Teaching Advanced Mathematical Concepts with Origami and GeoGebra Augmented Reality

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Abstract

This paper offers an educational approach connecting hands-on and computer-based activities for exploring profound mathematical concepts and their applications. The described approach was applied in the final year of high school lessons in Serbia. During the application we observed that the visual representation of objects in real and virtual worlds benefits students' understanding of advanced mathematical concepts. We combined realistic, origami-made models to investigate calculus applications and modeling in augmented reality with the educational software GeoGebra.

Introduction

In this paper, we present an educational approach that combines hands-on activities and computer-based visualizations. During the past year, we applied this approach in numerous mathematical lessons and reported initial results of these activities in other publications [5],[6]. In our approach, hands-on activities are based on origami and paper folding, while computer-based activities utilize the educational software GeoGebra (www.geogebra.com).

Joining these two approaches could help us overcoming weaknesses of single approach and complement possibilities that could enrich students' experiences in learning advanced mathematical concepts. On the one hand, origami has strong connection to mathematics [1], [8], [13] and valuable educational potentials [4], [5], [6], [7]; on the other hand, possibilities of connecting a computer software and its utilization in classrooms have not extensively examined. In the technological side, besides utilizing basic GeoGebra features, such as computer-algebra system or 3D graphics, there are now possibilities of involving mathematical modelling with augmented reality (AR). Augmented reality is a current technological trend heading into educational uses, but still not commonly applied in classrooms [2], [3], [9], [10], [11], [12], [20]. In order to explore the possibilities of AR in classroom activities and offer concepts for its applications we organized classroom activities that could illustrate the potentials of AR in mathematical lessons in association with hands-on activities as well as show new aspects of teaching mathematical concepts.

Maximizing the Volume of a Box to Illustrate Mathematical Concept

In order to demonstrate the approach, we chose the problem of the maximizing the volume of a box. The problem is known as a good example for illustrating practical applications of advanced mathematical concepts and calculus [14], [18], [19]. We applied this approach with the final year (18–19 years old students) of high school "Petro Kuzmjak" in Serbia. Participating students had strong background in calculus, due to the curriculum requirements for the final year of high school in Serbia [15].

The two joint lessons were organized in three segments: making boxes with origami techniques, investigating calculus applications with real-world problems based on the volume of the box and with the use of paper folding and computer-algebra, 3D graphics and modeling with augmented reality. During the activities, students received several tasks: folding a paper masu box with origami techniques; finding the height of the maximal volume masu box; and discovering mathematical equations that would model the shape of masu box in augmented reality. Traditionally, masu boxes were used for measuring rice in the feudal period of Japan. Masu box could be easily folded from a square sheet of paper, which was the starting activity for students. Figure 1 shows the masu boxes folded by students and crease pattern is showed in Figure 2.

Figure 1: *Paper masu boxes folded by students*

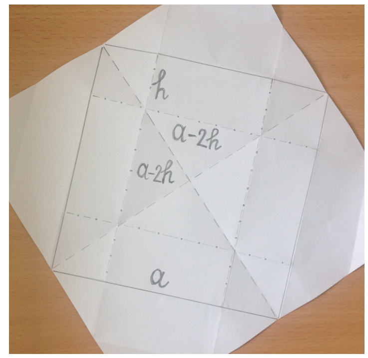

Figure 2: *Crease pattern of masu box*

Modeling Shapes of the Box by GeoGebra Augmented Reality Application

In order to find the solution of the problems, students analyzed the crease pattern of the masu box and used GeoGebra's computer-algebra support. The analysis of the masu box crease pattern showed that the volume of a box that was folded from a square sheet of paper with the side length *a* could be described with the mathematical function (1) $V(h)=4h^3-4ah^2+a^2h$, where *h* was the height of the box. Students obtained formula by applying prior knowledge in calculating the volume of a prism and multiplying the height and the area of box base which in this case was square with side length $a-2h$. Further on, students graphically represented (1), calculated the first derivative of (1) which was (2) $V'(h)=12h^2-8ah+a^2$ and found the local extremes with help of GeoGebra. As (2) is the quadratic equations, they obtained two solutions where one represented the maximum height value of the box. In Figure 3 we can see the application of GeoGebra and the solution created by students.

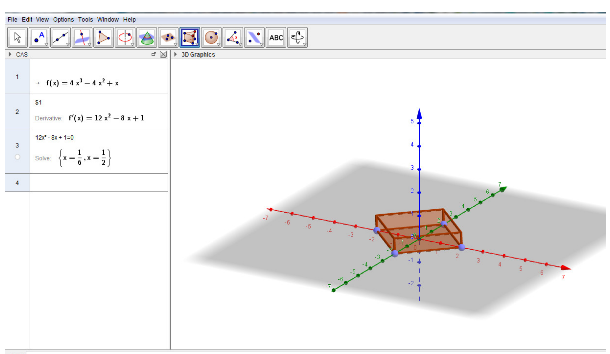

Figure 3: *Application of GeoGebra in calculating maximum volume*

After finishing the task, students briefly explored the GeoGebra AR application and its input functions to discover graphs that would model a static masu box. Students first explored the basic embedded shapes, and later on they were encouraged to model the surface that would correspond to the masu box. Results were obtained by trial and error, and after several attempts students successfully obtained AR models that could overlap parts of the masu box. To obtain results students needed to connect previously learned mathematical concepts with objects in real life in order to create AR-supported models. This offered students insights into understanding physical objects, such as a box previously made from paper that could be modeled with the knowledge of mathematics. In this case, students needed to know how to analytically write equations of lines or planes, to understand their coordinate representations, and to describe plane intersections. In Figure 4, we can observe parts of the modeling process using the GeoGebra augmented reality app.

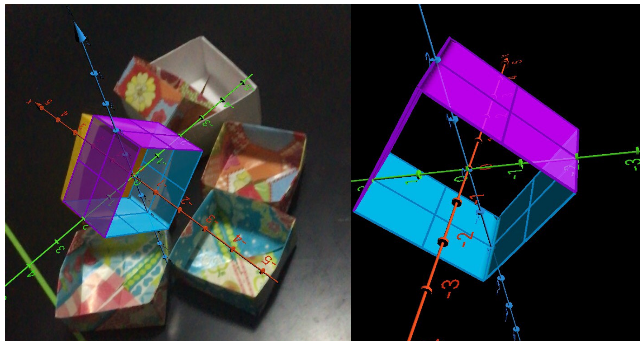

Figure 4: *Modeling the shape of the masu box using GeoGebra augmented reality app*

Conclusions

It is true that students like technology, but they also like physical models, and combining different approaches could enrich both the teaching and the learning processes. Activities performed in this approach offered students opportunities to visualize mathematical ideas in real and virtual environments, which made their previous knowledge applicable and significant. Each of the elements–origami, GeoGebra and augmented reality–contributed to the lesson quality and meets different requirements of students.

In this approach we engaged students in mathematical and logical thinking, spatial reasoning, and manual activities. The AR application was especially valuable in helping to introduce collaboration, combination of digital information, and mobile computing, but also it was intuitive interaction together with the mathematical concepts. It allowed students to receive computer generated ideas that supported their hands-on activities based on origami. Students practiced applications of theories in the real-world situations by making paper-folded masu boxes and experienced physical models, but also had a virtual experience in technology supported environments. It was the first time that students learned about AR, but they immediately showed positive attitudes towards the learning activities and exhibited great interest.

The process of building virtual objects, we believe, complemented and expanded students' knowledge, supported interaction between real and virtual worlds and provided dynamic tasks overlay, which would be difficult otherwise. Our work also had some constraints, such as GeoGebra AR application availability on students' phones, thus students had to work collaboratively in groups. Also, in some cases students could only deal with two functions in the z=f(x,y) form due to restrictions of the software. On the one hand, this was an obstacle because some adjustments were needed. On the other hand, making adjustments served as a good mathematical exercise. Nevertheless, the new version of GeoGebra AR this will be resolved and new challenges could be offered for students.

Using augmented reality features helped students to visualize mathematical object such as a prism, which in real life could be connected to a box. Also, it was possible to walk inside the virtual box and take screenshots from different points of views, which was a unique experience for students. It is likely that AR's influence on technology development is growing as it is considered as a technology that could have substantial impact on education, but it needs to have a meaningful connection to the educational content [2], [3], [11]. In this example, we added the use of AR to the curriculum content by following current trends in this segment of education [16], [17].

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