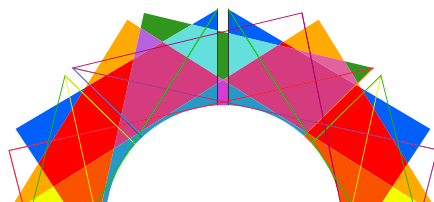


Bridges Stockholm 2018

Mathematics | Art | Music | Architecture | Education | Culture

2018 Conference Proceedings



BRIDGES
STOCKHOLM 2018

TEKNISKA MUSEET



Polismuseet

SJÖHISTORISKA



GÖTEBORGS UNIVERSITET



NobelCenter



Nobel Museum



SWEDISH NATIONAL HERITAGE BOARD
RIKSANTIKVARIEÄMBETET

Editors

Program Chairs

Eve Torrence

Department of Mathematics
Randolph-Macon College
Ashland, Virginia, USA

Bruce Torrence

Department of Mathematics
Randolph-Macon College
Ashland, Virginia, USA

Short Papers Chair

Carlo H. Séquin

Computer Science Division
University of California
Berkeley, USA

Workshop Papers Chair

Kristóf Fenyvesi

Department of Music, Art and Culture Studies
University of Jyväskylä
Jyväskylä, Finland

Production Chair

Craig S. Kaplan

Cheriton School of Computer Science
University of Waterloo
Waterloo, Ontario, Canada

Bridges Stockholm 2018 Conference Proceedings (www.bridgesmathart.org). All rights reserved. General permission is granted to the public for non-commercial reproduction, in limited quantities, of individual articles, provided authorization is obtained from individual authors and a complete reference is given for the source. All copyrights and responsibilities for individual articles in the 2018 Conference Proceedings remain under the control of the original authors.

ISBN: 978-1-938664-27-4

ISSN: 1099-6702

Published by Tessellations Publishing, Phoenix, Arizona, USA (© 2018 Tessellations)
Distributed by *MathArtFun.com* (mathartfun.com).

Cover design: Margaret Kepner, Washington, DC, USA

Bridges Organization Board of Directors

Kristóf Fenyvesi

Department of Music, Art and Culture Studies
University of Jyväskylä, Finland

George W. Hart

Stony Brook University
New York, USA

Craig S. Kaplan

Cheriton School of Computer Science
University of Waterloo, Ontario, Canada

Carlo H. Séquin

Computer Science Division
University of California, Berkeley, USA

Sujan Shrestha

Science, Information Arts & Technologies
Division
University of Baltimore, Maryland, USA

Eve Torrence

Department of Mathematics
Randolph-Macon College, Ashland, Virginia, USA

Artistic and Scientific Committee Members and Coordinators

Steve Abbott

Department of Mathematics
Middlebury College, Vermont, USA
Theater Event

Robert Fathauer

Tessellations
Phoenix, Arizona, USA
Art Exhibition

Kristóf Fenyvesi

Department of Music, Art and Culture Studies
University of Jyväskylä, Finland
Family Day

Susan Gerofsky

University of British Columbia
Vancouver, Canada
Theater Event

Sarah Glaz

University of Connecticut
Storrs, Connecticut, USA
Poetry Reading

Tiffany Inglis

D2L
Waterloo, Ontario, Canada
Technical Support

Katie McCallum

University of Brighton
Brighton, England, UK
Art Exhibition

Nathan Selikoff

Digital Awakening Studios
Orlando, Florida, USA
Technical Support

Bianca Violet

IMAGINARY
Berlin, Germany
Short Film Festival

Conference Organization

The Managing Board of the National Museum of Science and Technology

Peter Skogh, Museum Director
Stockholm, Sweden

Mariana Back

Curator
National Museum of Science and Technology
Stockholm, Sweden

Annika Brantingson

Coordinator Conferences/Events
National Museum of Science and Technology
Stockholm, Sweden

Lars Paulsson

Curator
National Museum of Science and Technology
Stockholm, Sweden

Linda Sandberg

Program Manager
National Museum of Science and Technology
Stockholm, Sweden

Cecilia Sommer

Host Manager
National Museum of Science and Technology
Stockholm, Sweden

Katrin Amberntsson

Strategic program coordinator
The National Museums of World Culture/
Ethnographic, Stockholm, Sweden

Annika Callen

Museum & Bridges Conference host
National Museum of Science and
Technology
Stockholm, Sweden

Björn Camitz

Technician
National Museum of Science and
Technology
Stockholm, Sweden

Christopher Einarsson

Museum Educator
National Museum of Science and
Technology
Stockholm, Sweden

Christopher Eliasson

Educator
Police Museum
Stockholm, Sweden

Karolina Furtenbach

Communicator
National Museum of Science and
Technology
Stockholm, Sweden

Annika Gooch

Program director
Museum of Performing Arts
Stockholm, Sweden

Sara Grane

Conference Coordinator
The National Museums of World Culture/
Ethnographic, Stockholm, Sweden

Lina Granér

Museum Host, Maritime Museum
Stockholm, Sweden

Kristina Hogvall

Event Coordinator
National Museum of Science and
Technology
Stockholm, Sweden

Britta Isaksson-Bergholm

Museum Educator
National Museum of Science and
Technology
Stockholm, Sweden

Marko Klemetti

Maintenance Manager
National Museum of Science and
Technology
Stockholm, Sweden

Cecilia Kozma

Head of House of Science
Royal Institute of Technology
Stockholm University, Sweden

Kicki Miles

Web-manager
National Museum of Science and
Technology
Stockholm, Sweden

Josefina Nilsson

Communicator
National Museum of Science and
Technology
Stockholm, Sweden

Dan Nordström

Technician
National Museum of Science and
Technology
Stockholm, Sweden

Peter Nyström

National Center of Mathematic
Education (NMC)
University of Gothenburg
Gothenburg, Sweden

Maria Olsson

Project leader/ Accessibility advisor
National Museum of Science and
Technology
Stockholm, Sweden

Gunilla Stillström

Curator
National Sports Museum
Stockholm, Sweden

Anders Wallenthin

Museum & Bridges Conference host
National Museum of Science and
Technology
Stockholm, Sweden

National Museum of Science and Technology Bridges Advisory Committee

Cissi Askwall
Science for All
Stockholm, Sweden

Betsy Devine
Nobel Center
Stockholm, Sweden

Annika Hedås Falk
Nobel Center
Stockholm, Sweden

Anki Hellberg
ArtsAdventure
Helsinki, Finland

Tom Callen
Nordic4DFrame
Vaxholm, Sweden

Lena Gumaelius
Royal Institute of Technology
Stockholm University, Sweden

Ann-Catherine Fröjdå
National Travelling Theater
Stockholm City, Sweden

Elisabeth Söder
Media Center
Stockholm City, Sweden

Mikal Vejdemo-Johnsson
Royal Institute of Technology
Stockholm, Sweden

Proceedings Program Committee

Steve Abbott
Middlebury College
Vermont, USA

Ellie Baker
Lexington, Massachusetts, USA

Christopher Brownell
Fresno Pacific University
California, USA

David Chappell
University of La Verne
California, USA

Erik D. Demaine
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Carol Dorf
Berkeley High School
California, USA

Frank Farris
Santa Clara University
California, USA

James Forren
Dalhousie University
Halifax, Nova Scotia, Canada

Sarah Glaz
The University of Connecticut
Storrs, Connecticut, USA

Abdalla G. M. Ahmed
Khartoum, Sudan

Debra K. Borkovitz
Wheelock College
Boston, Massachusetts, USA

Doug Burkholder
Lenoir-Rhyne University
Hickory, North Carolina, USA

Andrew Cooper
North Carolina State University
Raleigh, North Carolina, USA

Martin Demaine
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Mircea Draghicescu
ITSPHUN LLC
Portland, Oregon, USA

Robert Fathauer
Tessellations
Phoenix, Arizona, USA

Paul Gailiunas
Newcastle, England

Susan Goldstine
St. Mary's College of Maryland
Maryland, USA

Mara Alagic
Wichita State University
Kansas, USA

Robert Bosch
Oberlin College
Ohio, USA

Christopher Carlson
Wolfram Research
Champaign, Illinois, USA

Kelly Delp
Cornell University
Ithaca, New York, USA

Neil Dodgson
Victoria University of Wellington
New Zealand

Doug Dunham
University of Minnesota
Duluth, USA

Kristóf Fenyvesi
University of Jyväskylä
Finland

Susan Gerofsky
University of British Columbia
Vancouver, Canada

Chaim Goodman-Stauss
University of Arkansas
Fayetteville, Arkansas, USA

Gary Greenfield University of Richmond Virginia, USA	Emily Grosholz Pennsylvania State University University Park, Pennsylvania, USA	George Hart Stony Brook University New York, USA
Andrea Hawksley eleVR, HARC, YCR San Francisco, California, USA	Elisabeth Heathfield Bluewater District School Board Ontario, Canada	Judy Holdener Kenyon College Ohio, USA
Patrick Honner Brooklyn Technical High School New York City, NY, USA	Tiffany Inglis D2L Waterloo, Ontario, Canada	Veronika Irvine University of Waterloo Ontario, Canada
Craig S. Kaplan Cheriton School of Computer Science University of Waterloo, Ontario, Canada	Karl Kattchee University of Wisconsin La Crosse, USA	Eva Knoll Mount Saint Vincent University Halifax, Nova Scotia, Canada
Darci Kracht Kent State University Ohio, USA	Zsolt Lavicza Johannes Kepler University Linz, Austria	Kirsi Peltonen Aalto University Helsinki, Finland
Peter J. Lu Harvard University Cambridge, Massachusetts, USA	Penousal Machado University of Coimbra Portugal	Alice Major Edmonton, Alberta, Canada
Vincent J. Matsko Educational Consultant San Francisco, California, USA	Elisabetta Matsumoto Georgia Institute of Technology Atlanta, Georgia, USA	Dan May Black Hills State University Spearfish, South Dakota, USA
Doug McKenna Mathemaesthetics, Inc. Boulder, Colorado, USA	Kerry Mitchell Phoenix, Arizona, USA	Mike Naylor Matematikkbølgen Math Creativity and Competency Center Vanvikan, Norway
Doug Norton Villanova University Pennsylvania, USA	Osmo Pekonen University of Jyväskylä Finland	Rinus Roelofs Hengelo, The Netherlands
Radmila Sazdanovic North Carolina State University Raleigh, North Carolina, USA	Karl Schaffer De Anza College and MoveSpeakSpin Scotts Valley, California	Henry Segerman Oklahoma State University Stillwater, Oklahoma, USA
Carlo H. Séquin University of California, Berkeley USA	Sujan Shrestha University of Baltimore Maryland, USA	Donald Spector Hobart & William Smith Colleges Geneva, New York, USA
Stacy Speyer Cubes and Things Alameda, California, USA	David Swart Waterloo, Ontario, Canada	John Sullivan Technische Universität Berlin Berlin, Germany
Felicia Tabing University of Southern California Los Angeles, California, USA	Briony Thomas University of Leeds England	Bruce Torrence Randolph-Macon College Ashland, Virginia, USA
Eve Torrence Randolph-Macon College Ashland, Virginia, USA	Godfried Toussaint New York University Abu Dhabi, United Arab Emirates	Tom Verhoeff Eindhoven University of Technology The Netherlands
Charles Wampler General Motors Research and Development Warren, Michigan, USA	Phil Webster Phil Webster Design Scotts Valley, California, USA	Carolyn Yackel Mercer University Macon, Georgia, USA
	Rosa Zwier Melbourne, Victoria, Australia	

Art Exhibition and Catalog Program Committee

Robert Fathauer

Tessellations Company
Phoenix, Arizona, USA

Conan Chadbourne

San Antonio, Texas, USA

Kristóf Fenyvesi

University of Jyväskylä
Jyväskylä, Finland

Abdalla G. M. Ahmed

Khartoum, Sudan

Gabriele Meyer

Department of Mathematics
University of Wisconsin, Madison, USA

Katie McCallum

University of Brighton
England

Lars Paulsson

National Museum of Science and Technology
Stockholm, Sweden

Nathan Selikoff

Digital Awakening Studios
Orlando, Florida, USA

Karl Kattchee

Department of Mathematics
University of Wisconsin, LaCrosse, USA

Marty Levin

Portland, Oregon, USA

Short Film Festival Program Committee

Evelyn Lamb

Salt Lake City, Utah, USA

Giulio Rasi

Technical University
Berlin, Germany

Francesco Mancini

The Garden of Archimedes Museum
Florence, Italy

Bianca Violet

IMAGINARY
Berlin, Germany

Contents

Preface *xix*

Invited Papers

Marjorie Rice and Her Pentagonal Tilings 1
Doris Schattschneider

Some Memories of Koos Verhoeff (1927 – 2018) 3
Tom Verhoeff

A Personal Approach to the Art of Mathemagic with a Deck of Cards 7
Colm Mulcahy

Regular Papers

Art and Recreational Math Based on Kite-Tiling Rosettes 15
Robert Fathauer

Re-Generating Continuous Rumî Compositions 23
Nadide Ebru Yazar and Tuğrul Yazar

4-3 Dissection Tiling System 31
Andrew Sniderman

Multi-Scale Truchet Patterns 39
Christopher Carlson

Monochrome Map Weaving with Truchet-Like Tiles 45
Abdalla Gaafar Merghani Ahmed

Phoenix – Symbol of Mathematics 53
Tuomas Nurmi

A Poor Man’s Hyperbolic Square Mapping 59
Chamberlain Fong and Douglas Dunham

Kaleidoscopes for Non-Euclidean Space 67
Peter Stampfli

<i>New Metamorphosis Patterns</i>	75
Douglas Dunham and John Shier	
<i>A Minimal Art Object with Four Famous Fabulous Faces</i>	83
Walt van Ballegooijen and Hans Kuiper	
<i>Constructing 3D Perspective Anamorphosis via Surface Projection</i>	91
Tiffany Inglis	
<i>Mondrian Revisited: A Peek Into The Third Dimension</i>	99
Martin Skrodzki and Konrad Polthier	
<i>The Art and Mathematics of Self-Interlocking SL Blocks</i>	107
Shen-Guan Shih	
<i>Torus and Klein Bottle Tessellations with a Single Tile of Pied de Poule (Houndstooth)</i>	115
Loe M.G. Feijs	
<i>Homage to Charles O. Perry</i>	123
Carlo H. Séquin	
<i>Modular Toroids Constructed from Nonahedra</i>	131
Yifat Amir and Carlo H. Séquin	
<i>Weaving Double-Layered Polyhedra</i>	139
Rinus Roelofs	
<i>Two-Layer Woven Surfaces with Planar Faces</i>	147
Ulrich Reitebuch, Eric Zimmermann and Konrad Polthier	
<i>Laminar Reciprocal Structures</i>	155
Javier Barrallo, Francisco González-Quintial and Antonio Sánchez-Parandiet	
<i>The Beauty of the Symmetric Sierpinski Relatives</i>	163
Tara Taylor	
<i>Sphairahedra and Three-Dimensional Fractals</i>	171
Kento Nakamura and Kazushi Ahara	
<i>Plane-Filling Folding Curves on the Square Grid</i>	179
Jörg Arndt and Julia Handl	
<i>On a Better Golden Rectangle (That is Not 61.8033... % Useless!)</i>	187
Douglas McKenna	

<i>Loop-Forms: From Construction to Composition</i>	195
James Mai	
<i>The Pentagon and the Golden Angle in Almada Negreiros’ Mural “Começar”</i>	203
Pedro J. Freitas and Simão Palmeirim	
<i>Illustrating the Theory of Numbers</i>	211
Martin Weissman	
<i>Walking Around Trees: A 6-Letter ‘DNA’ for Baskets with Handles</i>	219
James Mallos	
<i>A New Family Satisfying the Intersection Graph Conjecture</i>	225
Ally Stacey	
<i>Textures in Simulations of Biological Cell Growth</i>	231
David Chappell	
<i>A Class of Spherical Penrose-Like Tilings with Connections to Virus Protein Patterns and Modular Sculpture</i>	237
Hamish Todd	
<i>Hex-Chaos Compositions and Equivalence Classes of Packing Problems</i>	245
Gary Greenfield	
<i>Combinatorial Greetings from Georges Perec</i>	253
Tatiana Bonch-Osmolovskaya	
<i>Hyperspace, Poetic Science Fiction and Algebraic Topology</i>	259
Emily Grosholz	
<i>Geometry of Quadrangles in Almqvist’s “The Queen’s Tiara”</i>	265
Tiina Katriina Kukkonen	
<i>The Mathematical Center of Attention, its Attributes and Motion Analyses in Dance Choreography</i>	273
Karl Schaffer, Joseph Thie and Kasia Williams	
<i>The Mathematical Circus Project</i>	281
Andreia Hall and Sónia Pais	
<i>Music from Vibrating Wallpaper</i>	287
Frank Farris	

<i>The Checkerboard of Tunes</i>	295
Andrew Crompton	
<i>A Generalized Dual of the Tonnetz for Seventh Chords: Mathematical, Computational and Compositional Aspects</i>	301
Sonia Cannas and Moreno Andreatta	
<i>From the Cheesegrater to the Parthenon: A Musical Odyssey</i>	309
Terry Trickett	
<i>Math in the Studio</i>	317
Judy Holdener and Karen Snouffer	
<i>Catenary Arch Constructions</i>	325
George Hart and Elisabeth Heathfield	
<i>Teaching Mathematics and Physics for Animation in Processing</i>	333
Lali Barrière	
<i>When Mathematics Meets Art: How Might Art Contribute to the Understanding of Mathematical Concepts?</i>	341
Liora Nutov	
Short Papers	
<hr/>	
<i>Dance Art, Math, Education – an Eternal Triangle.</i>	347
Paul Moerman	
<i>Turning Math into Dance: Lessons from Dancing My PhD</i>	351
Nancy Scherich	
<i>Voronoi Diagrams: Didactical and Artistic Applications</i>	355
Sandra Bento, Helena Ferreira and Andreia Hall	
<i>Play Truchet: Using the Truchet Tiling to Engage the Public with Mathematics</i>	359
Cindy Lawrence	
<i>Visualizing Symmetry Subgroup Structures Using Simple Motifs</i>	363
David Reimann	
<i>The Marvellous Bridging of Maths and Art Education and its Relation to Cognitive and Emotional Development</i>	367
Gunnel Berlin	

<i>Don't Preach Facts – Stimulate Acts</i>	371
Peter Baptist and Carsten Miller	
<i>Geometry and Origami to Share Cultural Heritage: Results of the Experimentation</i> <i>“The King and the Origami” at the Royal Residence of Venaria</i>	375
Paolo Armand, Caterina Cumino, Martino Pavignano, Maria Luisa Spreafico and Ursula Zich	
<i>Developing Mathematical and Technological Competencies of Students Through</i> <i>Remodeling Games and Puzzles</i>	379
Diego Lieban, Marina Menna Barreto, Sandra Reichenberger, Zsolt Lavicza and Ruana M. Schneider	
<i>Science Spaces: An Open Workshop Concept to Create Science Exhibits</i>	383
Bianca Violet and Milena Damrau	
<i>Musical Scales and Multiplicative Groups</i>	387
Donald Spector	
<i>Rock Me Fibonacci: Using Recurrence Relations and State-Transition Matrices</i> <i>to Count Rock Drum Fill Patterns</i>	391
Joshua Holden	
<i>Combinatorics, Probability and Choice in Music Composition: Towards an</i> <i>Aesthetics of Composing Systems for Non-Musicians</i>	395
Giovanni Albini	
<i>‘Dreamlinks’: Link Theory Meets Music Composition. An Introduction to Compositional</i> <i>Methods Related to Primary Links</i>	399
Saverio Tesolato	
<i>Just Intonation Keyboard: Isomorphic Keyboard Reimagined</i>	403
Marek Źabka	
<i>The Area Set Value Relationships in Atonal Music</i>	407
Nikita Mamedov	
<i>Landmarks in Algebra Quilt</i>	411
Elaine Ellison	
<i>Algoritmisch Ritme: Algorithmic Art as Material in an Interactive Dance-Projection</i>	415
Daphne Muller and Loe M.G. Feijs	

<i>A Methodology of Leaping Iteration for Drawing</i>	419
Ming Jang Chen	
<i>Surfing the Möbius Band: An Example of the Union of Art and Mathematics</i>	423
Francisco Saez de Adana	
<i>Capturing the Visual Traits of a Mathematician – On Anders Johan Lexell’s Futile Studies in Physiognomy</i>	427
Johan Stén and Martina Reuter	
<i>Representing the Undecidable</i>	431
Michel Tombroff	
<i>Observations on Concave Perspective</i>	435
Stephen Campbell	
<i>Which Integer Is the Most Mysterious?</i>	439
Osmo Pekonen	
<i>Mapping from e to Metaphor</i>	443
Alice Major	
<i>Art of de Bruijn Sequences</i>	447
Karl Kattchee	
<i>Card Shuffling Visualizations</i>	451
Roger Antonsen	
<i>The Art and Mathematics of Cycling: Using Old Bicycles to Draw Spirograph Patterns</i>	455
Nick Sayers	
<i>Juan Sánchez Cotán: Seeing Mathematically</i>	459
Paul Zorn	
<i>Mathematics in Drafting Japanese Crest Designs</i>	463
Felicia Tabing	
<i>Glass Mosaics Using Right-Triangle Subdivision</i>	467
Thomas Denker	
<i>A Plethora of Patterns from Discrete Spiraled Sequences</i>	471
Tom Bates	

<i>Computationally Intensive Puns with Figurative Subgraphs</i>	475
Robert Bosch	
<i>To the World's End/ A Circle Bundle Over a Circle</i>	479
Zachary Treisman and Lun-Yi Tsai	
<i>Folding Space-Filling Bisymmetric Hendecahedron for a Large-Scale Art Installation</i>	483
Jiangmei Wu and Guy Inchbald	
<i>A Threefold Möbius Band with Constant Twist and Minimal Bending as the Limit of Tetrahedral Rings</i>	487
Johannes Schönke, Michael Grunwald and Eliot Fried	
<i>3D Printing Chaos</i>	491
Micahel Gagliardo	
<i>Compound Parallelohedra Building Blocks with Creature-Like Morphologies</i>	495
Akihiro Matsuura	
<i>Method for Designing a Hinged Cube Puzzle</i>	499
Chirag Mehta	
<i>A Design Method Based on Close-Packing Circles and Spheres of Multiple Sizes for Designers and Architects</i>	503
Roger Burrows	
<i>Knot Designs Based on Rhombille Tiling Notations</i>	507
Nithikul Nimkulrat and Tuomas Nurmi	
<i>Girih Tiles in 3D</i>	511
Ulrich Reitebuch, Henriette-Sophie Lipschütz and Konrad Polthier	
<i>Origami Explorations of Convex Uniform Tilings Through the Lens of Ron Resch's Linear Flower</i>	515
Uyen Nguyen and Ben Fritzson	
<i>Self-Diagramming Lace</i>	519
Susan Goldstine	
<i>Möbius Cellular Automata Scarves</i>	523
Elisabetta Matsumoto, Henry Segerman and Fabienne Serriere	
<i>Lizardy Loops: The Savvy Selection of Sinuous Sequences of Circular Sectors</i>	527
Phil Webster	

<i>Horosphere, Cyclide and 3d Hyperbolic Tilings</i>	531
Vladimir Bulatov	
<i>Folding the Vesica Piscis</i>	535
Klara Mundilova and Tony Wills	
<i>Realizations and Constructions of Minimally Rigid Graphs</i>	539
Georg Grasegger	
<i>Polyhedral Models of the Projective Plane</i>	543
Paul Gailiunas	
<i>Extending Mandelbox Fractals with Shape Inversions</i>	547
Gregg Helt	
<i>Virtual Crocheting of Euclidean Planes in a 3-Sphere</i>	551
Eryk Kopczyński and Dorota Celińska	
<i>Designing Beaded Sculptures Inspired by Clathrate Hydrates</i>	555
Yuan-Jia Fan, Bih-Yaw Jin and Chia-Chin Tsou	
<i>Desargues Configuration as a Gnomonic Projection</i>	559
Taneli Luotoniemi	
<i>Action Modular Origami</i>	563
Tung Ken Lam	
Workshop Papers	
<hr/>	
<i>Sevenfold and Ninefold Möbius Kaleidocycles</i>	567
Michael Grunwald, Johannes Schönke and Eliot Fried	
<i>Self-Similarity and the Tumbling Square</i>	575
Susan Happersett	
<i>Adopt a Polyhedron – A Citizen Art Project in Mathematics</i>	579
Anna Maria Hartkopf and Günter M. Ziegler	
<i>Folding Curlicue and Exploring Its Mathematical Properties</i>	585
Natalija Budinski	
<i>Geometry and Origami to Make Dynamic Street Art</i>	589
Sara De Grandis, Silvia Fiore, Maria Luisa Spreafico, Marco Torredimare, Margherita Truffa and Ursula Zich	

<i>Geometrical Object Making For Design Thinking</i>	597
Gizem Aytaç	
<i>Open Geoboard – a Platform for Art, Math and Inspiration</i>	603
Yordan Hodzhev and Nikola Chernev	
<i>Creating Painting Puzzles: Math, Art, Games and Technology</i>	609
Marina Menna Barreto, Diego Lieban and Barbara Kimeswenger	
<i>Constructing Linkages for Drawing Curves</i>	613
Barbara Kimeswenger, Georg Grasegger and Sandra Reichenberger	
<i>A Hands-on Laboratory with Mathematical Mechanical Drawing Machines</i>	617
Laura Farroni and Paola Magrone	
<i>Renewable Energy Resources for Mathematics Learning: Windmills and Water Wheels at the Math Class</i>	623
Kristóf Fenyvesi, Ho-Gul Park, Kwang-Cheol Song, Zsolt Lavicza and Mariana Back	
<i>Engaging Community Through the Integration of Art and Mathematics</i>	629
Ellie Balk and Tricia Stanley	
<i>Let’s Sketch in 360°: Spherical Perspectives for Virtual Reality Panoramas</i>	637
António Araújo	
<i>Poetry Puzzles</i>	645
Lisa Lajeunesse	
<i>The Theory-Headed Poem</i>	649
Carol Dorf	
<i>Reimagining the Mathematical Paper</i>	651
Katie McCallum	
<i>Experiencing Group Structure: Observing, Creating and Performing the Plain Hunt on 4 via Music, Poetry, Visual and Culinary Arts</i>	659
Susan Gerofsky, Eva Knoll, Tara Taylor and Avalon Campbell-Cousins	
<i>Exploring the Geometry of Music with Technology: Rhythm, Scales and Temperament via Geogebra</i>	667
Steven Bleiler, Ewan Kummel and Michael Wells	
<i>Using Math to Create a Musical Sandbox</i>	673
Esmee Verschoor, Alyssa Eggersgluss, Collin Goldbach, Annmarie Thomas and Allison Knoph	

Human Encryptable Visual Cryptography.....675
Andrea Hawksley and Andrew Lutomirski

Mathematical Magic With a Deck of Cards.....681
Jorge Nuno Silva, Pedro J. Freitas and Tiago Hirth

Author Index.....687

Preface

Welcome to the 21st Bridges conference! This year the conference is being held in Stockholm, Sweden at the National Museum of Science and Technology (Tekniska museet). Stockholm is known for its ancient palaces, cathedrals, and cobbled lanes, and boasts one of Europe's best-preserved medieval city centers. Stockholm is also a dynamic modern city which is famous for its higher learning and scientific research institutions, such as the Royal Institute of Technology, Karolinska Institute, and the Nobel Center, as well as its museums, theaters, art galleries and concert venues. Located beside an archipelago of over 30,000 islands and containing a large number of parks, Stockholm is one of the greenest cities in the world and a particularly lovely city for enjoying the long twilight summer evenings found at this high northern latitude.

The National Museum of Science and Technology was one of the first museums of its kind when it opened in 1936. The museum attracts over 300,000 visitors every year and is the home of Sweden's first science center, which was completely renovated into the highly interactive learning environment, *MegaMind*, in 2015. The museum was named the *Swedish Museum of the Year* in 2016 and received the international *Children in Museums Award* for *MegaMind* in 2017.

The museum's *Mathematical Garden*, which opened in autumn 2017, is an accessible playground that covers the entire area in front of the museum. The garden's five zones are each based on a different mathematical theme. The center of the garden features a tessellation by the U.S. amateur mathematician Majorie Rice on the ramp leading to the museum's entrance. With the opening of *MegaMind* and the *Mathematical Garden* the museum has undergone a transformation from a more traditional technology museum into an arena of discovery, with a clear focus on children and their families.

This year's Bridges Program Co-chairs are Eve and Bruce Torrence. They coordinated an international Program Committee of over 70 experts who provided extensive reviews and editorial comments on submissions. They also served as chairs of the regular paper track. Carlo Séquin chaired the short papers track and Kristóf Fenyvesi chaired the workshop submissions. Many thanks go to the members of the Program Committee who reviewed the large number of papers received. Special thanks to Bianca Violet, our Short Film Festival Chair; Evelyn Lamb, Giulio Rasi, and Francesco Mancini, who served as the jury for the Film Festival; Sarah Glaz, the Poetry Reading Chair; Steve Abbott and Susan Gerofsky, the Theater Event Chairs; Kristóf Fenyvesi for organizing the public Family Day; Margaret Kepner for designing our beautiful book covers and Phil Webster for the 2018 postcard design. This year for the first time, the conference will include a Mathematical Fashion Show, organized by Susan Goldstine, on the evening of Informal Music Night, which is hosted by Mike Naylor.

The 2018 edition of the Bridges proceedings includes 3 invited papers, 44 regular papers, 55 short papers, and 21 workshop papers. You will find new work on using the arts to explore topics in tiling, cellular automata, fractals, knot theory, number theory, polyhedra, topology, space-filling curves, as well as mathematical concepts in dance, music, biology, chemistry, architecture and art history. There are papers on mathematics in literature, including historical novels, poetry, and comic books. Other fascinating papers explore card shuffling patterns and using 3D printing to present chaos, musical concepts, and portraits. There is even a witty discourse on the golden rectangle. Fiber arts are well represented with papers on weaving, quilting, lacemaking, knitting, crochet, and beading. Examples of public outreach through dance and magic performances, a mathematical circus, large scale art installations, a bicycle parts spirograph, and street art are explored as well as ways to engage students of all ages in exciting classroom activities. Particularly intriguing are two papers by graduate students in mathematics who used painting and dance to understand and express patterns related to concepts in their dissertations. Thank you to all of the authors and reviewers for their generous contributions to this year's proceedings, with special thanks to David Swart, George Hart, and Craig S. Kaplan for their endless advice and support.

This year we were able to offer \$10,000 USD in student travel scholarships to 14 students from around the world who authored accepted papers and created mathematical artworks. We are very grateful

to Jade Vinson for his generous contribution, which made this program possible, and to Sujan Shrestha who managed the program.

An exhibition of mathematical art has been an annual feature of Bridges since 2001. Artists from Europe, Africa, Asia, and North and South America will be represented. A wide variety of artistic media are included in the exhibition, including 2D and 3D digital prints, painting, beadwork, ceramics, wood, metal, quilting, and paper folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, number theory, and more. This year Katie McCallum and Robert Fathauer served as co-curators of the exhibition. The jury considering the artworks consisted of Abdalla Ahmed, Karl Kattchee, Martin Levin and Gabriela Meyer. The print catalog was prepared by Conan Chadbourne with cover art by Margaret Kepner, and the art submission website was created and administered by Nathan Selikoff. Lars Paulsson, the Curator/Project Leader for the *Mathematical Garden*, was the local coordinator in Stockholm for the art exhibition.

Many people contributed to the organization of this year's conference. A debt of gratitude and thanks to Museum Director Peter Skogh, who supported the conference planning throughout its various phases. We would also like to thank the entire Museum's Board of Directors for their enthusiasm for the project. Special thanks to Mariana Back, General Chair for Bridges 2018. Mariana is a Curator and Development Leader for the Museum's highly interactive science center, *MegaMind*. She took the initiative to have Bridges 2018 in Stockholm after participating in Bridges 2016 in Jyväskylä, Finland. Following that meeting, she realized that the combination of *MegaMind* and the recently opened *Mathematical Garden* would make the National Museum of Science and Technology an interesting venue for a Bridges conference. Kristóf Fenyvesi provided Mariana with endless encouragement, without which she would not have taken the idea any further. Event Managers Annika Brantingson and Kristina Hogvall have been a tremendous support during the planning process as well as during the conference. Staff Manager Cecilia Sommer, together with Head Technician Marko Klemetti, Conference Technician Björn Camitz, Conference Hosts Annika Callen and Anders Wallenthin, and Museum Educators Britta Isaksson-Bergholm and Christopher Einarsson strove to ensure everything would run smoothly during the conference. Program Manager Linda Sandberg and Communication and Marketing's Josefina Larsson and Karolina Furtenbach all played an important role in the local project group. Sebastian Flavet had the social media channels under control throughout the conference.

We would also like to thank the National Museum of Science and Technology's local Advisors: Annika Hedås Falk and Betsy Devine from the Nobel Center, Stockholm; Elisabeth Söder from the Media Center, Stockholm City; Tom Callen of Nordic4DFrame AB, Vaxholm; Mikal Vejdemo-Johnsson from the Royal Institute of Technology, Stockholm; Lena Gumaelius from Stockholm University/Royal Institute of Technology; Cissi Askwall of Science for All (Vetenskap & Allmänhet), Stockholm; Anki Hellberg from ArtsAdventure, Helsinki; and Ann-Catherine Fröjdå of the National Touring Theater, Stockholm. Visit Stockholm has been most helpful, and we would also like to thank the publishing company, Natur och kultur, for support in connection to activities in the Mathematical Garden.

We gratefully acknowledge the support of our sponsors in making Bridges 2018 possible. Heartfelt thanks to the Foundation for the National Museum of Science and Technology's 1974 Jubilee Fund, the Ethnographic Museum, the Maritime Museum/Sjöhistoriska, the Swedish Sports Museum, the Police Museum, and the Swedish Museum of Performing Arts. Many thanks to the Royal Institute of Technology, Stockholm University, the National Center of Mathematics Education (NCM), the Nobel Museum and Nobel Center, the National Touring Theater, ArtsAdventure, Swedish National Heritage Board and many others who helped with advice and to spread information about the Conference.

We hope you enjoy the 2018 Bridges Conference and this wonderful collection of new ideas. The joyful and creative spirit of our conference's founder, Reza Sarhangi, lives on in Bridges and continues to bring inspiration to mathematicians and artists around the world.

The Bridges Organization Board of Directors
www.bridgesmathart.org