Creating Deltahedra with Unfolded Net of Tetrahedron

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Abstract

This paper presents a new origami-inspired design to create polyhedra. By using the unfolded net of tetrahedron as modular units, one can assemble a variety of deltahedra. This design was used to engage students in geometry classes by having them build polyhedral models from specially designed cardboards. Through this exercise, students learned about the alternative features of polyhedra in a hands-on manner and appreciated the dynamic interaction among math, origami, and art.

Introduction

Origami designers typically need to pre-fold many identical modular units [4,5,6], a very time-consuming process unsuitable for classroom use. I developed special cardboard units for students to efficiently make deltahedra [1,8], incorporating origami techniques. This design is intended to encourage students to think spatially while making geometry-based artwork. It was used to promote the Math-Maker course of The Affiliated Senior High School of National Taiwan Normal University. Students learned the relationship within polyhedral structures by being makers themselves. The exhibition of student's works had the slogan of "Math, Origami, and Art."

Basic Design

Figure 1*a*, a simple origami-inspired design, shows the initial step of using such modular units. One can create bipyramid, octahedron, and some elevated polyhedra [7], by assembling these basic units, as shown in Fig. 1*b*.

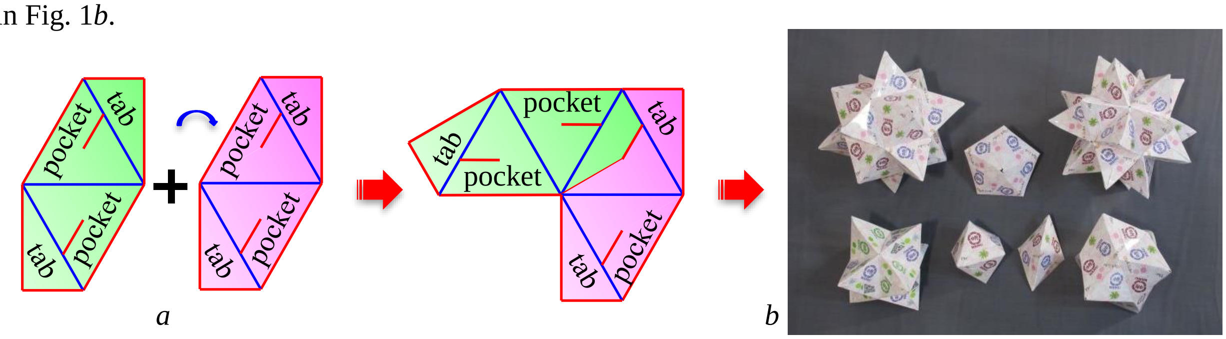

Figure 1: *Unit diagrams (a) and the works of deltahedra (b).*

Improved Design

For regular tetrahedron and icosahedron, the mirror-symmetrical part unit from the basic design above is used. I modified the tabs and pockets of Fig. 2*b* so it can be combined with Fig. 2*a* to form a new unit, Fig. 2*c*. Since Fig. 2*c* is the zipper unfolding of tetrahedron [3], and the tabs and pockets are arranged to interlock, according to the zipper principle of unfolded polyhedron [2], it is now easy to complete the models, following the equal segmentation of the unfolded nets and the interlocking tabs and pockets. Fig. 2*d* shows the net of octahedron combined with two units; Fig. 2*e* shows the net of icosahedron combined with five units.

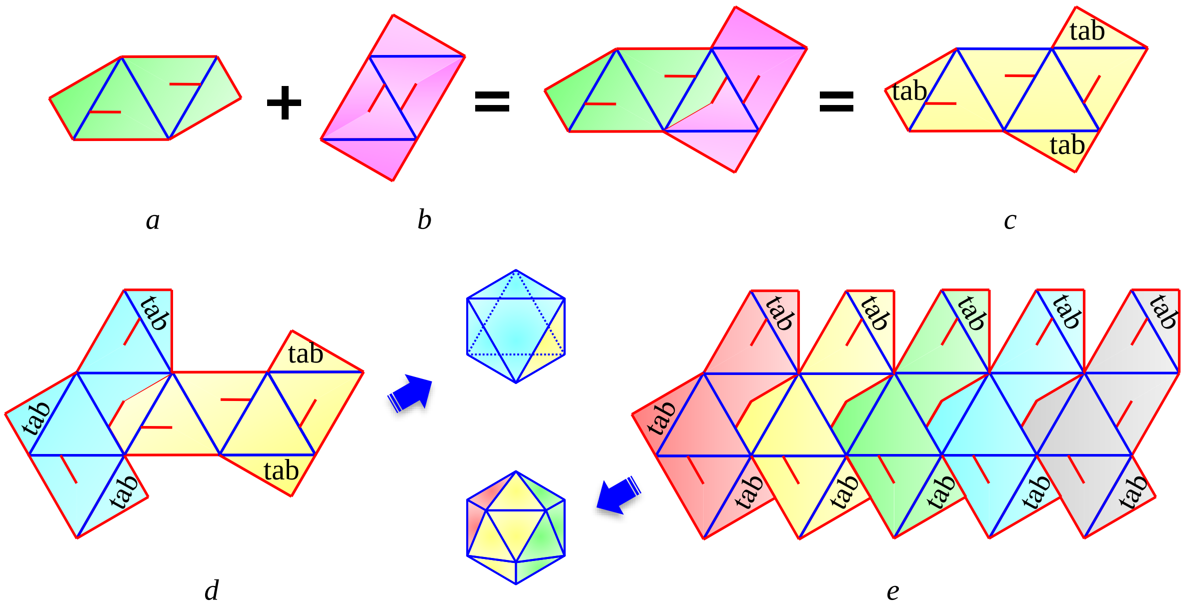

Figure 2: *The nets of octahedron (d) & icosahedron (e) made up with the nets of tetrahedron (c).*

Assembling an elevated polyhedron with these units requires more spatial thinking. One strategy is to first fold the units into Fig. 3*a* and its mirror-symmetrical counter-part Fig. 3*b*, and then further combine them into an elevated tetrahedron (Fig. 3*c*) and an elevated octahedron (Fig. 3*d*), similar to the common way of combining origami units with the interlocking tabs and pockets. If the final assembled structure is loose, the cardboard is over-stressed by the valley-folded structure. In this case, glue or double-sided tapes can be used to fix the problem.

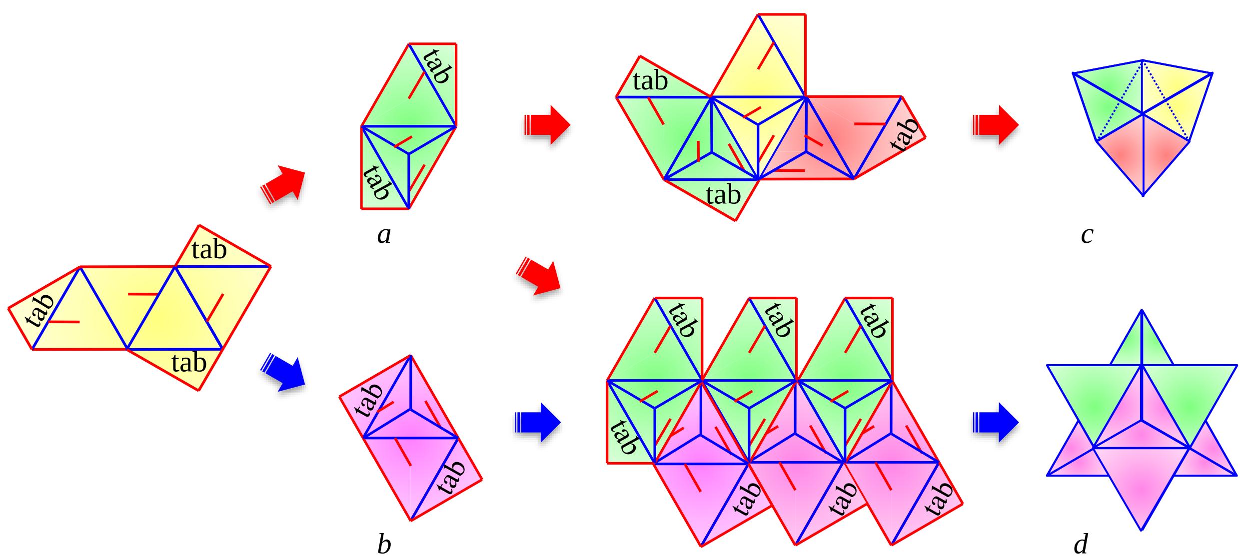

Figure 3: *Assemble elevated polyhedron based on unit origami.*

Even more challenging is the elevated icosahedron. One can first make up a “southern hemisphere” (Fig. 4*b*) and a “northern hemisphere” (Fig. 4*a*) with 5 units (Fig. 3*a*) and 5 mirror-symmetrical units (Fig. 3*b*), respectively. Then the “equator” (Fig. 4*c*) can be created with 5 unfolded units. What remains is to connect the two hemispheres with the equator with the interlocking tabs and pockets.

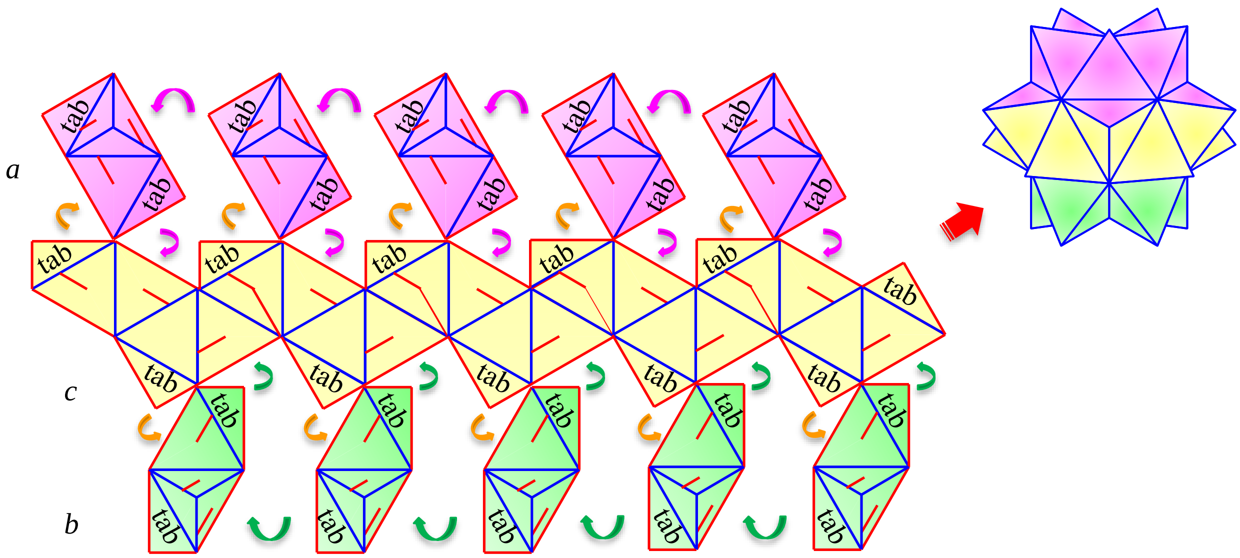

Figure 4: *Assemble elevated icosahedron based on a globe.*

Improvement

After several iterations of design, the unit is more multifunctional. I added more fold lines to the schematic diagram to create two types of tiling units, which bear consistent mirror symmetry with each other for most economical use of the layout. One unit can be folded at the trisection of edges (Fig. 5*a*); the other can be folded at the bisection of edges (Fig. 5*b*). With these multifunctional units, it is now possible to construct variations of Archimedean solids, with craters instead of “vertices” (Fig. 5*c*). The template of the final design, used for mass production, with Sierpinski triangle or Koch snowflake as decoration, is attached below (Fig. 6).

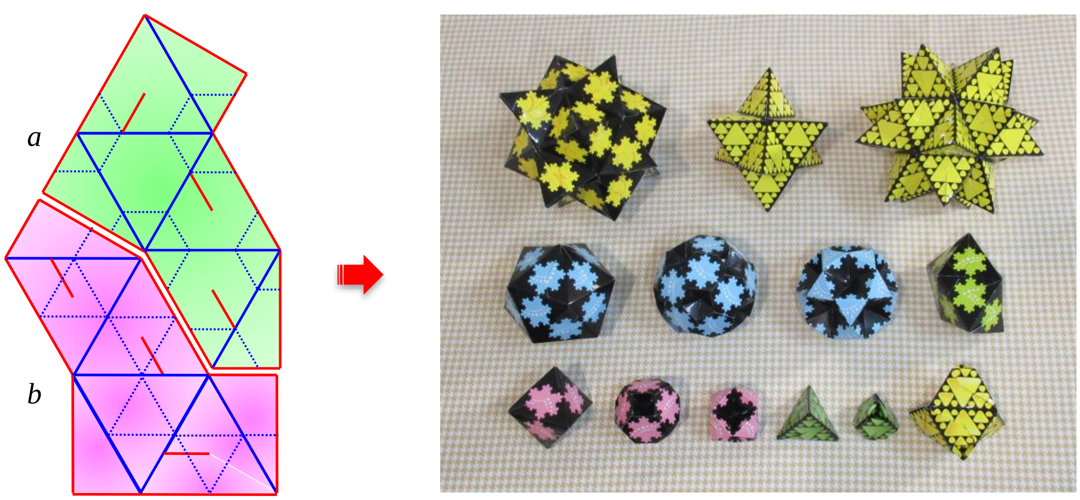

Figure 5: *Multifunctional units (a, b) for more polyhedra.*

Conclusion

This paper explained how to use multifunctional units to construct various types of deltahedra. The process of folding and combining the units into deltahedral structures engages the learners to think spatially while they make satisfying art with their own hands.

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Attachments

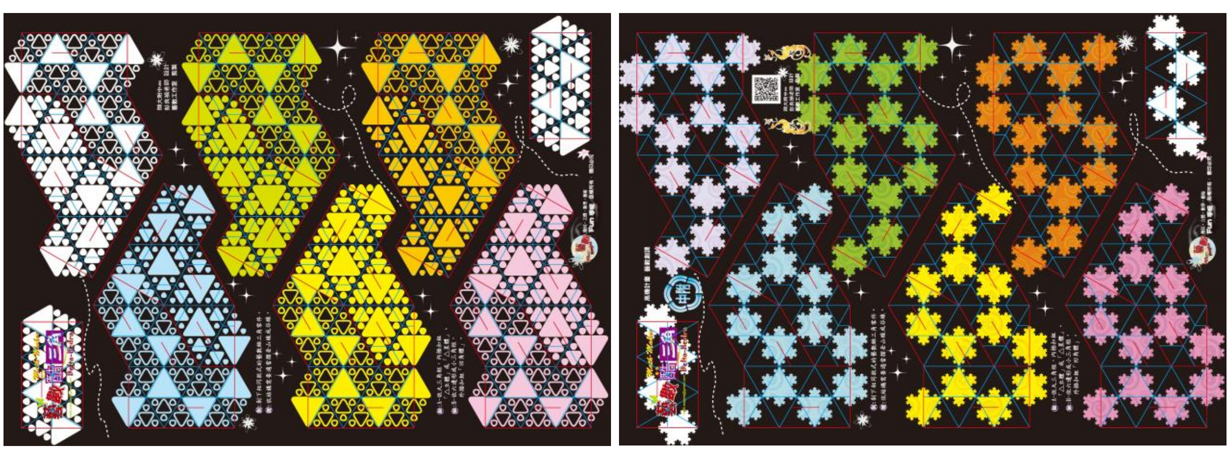

Figure 6**:** *The final design cardboard with Sierpinski triangle or Koch snowflake as decoration.*