Bridges Stockholm 2018

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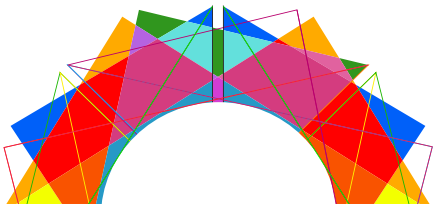
2018 Conference Proceedings



BRIDGES
STOCKHOLM 2018

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Preface

Welcome to the 21st Bridges conference! This year the conference is being held in Stockholm, Sweden at the National Museum of Science and Technology (Tekniska museet). Stockholm is known for its ancient palaces, cathedrals, and cobbled lanes, and boasts one of Europe's best-preserved medieval city centers. Stockholm is also a dynamic modern city which is famous for its higher learning and scientific research institutions, such as the Royal Institute of Technology, Karolinska Institute, and the Nobel Center, as well as its museums, theaters, art galleries and concert venues. Located beside an archipelago of over 30,000 islands and containing a large number of parks, Stockholm is one of the greenest cities in the world and a particularly lovely city for enjoying the long twilight summer evenings found at this high northern latitude.

The National Museum of Science and Technology was one of the first museums of its kind when it opened in 1936. The museum attracts over 300,000 visitors every year and is the home of Sweden's first science center, which was completely renovated into the highly interactive learning environment, *MegaMind*, in 2015. The museum was named the *Swedish Museum of the Year* in 2016 and received the international *Children in Museums Award* for *MegaMind* in 2017.

The museum's *Mathematical Garden*, which opened in autumn 2017, is an accessible playground that covers the entire area in front of the museum. The garden's five zones are each based on a different mathematical theme. The center of the garden features a tessellation by the U.S. amateur mathematician Majorie Rice on the ramp leading to the museum's entrance. With the opening of *MegaMind* and the *Mathematical Garden* the museum has undergone a transformation from a more traditional technology museum into an arena of discovery, with a clear focus on children and their families.

This year's Bridges Program Co-chairs are Eve and Bruce Torrence. They coordinated an international Program Committee of over 70 experts who provided extensive reviews and editorial comments on submissions. They also served as chairs of the regular paper track. Carlo Séquin chaired the short papers track and Kristóf Fenyvesi chaired the workshop submissions. Many thanks go to the members of the Program Committee who reviewed the large number of papers received. Special thanks to Bianca Violet, our Short Film Festival Chair; Evelyn Lamb, Giulio Rasi, and Francesco Mancini, who served as the jury for the Film Festival; Sarah Glaz, the Poetry Reading Chair; Steve Abbott and Susan Gerofsky, the Theater Event Chairs; Kristóf Fenyvesi for organizing the public Family Day; Margaret Kepner for designing our beautiful book covers and Phil Webster for the 2018 postcard design. This year for the first time, the conference will include a Mathematical Fashion Show, organized by Susan Goldstine, on the evening of Informal Music Night, which is hosted by Mike Naylor.

The 2018 edition of the Bridges proceedings includes 3 invited papers, 44 regular papers, 55 short papers, and 21 workshop papers. You will find new work on using the arts to explore topics in tiling, cellular automata, fractals, knot theory, number theory, polyhedra, topology, space-filling curves, as well as mathematical concepts in dance, music, biology, chemistry, architecture and art history. There are papers on mathematics in literature, including historical novels, poetry, and comic books. Other fascinating papers explore card shuffling patterns and using 3D printing to present chaos, musical concepts, and portraits. There is even a witty discourse on the golden rectangle. Fiber arts are well represented with papers on weaving, quilting, lacemaking, knitting, crochet, and beading. Examples of public outreach through dance and magic performances, a mathematical circus, large scale art installations, a bicycle parts spirograph, and street art are explored as well as ways to engage students of all ages in exciting classroom activities. Particularly intriguing are two papers by graduate students in mathematics who used painting and dance to understand and express patterns related to concepts in their dissertations. Thank you to all of the authors and reviewers for their generous contributions to this year's proceedings, with special thanks to David Swart, George Hart, and Craig S. Kaplan for their endless advice and support.

This year we were able to offer $10,000 USD in student travel scholarships to 14 students from around the world who authored accepted papers and created mathematical artworks. We are very grateful

to Jade Vinson for his generous contribution, which made this program possible, and to Sujan Shrestha who managed the program.

An exhibition of mathematical art has been an annual feature of Bridges since 2001. Artists from Europe, Africa, Asia, and North and South America will be represented. A wide variety of artistic media are included in the exhibition, including 2D and 3D digital prints, painting, beadwork, ceramics, wood, metal, quilting, and paper folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, number theory, and more. This year Katie McCallum and Robert Fathauer served as co-curators of the exhibition. The jury considering the artworks consisted of Abdalla Ahmed, Karl Kattchee, Martin Levin and Gabriela Meyer. The print catalog was prepared by Conan Chadbourne with cover art by Margaret Kepner, and the art submission website was created and administered by Nathan Selikoff. Lars Paulsson, the Curator/Project Leader for the *Mathematical Garden*, was the local coordinator in Stockholm for the art exhibition.

Many people contributed to the organization of this year's conference. A debt of gratitude and thanks to Museum Director Peter Skogh, who supported the conference planning throughout its various phases. We would also like to thank the entire Museum's Board of Directors for their enthusiasm for the project. Special thanks to Mariana Back, General Chair for Bridges 2018. Mariana is a Curator and Development Leader for the Museum's highly interactive science center, *MegaMind*. She took the initiative to have Bridges 2018 in Stockholm after participating in Bridges 2016 in Jyväskylä, Finland. Following that meeting, she realized that the combination of *MegaMind* and the recently opened *Mathematical Garden* would make the National Museum of Science and Technology an interesting venue for a Bridges conference. Kristóf Fenyvesi provided Mariana with endless encouragement, without which she would not have taken the idea any further. Event Managers Annika Brantingson and Kristina Hogvall have been a tremendous support during the planning process as well as during the conference. Staff Manager Cecilia Sommer, together with Head Technician Marko Klemetti, Conference Technician Björn Camitz, Conference Hosts Annika Callen and Anders Wallenthin, and Museum Educators Britta Isaksson-Bergholm and Christopher Einarsson strove to ensure everything would run smoothly during the conference. Program Manager Linda Sandberg and Communication and Marketing's Josefina Larsson and Karolina Furtenbach all played an important role in the local project group. Sebastian Flavet had the social media channels under control throughout the conference.

We would also like to thank the National Museum of Science and Technology's local Advisors: Annika Hedås Falk and Betsy Devine from the Nobel Center, Stockholm; Elisabeth Söder from the Media Center, Stockholm City; Tom Callen of Nordic4DFrame AB, Vaxholm; Mikal Vejdemo-Johnsson from the Royal Institute of Technology, Stockholm; Lena Gumaelius from Stockholm University/Royal Institute of Technology; Cissi Askwall of Science for All (Vetenskap & Allmänhet), Stockholm; Anki Hellberg from ArtsAdventure, Helsinki; and Ann-Catherine Fröjdå of the National Touring Theater, Stockholm. Visit Stockholm has been most helpful, and we would also like to thank the publishing company, Natur och kultur, for support in connection to activities in the Mathematical Garden.

We gratefully acknowledge the support of our sponsors in making Bridges 2018 possible. Heartfelt thanks to the Foundation for the National Museum of Science and Technology's 1974 Jubilee Fund, the Ethnographic Museum, the Maritime Museum/Sjöhistoriska, the Swedish Sports Museum, the Police Museum, and the Swedish Museum of Performing Arts. Many thanks to the Royal Institute of Technology, Stockholm University, the National Center of Mathematics Education (NCM), the Nobel Museum and Nobel Center, the National Touring Theater, ArtsAdventure, Swedish National Heritage Board and many others who helped with advice and to spread information about the Conference.

We hope you enjoy the 2018 Bridges Conference and this wonderful collection of new ideas. The joyful and creative spirit of our conference's founder, Reza Sarhangi, lives on in Bridges and continues to bring inspiration to mathematicians and artists around the world.

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