Shape-Partitions: New Elements, New Artworks

James Mai

School of Art, Illinois State University, Normal, IL, USA; jlmai@ilstu.edu

Abstract

The development of shape-partitions is explained as graphic adaptations of integer partitions, producing forms that the author incorporates in artworks. Shape-partitions expand the possibilities of integer partitions by producing multiple graphic appearances for single integer partitions. The complete sets of shape-partitions for a triangle containing 6 points and a rhombus containing 9 points are described, categorized, and illustrated. Artworks utilizing these shape-partitions are reproduced, accompanied by discussion of the color and composition strategies used to foster a fundamentally visual apprehension of mathematical order.

Introduction

Much of my work as an artist involves the combinatoric development of sets of forms which are used to populate my compositions. I systematically vary certain properties of a base shape, which yields a set of forms whose members are at once related and distinct. One of the most important goals of this work is to generate complete form-sets which include all forms allowable within the combinatoric parameters. Equally important, I reduce the set to its minimum number of distinct forms by eliminating any member that is identical to another after reflection or rotation. The result is a *minimum-complete* form-set. Forms from the set are then selected and composed in artworks. The structural features of the forms and their coherence as a set are themselves put forward as aesthetic content in my artworks—they are intended to be seen and known. The visual apprehension of order is the central priority that guides decisions about color, scale, position, and more in the artworks. In short, the mathematical features become aesthetic features.

Integer partitions have become particularly interesting to me as a new approach to producing form-sets for my studio work. Although integer partitions are mathematical relationships of numbers, they are in many ways like my graphic form-sets: each partition is a distinct combination of integer components, all partitions are similar in arriving at a common sum, and the set of all partitions of a given number is exhaustive and complete. These characteristics seem appropriate and adaptable to the purposes of my artworks provided the number relationships can be translated into shape relationships. Most of my work begins with drawing and diagramming and only later does mathematics emerge to organize and describe the results; I typically start with graphic form and let the mathematics follow. This paper presents a different approach, proceeding in the opposite direction, where I start from a mathematical idea and produce an aesthetic realization of it—and perhaps an expanded interpretation of the mathematical idea itself

Ring-Partitions

I began to work with *ring-partitions* before I was aware that integer partitioning was a formal mathematical subject. I was interested in finding the different ways to divide a series of six points arranged equidistantly around a circle. The composition *"Ring Partitions"* (Figure 1) displays the complete set of ring-partitions of 6, here shown as loops encompassing black dots (for better understanding of the illustrations, the reader is encouraged to consult the electronic, color version of this paper). Each of the five larger circles hold ring partitions with the same degree of symmetry (6-, 3-, 2-, 1-, and 0-axis reflections). Alternatively, forms may be grouped by the number of elements in the partitions and these are denoted by common colors (e.g., the three ring-partitions with three components [3,3,3], [3,2,1], and [4,1,1] are colored violet). In total, I found twelve distinct ring-partitions, but I subsequently learned that there are eleven integer partitions of 6—had I made a mistake? The difference between my ring-partitions and mathematical integer partitions lay in there being two graphic expressions of the single integer partition [2,2,1,1]; the two forms, one upper right and one lower left, are distinct arrangements of the same component parts, each with a different symmetry and a different sequential order: upper right [2,1,2,1] with two axes and lower left [2,2,1,1] with one axis. This suggested an interesting opening to expand integer partition sets by converting them to shapes. However, the ring-partitions yielded only one additional form. I assumed this minimal increase was due to the essentially linear nature of ring-partitions (a linear sequence of partitions bent into a ring) and sought to apply these partitions to more fully 2-dimensional forms and compositions.

A clarification of terminology is probably called for here: The word *composition* is also used in mathematics to describe different permutations of numbers in an integer partition, and in that sense there are 32 distinct compositions for the integer partitions of 6. However, in this paper *composition* refers to the visual artist's concern with 2-dimensional relationships of shapes and colors, both in the arrangement of component shapes of a graphic partition and in the organization of elements in an artwork. In this sense of composition, I am guided by both the principles of visual perception and the capacity for color, texture, scale, location, and more, to encode multiple mathematical features in the same artwork.

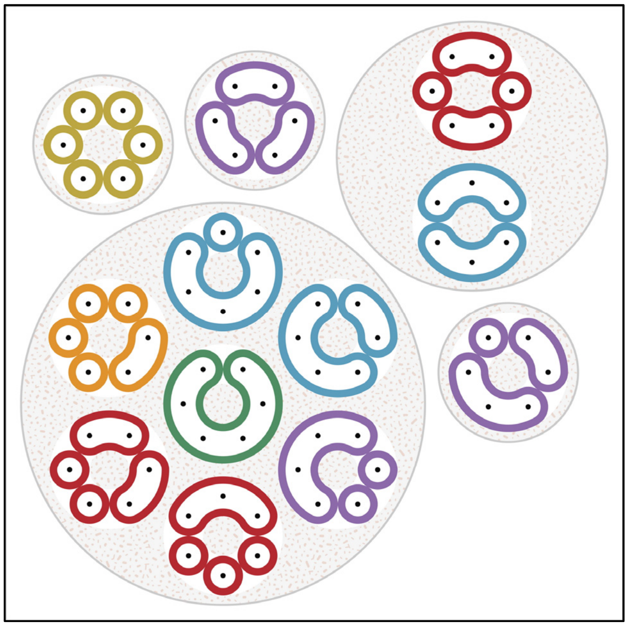


Figure 1: *"Ring Partitions" 2016.*

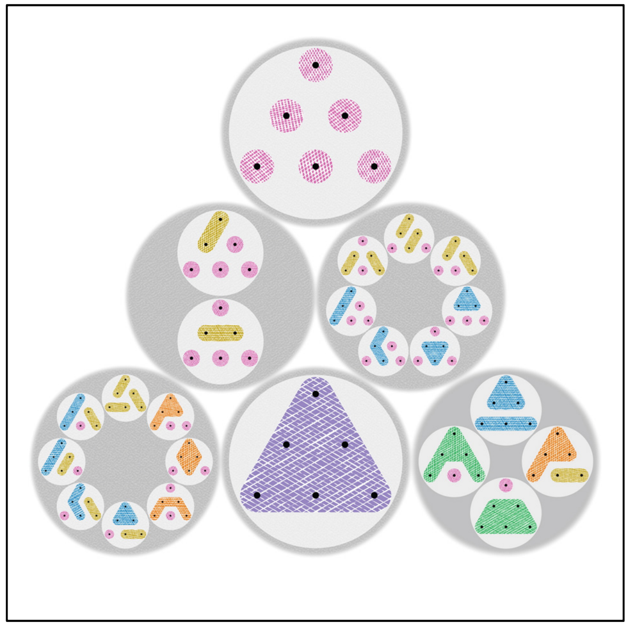


Figure 2: *"Stack (partitions of 6)" 2017.*

Shape-Partitions of Triangle-6

Intending to avoid the limitations of the ring-partitions, I distributed six points in a triangle (a *Triangle-6*) instead of a circle (see Figure 2). This produced graphic partitions both more varied and more fully two-dimensional (now called *shape-partitions* to replace *ring-partitions*). The triangle has three points along each edge, creating a triangular grid whose grid lines determine the shapes that envelop points. The combination of points determines the component shapes and the combinations and arrangements of component shapes determine the shape-partition. The set of component shapes can be considered as a "vocabulary," and the compositional arrangements of those shapes (the shape-partitions) can be considered as a "syntax." While the former must be understood, the latter is my ultimate concern as I seek the complete set of distinct shape-partitions for a given polygon with a given grid frequency.

The rules for making the component shapes in a Triangle-6 are simple: (A) A component shape can encompass between one and six points (*1-Shape*, *2-Shape*, *3-Shape*, etc.). (B) A component shape may encompass only adjacent points on the triangular grid. (C) The edges of a component shape are constrained to the angles of the triangular grid. Similarly, the rules for the shape-partitions, the arrangements of component shapes in the Triangle-6, are straightforward: (A) Component shapes cannot share points or overlap. (B) Each of the six points must be used in a shape-partition. (C) Component shapes may be rotated or reflected within the constraints of the triangular grid. (D) No shape-partition should repeat another after reflection or rotation.

There are eleven possible component shapes for the Triangle-6 (see Figure 3). Combining these shapes within the same Triangle-6 according to the previously stated rules yields 23 distinct shape-partitions for the Triangle-6. In *"Stack (partitions of 6)"* (Figure 2), I used color, size, and clustering to visually encode multiple levels of structure in this shape-partition set. Each color denotes the value of the component shape (e.g., all 2-Shapes are yellow, all 3-Shapes are blue, etc.). Each of the 6 larger circles holds partitions with the same number of parts (e.g., top circle, 6 parts; middle-left circle, 5 parts; etc.). Fortunately, the 23 shape-partitions form six groups when selected by the number of parts, which restates at a larger compositional level the Triangle-6 of the smaller shape-partitions themselves. Although the large-scale composition need not be a triangle, I usually favor such a correspondence between parts and whole because it possesses an integrity of form across scales. For my sensibility as an artist, such self-similarity is as much an aesthetic value as a mathematical property.

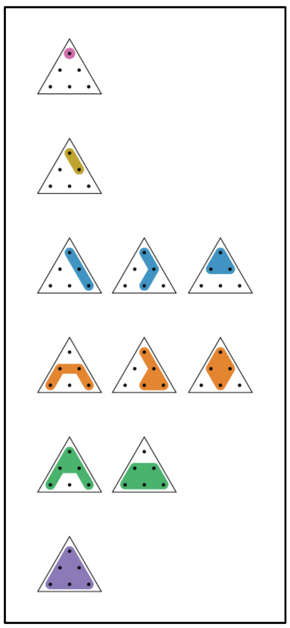


Figure 3: *Triangle-6 component shapes.*

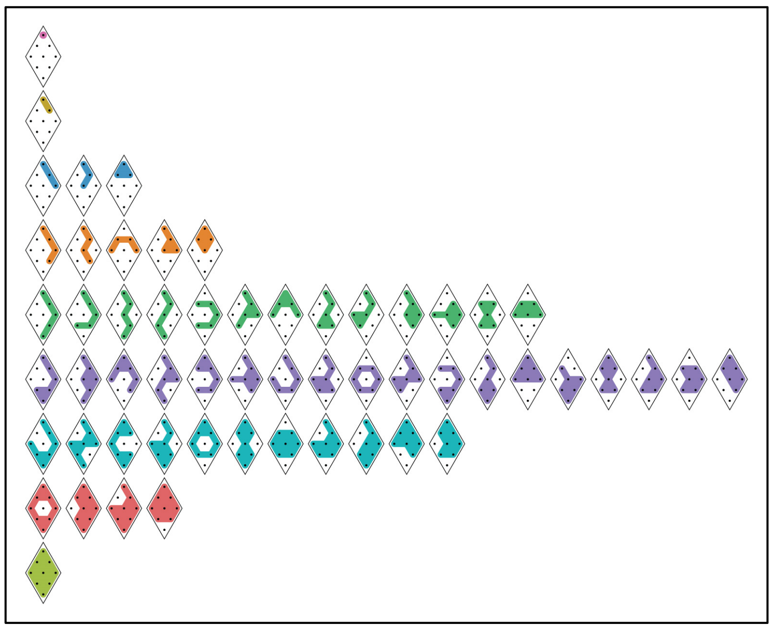


Figure 4: *Rhombus-9 component shapes.*

The graphic advantages of shape-partitions over ring-partitions were clear: there were both more varied component shapes and more varied arrangements of the component shapes. The 23 shape-partitions of Triangle-6 represented a marked increase over the twelve ring-partitions and the eleven integer partitions of 6. The larger number and variety occur for two reasons: (A) There are multiple shape possibilities for the 3-Shape, 4-Shape, and 5-Shape (e.g., the 3-Shape can be a straight line, a "bent" line, or a triangle—see the third row of 3-Shapes in Figure 3). (B) There are multiple arrangements of the same combination of shape components (e.g., there are four possible arrangements of the component shapes for partition [3,2,1]—see the four left-most forms in the lower left circle in Figure 2).

Shape-Partitions of Rhombus-9

After the Triangle-6 it seemed a natural next step to examine a *Rhombus*-9 since it would preserve the triangular grid and continue to use the triangle's eleven component shapes as part of a new set of rhombus component shapes. The rules for making component shapes and for combining and arranging them in the Rhombus-9 are the same as those for the Triangle-6 described above. Predictably, the increase in points from six to nine expands the size, number, and variety of component shapes, which totals 57 (see Figure 4). Some of these component shapes are capable of rotation, reflection, and repositioning within the rhombus, which further expands the total number of distinct shape-partitions of the Rhombus-9.

After elimination of symmetrical and rotational redundancies, the minimum-complete set of Rhombus-9 shape-partitions is 593 (see Table 1). Given the 30 integer partitions of the number 9, this large increase to 593 shape-partitions of Rhombus-9 was a surprise. By comparison, the 11 integer partitions of the number 6 increased to 23 shape-partitions of Triangle-6. This greater increase in Rhombus-9 partitions, of course, results from the many distinct arrangements of shapes for 28 of the 30 integer partitions. As an example, the largest increase from an integer partition to shape-partitions is [3,2,2,1,1], with 67 distinct arrangements of shape components (see Table 1). The set of 593 forms, as a whole, is far too large and unwieldy for my purposes in visual composition, which requires much smaller groups of forms.

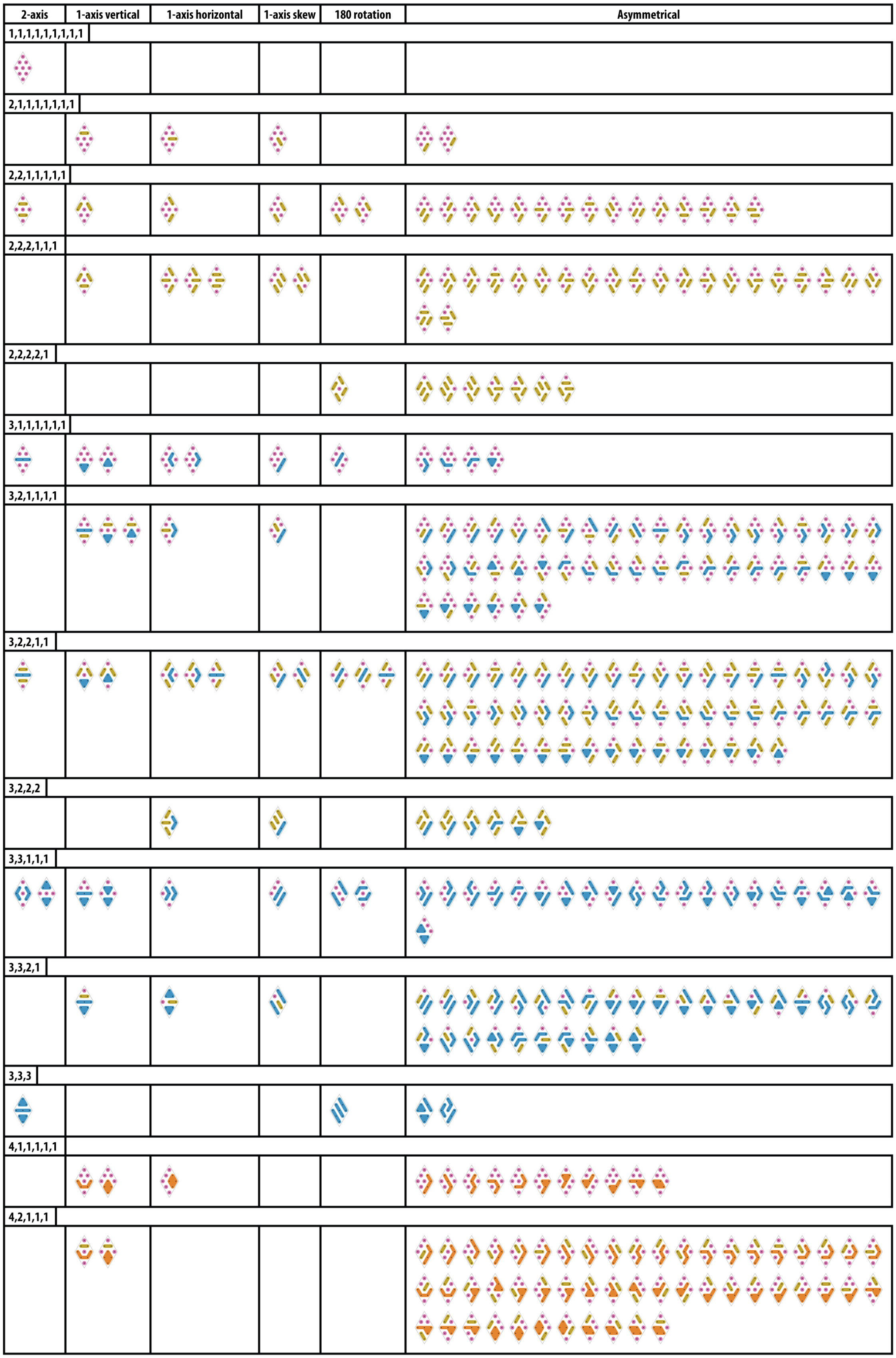
At this juncture I should explain briefly how the number of forms impacts decisions in visual composition. I have found that the typical upper limit is about 24 forms in a composition for clear perceptual recognition of the structural relationships; an ideal range is about 12 to 18 forms. Depending upon the complexity of the forms, it is sometimes possible to use 30 or more. These conclusions have been inductively derived over years of composing artworks with careful attention to the visual apprehension of multiple levels of order, but my conclusions are also informed by the experimental results of vision science and the gestalt principles of visual organization. To adapt a set this large to the needs of the artworks, I use various criteria to select subsets, and the Rhombus-9 form-set offers interesting subsets for composition.

Modes of Selection

The most obvious and direct Rhombus-9 subsets are the 30 partition groups (horizontal rows in Table 1). Some of these subsets are too large for my compositional purposes, but many of the partition subsets offer numbers of forms that are within the above-mentioned upper limits and ideal totals that I seek for compositions (21 of the 30 partition subsets possess 22 or fewer forms, and 17 of those possess 18 or fewer forms).

Perhaps the most important subsets for my purposes are those comprised of symmetrical shape-partitions (first five vertical columns in Table 1). The Rhombus-9 symmetrical shape-partition groups are: 2-axis (vertical and horizontal) mirror symmetry, 1-axis (vertical) mirror symmetry, 1-axis (horizontal) mirror symmetry, 180-degree rotation-only symmetry, and a "skewed" 1-axis mirror symmetry. The last of these, the skewed symmetry, includes those forms with an axis extending from the middle of one edge of the rhombus to its opposite edge (see the fourth column in Table 1). These subsets of symmetry and rotation offer smaller, coherent groups of forms that are especially useful for my artworks. Because our visual system so readily recognizes symmetry, the structural features of symmetrical forms are more easily distinguished and compared. In effect, each symmetrical form has a distinct identity, facilitating visual comparisons and recognition of both the combinatoric conditions of the system and the completeness of the set/subset in each composition. Still, the Rhombus-9 symmetrical and rotational partitions number 113 forms, which is larger than most other whole form-sets with which I have worked. While the symmetry subsets of 2-axis forms, 1-axis-skewed forms, and rotational forms have manageable totals (13, 13, and 17 forms, respectively) for my composition purposes, the 1-axis-vertical forms and the 1-axis-horizontal forms (36 and 34 forms, respectively) are probably too large to be used as such and will likely require further subdivision.

The asymmetrical forms also offer interesting potential, especially in presenting greater challenges to our perception of structure—that is, making us look more carefully for the order underlying the forms. This challenge to perception is desirable because it stimulates active rather than passive seeing—the viewer must actively *discover* the orders among the forms in an artwork rather than passively *accept* an obvious, illustrated order (for example, compare how we visually assess Figure 7 and Table 1).


2-axis
1-axis vertical
1-axis horizontal
1-axis skew
180 rotation
Asymmetrical
1,1,1,1,1,1,1,1,1
2,1,1,1,1,1,1,1
2,2,1,1,1,1,1
2,2,2,1,1,1
2,2,2,2,1
3,1,1,1,1,1,1
3,2,1,1,1,1
3,2,2,1,1
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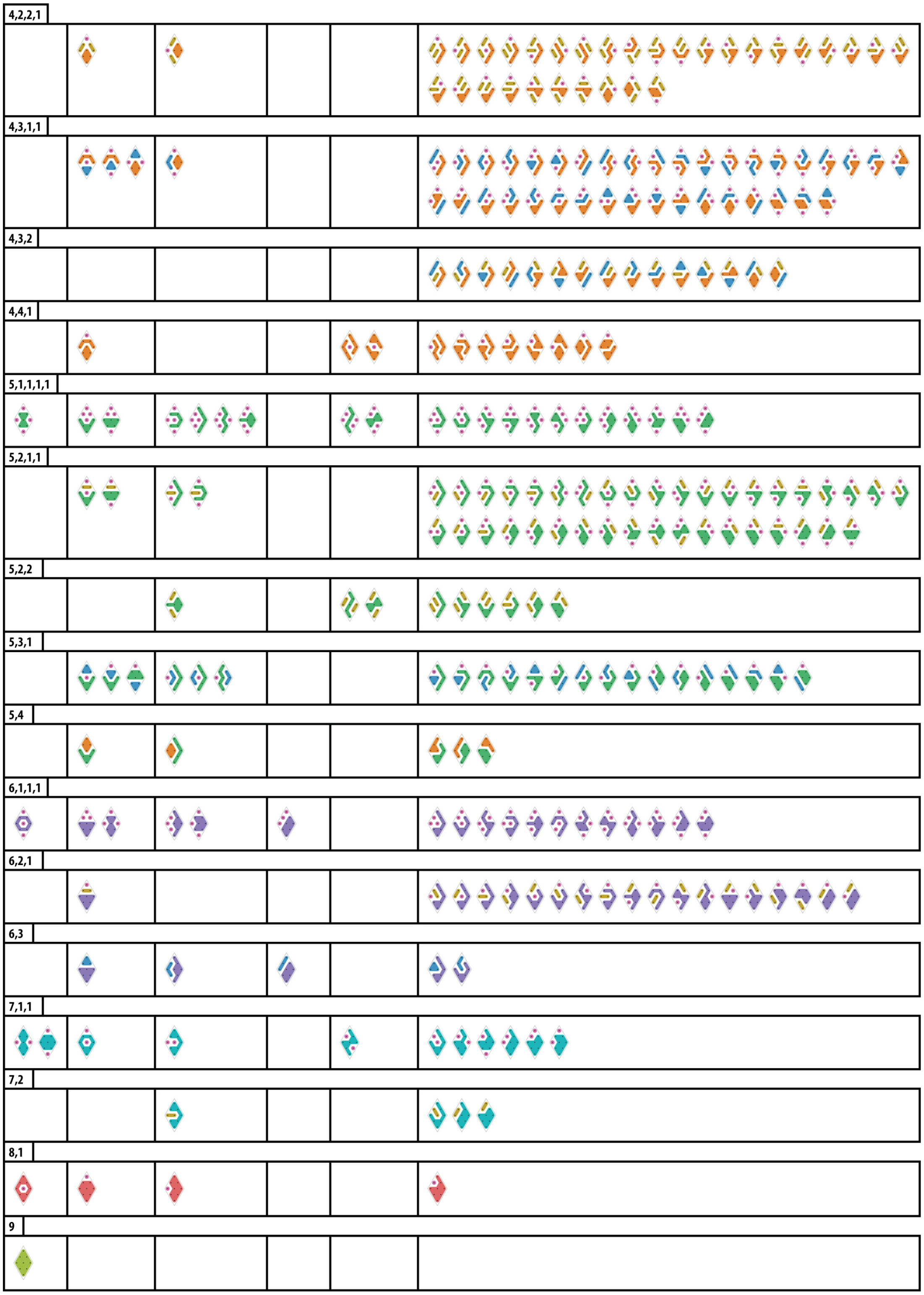

Table 1 (previous page and above): *Minimum-complete set of 593 shape-partitions of Rhombus-9.*

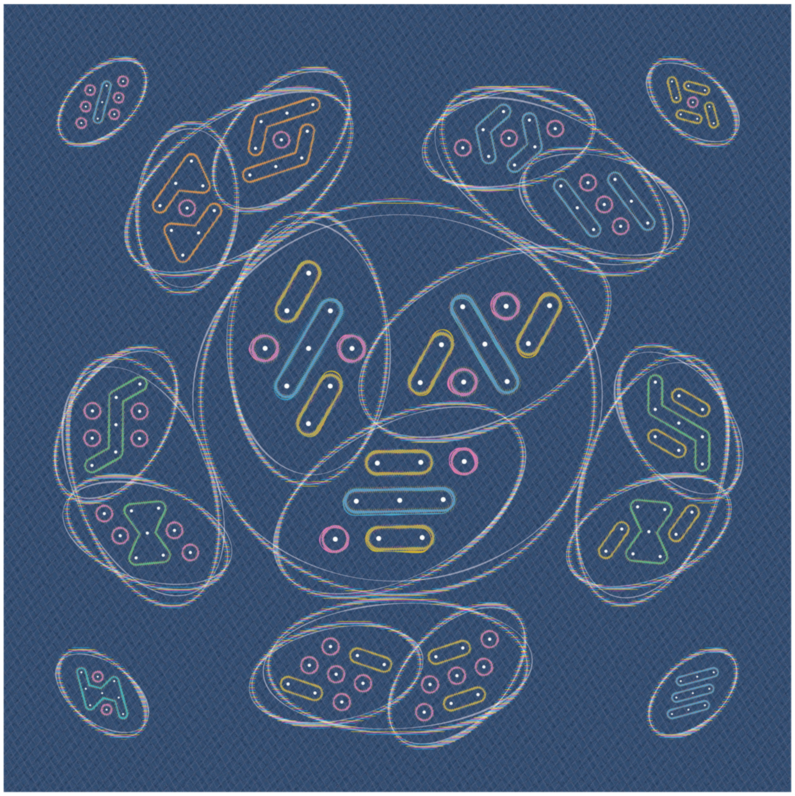

Figure 5: *"Whorls" 2019.*

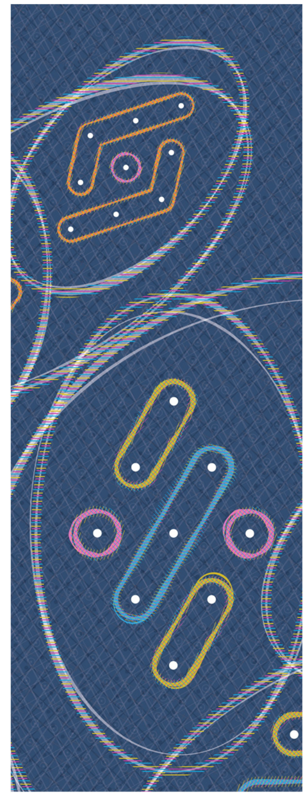

Figure 6: *Detail of Figure 5.*

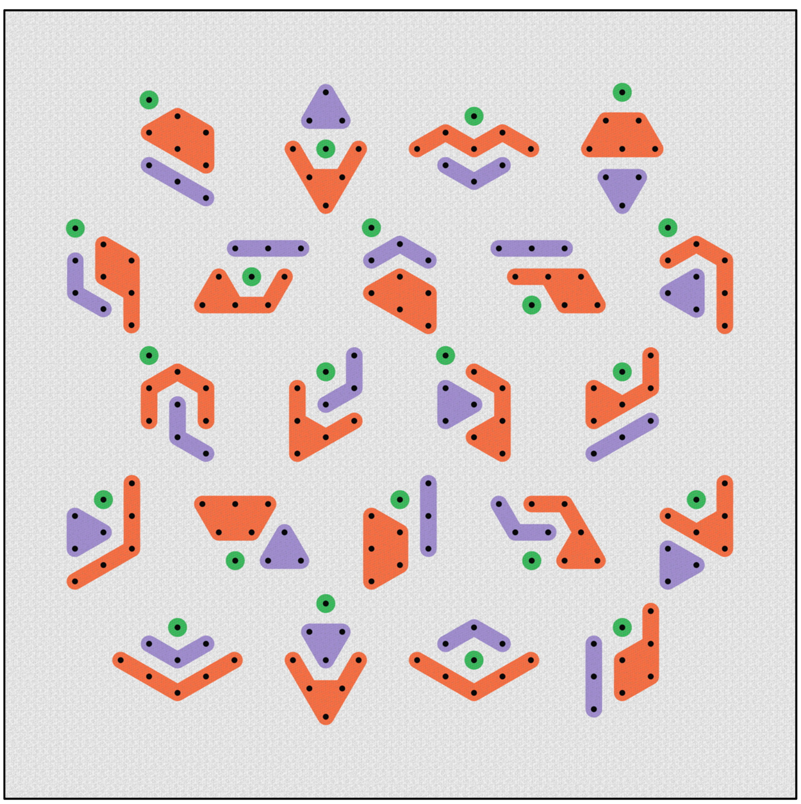

Figure 7: *"Scatter" 2019.*

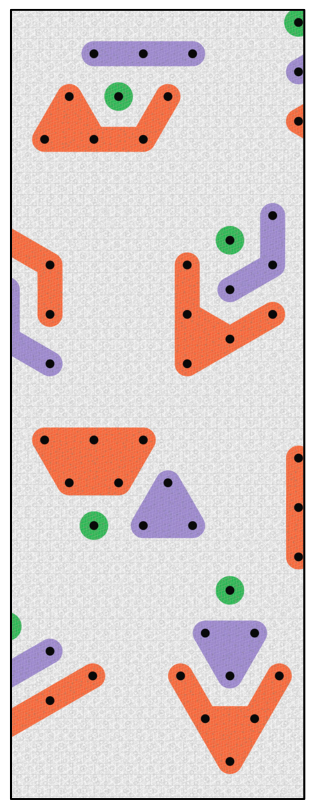

Figure 8: *Detail of Figure 7.*

Rhombus-9 Artworks

The Rhombus-9 form-set is a new development in my work, and the first artworks to utilize the forms are included here. *"Whorls"* (Figures 5 and 6) employs the complete subset of 17 rotation-only forms belonging to 10 partition groups. One partition has three rotational forms (the largest, central shapes); five partitions have pairs of rotational forms (the ten middle-size forms); and four partitions have a single rotational form (the smallest shapes). In the later stages of development of an artwork, I will sometimes imply figurative subjects by means of colors, shapes, positions, and titles. This is done with restraint, as I wish only to evoke and not to illustrate subject matter. The rotational forms in *"Whorls"* suggest spiral galaxies or vortices in water, while the centralized composition is reminiscent of a mandala. The former points outward to physical nature while the latter points inward to human consciousness—it is a visual musing on objective and subjective domains, on matter and psyche. The artwork's mathematical forms not only serve as the elements of an underlying logic, but they also offer associations for a playful meditation on the paradoxes and ambiguities of human experience.

"Scatter" (Figures 7 and 8) is comprised of all shape-partitions (symmetric and asymmetric) of [1,3,5]. The composition deliberately disguises each form by eliminating the bounding rhombuses, crowding the forms together, and varying their rotational angles. Yet there is order to be seen and discovered. A viewer uninitiated to shape-partitions will likely notice that the many shapes across the composition contain either 1, 3, or 5 black dots. Visual attention to spacing will confirm that triads of 1-Shapes, 3-Shapes, and 5-Shapes group together as 22 rhombuses lying at various angles (the complete [1,3,5] subset). Further examination will reveal that rhombuses tilted at common angles possess similar features (e.g., six forms tilted upper left to lower right are asymmetrical and have 1-Shapes at the top corner). An underlying structure is revealed through active and attentive seeing. The content of the artwork lies not in any subjects or symbols but in the experience it engenders—a decidedly perceptual encounter with mathematical order.

The compositions of *"Whorls"* and *"Scatter"* were developed in response to the specific characteristics of each shape-partition group—the compositional orders grew out of the subset orders. Over the years of working in this way, I have found that each form-set (or subset) offers different structural features that call for different compositional responses—the artist must be attentive to the specific mathematical order of the forms, both individually and as a group, and develop an appropriate visual expression for each. It is a rich and ongoing dialogue between the mathematics and the art.

Future Work and Conclusion

The Rhombus-9 shape-partitions are the most extensive form-set in my work to date. Although many of the forms will not find their way into my compositions, this large set has provided and will continue to provide valuable subsets for my work. Future work lies ahead for shape-partitions of other polygons and grid frequencies Work is underway on shape-partitions for the Square-9, which promises some interesting comparisons to the Rhombus-9 set because although the numbers of edges (four) and points (nine) are the same, the Square-9 possesses a square grid with four lines meeting at each point. This will result in a different variety and number of both component shapes and shape-partitions.

The aesthetic (that is, *perceptual*) apprehension of mathematical order has been a primary purpose in my work for most of my career as an artist. More particularly, I have been exploring the extent of that visual apprehension of order—how many distinct forms, how much complexity of variation, how many levels of enfolded order can be grasped through the language of color and shape, aside from verbal or mathematical explanation? Each artwork reveals to me something more about this endeavor, and shape-partitions are an important step in the search.

Acknowledgements

I wish to express my gratitude to Christopher Carlson for his generous and expert assistance in eliminating redundancies and verifying the final forms and partition groups in the Rhombus-9 form-set.