experience. In other words, the totality of a raaga alapana with unpredictable sequencing of emerging numerous known or unknown raaga phrases by the musician might be called as creativity in raaga alapana.

Equivalently, what encourages me to write down the first statement of a mathematical proof is highly dependent on my knowledge base about the topic and how I view the assumption at that particular time and context. Once the first statement has been written, it informs the second statement through aesthetic or logical deductions and successive statements thus emerge. As in the case of a raaga alapana, the individual statements in a proof may or may not be completely novel, but it is the totality of the proof that makes it a novel experience. And so, from my experience, I deduce that creative processes in raaga alapana could be analogous to mathematical proof writing and hence, proof writing is not just a mechanical activity, but a process that lies at the emergence of unpredictable logical statements.

To illustrate this idea, let us consider some different ways of approaching the proof of uncountability of the real number set R. One of the most common proofs that we come across is by contradiction, i.e., by

assuming [0,1] to be countable and then proceeding from this assumption using the diagonalization argument to show that [0,1] is uncountable and hence R is uncountable. However, there might be few other formal or intuitive ways to proceed with this proof. For example, one could try to write a topological proof, starting with the fact that [0,1] is a compact Hausdorff space with no isolated points. This assumption may lead to a different unfolding of the proof as a whole and would involve unpredictable statements emerging at each point, according to the respective assumption with which it was started. Another intuitive or informal way of proving this would be proceeding with the idea that rational numbers are countable. This would lead to a completely new path to envision this proof, much different from the earlier two proofs mentioned.

Thus, the initial assumption made by a mathematician plays a huge role in determining the path of the proof, that includes emergence of unpredictable statements at each step. This could be mapped to the description of raaga alapana described earlier, as the initial phrase sung by a musician informs the successive phrases and the emergence of these phrases in a particular sequence depends upon the knowledge base, personality, mood and other external or internal factors affecting the musician at any given space and time. Connecting both these ideas, we could say that writing proofs also depends on a lot of factors involving the mathematician and the situation that they are situated in, making it as a result of their mutual interaction, as suggested by the enactivist theory.

Conclusion

This paper is an endeavour to view creativity through an enactivist perspective by relating ideas of creativity in raaga improvisation to mathematical proof writing. It gives an idea that though mathematical proofs appear very static and organized at the outset, their inner workings are much like the melodic expansion and improvisation of a raaga, whose heart lies at the emergence of successive musical phrases by a musician. I conclude that creativity in both proof writing and raaga alapana, depends on how a particular individual is impacted by intellectual, aesthetic and emotional dimensions of each part of the respective activity and how he/she further deforms the environment and is inspired at each step.

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