

# Mondrian Revisited: A Peek into the Third Dimension

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## Abstract

The artist Piet Mondrian (1872 – 1944) is most famous for his abstract works utilizing primary colors and axes-parallel black lines. A similar structure can be found in visualizations of the KdTree data structure used in computational geometry for range searches and neighborhood queries. In this paper, we systematically explore these visualizations and their connections to Mondrian’s work and give a dimension-independent generalization of Mondrian-like pieces.

## The Artist Piet Mondrian and His Work

Pieter Cornelis Mondriaan was born on March 7th, 1872 in Den Haag. His father was principal of a protestant primary school in Amersfoort. At the Rijksacademie Amsterdam he studied painting and drawing from 1892 till 1897. In 1911, he moved to Paris, where he was influenced by Cubism. Starting from this stay in France he was known as Piet Mondrian. His first solo exhibition took place at Walrecht Gallery, Den Haag, in 1914. In 1920, the “De Stijl Manifesto on Literature” is written by the artist van Doesburg and signed — amongst others — by Mondrian. After 1922, Mondrian used only primary red, yellow, and blue against white and black. Gray becomes rare. Toward the end of the 1920s, Mondrian’s canvases tend to include fewer elements, notably in the diamond compositions of 1926–33. Mondrian died on February 1st, 1944 in Manhattan, New York. The preceding biographical information is taken from [4].

Mondrian’s oeuvre ranges from early impressionist drawings of landscapes, over cubist paintings, to neo-plasticistic pieces influenced by and influencing the “De Stijl” movement. His most prominent works are without doubt those utilizing only black, axes-parallel lines, as well as primary colors. Early studies for these start in 1910 [5, p.102] with the first characteristic painting in 1918 [5, p.107]. These pieces have inspired a multitude of products utilizing his designs. Available are for example flower vases<sup>1</sup>, T-shirts<sup>2</sup>, furniture<sup>3</sup>, jewelry<sup>4</sup>, and much more<sup>5</sup>. Even in theoretical computer science, Mondrian is present with his work. There exists a programming language called “Piet”, designed by David Morgan-Mar [1, pp. 6,7]. The input of the language is not a text file but a bitmap resembling the art of Piet Mondrian.

Despite the overwhelming reception of Mondrian’s work, he is a stranger to the Bridges community. In the archive [8], no publication is listed connecting the work of Piet Mondrian to mathematical concepts. This article aims to close this gap. We will not only connect the two-dimensional pieces of Mondrian with the concept of KdTrees from computational geometry [2], but also generalize this connection to arbitrary high dimension. Thereby, we continue our series of articles on generalization to higher dimensions [7, 6].

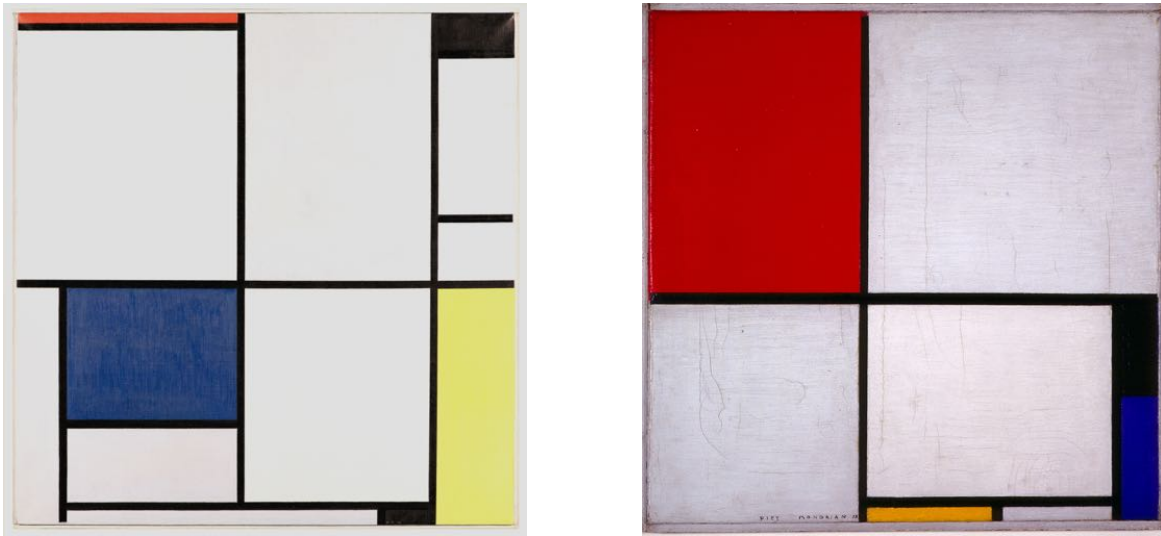
<sup>1</sup>[designisthis.com/blog/en/post/mondri-vase-frank-kerdil](http://designisthis.com/blog/en/post/mondri-vase-frank-kerdil)

<sup>2</sup>[redbubble.com/de/shop/piet+mondrian+t-shirts](http://redbubble.com/de/shop/piet+mondrian+t-shirts)

<sup>3</sup>[pinterest.de/atijoumana/modern-furnituremondriande-stijl/](http://pinterest.de/atijoumana/modern-furnituremondriande-stijl/)

<sup>4</sup>[etsy.com/search/jewelry?q=mondrian](http://etsy.com/search/jewelry?q=mondrian)

<sup>5</sup>[ifitshipitshere.com/mondrian-inspired-products/](http://ifitshipitshere.com/mondrian-inspired-products/)



**Figure 1:** Piet Mondrian, “Tableau I”, 1921, oil on canvas, Collection Gemeentemuseum Den Haag (Left) and “Composition with Red, Yellow, and Blue”, 1928, oil on canvas, Collection Wilhelm-Hack-Museum, Ludwigshafen (Right).

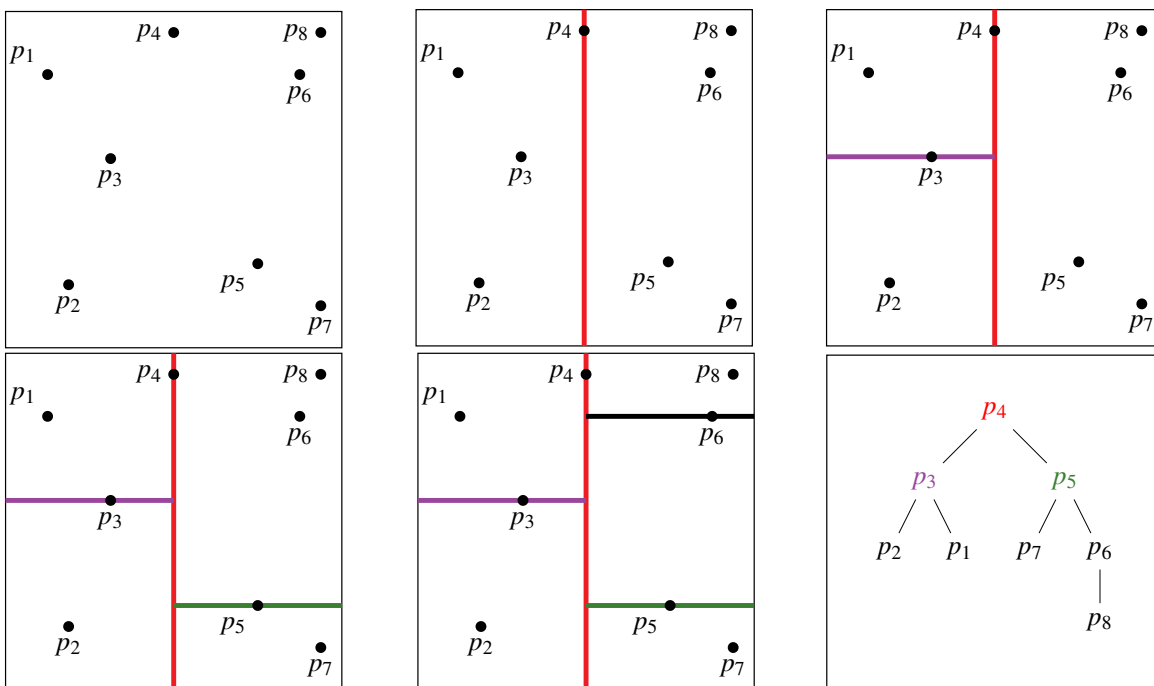
### **Tableau I, 1921**

We will now present a piece of Piet Mondrian in greater detail, namely his painting “Tableau I” from 1921, see Figure 1. The following elaborations and construction could also be modified to fit “Composition with Red, Yellow, and Blue”, also given in Figure 1. Concerning “Tableau I”, the original piece measures 103cm×100cm and is located in the Collection Gemeentemuseum Den Haag. The almost quadratic image is very simply structured by several axes-parallel black lines. These create thirteen rectangles of various sizes. One of them is colored blue and is located to the lower left of the center of the painting. Because of this almost central position, the rectangle draws the attention of the recipient with its blue color being in contrast to the surrounding white rectangles. Only two more rectangles are drawn in color, one in the lower right corner of the image, drawn in bright yellow and extending in height from the lower corner to the middle of the image. The other colored rectangle is of very narrow height and was squeezed in the upper left corner, extending horizontally to the middle of the picture. The upper right corner is filled with a completely black rectangle, another small black rectangle is placed next to the yellow one. All remaining other eight rectangles are white. The three used colors, yellow, blue, and red, are very strong in the contrast to the “non-colors” black and white. The recipient remains puzzled concerning the question whether the white rectangles are actually painted equally to the black and colored ones or whether they are merely showing the white canvas in the background. Thereby, Mondrian achieves perspective in his otherwise flat and two-dimensional piece.

Mondrian himself elaborated on his work in his only explicitly autobiographical essay “Towards the True Vision of Reality” [4, pp. 338–341]. He states (*italics by the author*):

It took me a long time to discover that particularities of form and natural color evoke subjective states of feeling, which obscure *pure reality*. The appearance of natural forms changes but reality remains constant. To create pure reality plastically, it is necessary to reduce natural forms to the *constant elements* of form and natural color to *primary color*.

In the following, we will establish a striking visual connection between the works of Piet Mondrian and visualizations of the KdTree data structure from computational geometry.



**Figure 2:** Recursively building a KdTree on eight points. The hyperplanes are shown in the first five figures, while the whole tree is shown in the last figure on the lower right.

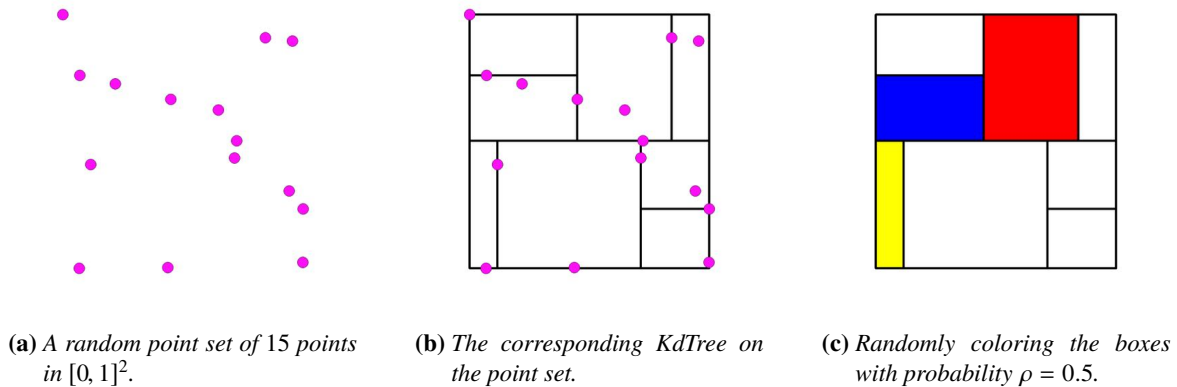
### The KdTree Data Structure

In this section we will give a brief introduction to the KdTree data structure. A more thorough description can be found in [2]. Given a finite point set  $P = \{p_1, \dots, p_n\} \subset \mathbb{R}^d$ ,  $p_i = (p_i(1), \dots, p_i(d))^T$  for  $i \in \{1, \dots, n\}$ , a KdTree is recursively defined. If  $P$  is empty, or only contains one point, nothing is done. Otherwise, it is determined in which of the  $d$  dimensions the points of  $P$  have the largest spread. Call this dimension  $d'$ . Now the points are sorted according to their values on dimension  $d'$ , i.e. they are rearranged to  $p'_1, \dots, p'_n$  such that  $p'_1(d') \leq p'_2(d') \leq \dots \leq p'_n(d')$ . Consider the median according to this sorting, i.e. the point with index  $m = \lceil \frac{n}{2} \rceil$ . Now, a hyperplane  $H = \{x \in \mathbb{R}^d \mid x(d') = p'_m(d')\}$  is introduced which splits the set  $P$  into two subsets  $P_1 = \{p'_1, \dots, p'_{m-1}\}$  and  $P_2 = \{p'_{m+1}, \dots, p'_n\}$  with  $P_1$  containing at most one more point than  $P_2$ . The procedure is then recursively applied to  $P_1$  and  $P_2$ . An example for a KdTree building procedure is given in Figure 2.

The data structure of KdTrees is significant in the context of geometry processing, as it can be built fast and can also be used for range searches and fast neighborhood queries. More precisely, the following results hold:

**Theorem 1.** A KdTree for a set of  $n$  points  $p_1, \dots, p_n \in \mathbb{R}^d$  can be built in time  $O(n \cdot \log(n))$ , takes  $O(n)$  storage, and has an average query time of  $\Theta(\log(n))$  to find the closest neighbor from the  $p_i$  to a given point  $q \in \mathbb{R}^d$ .

A proof for the building time and memory consumption can be found in [2]. The average asymptotic runtime of neighborhood queries is proven in [3]. Despite its theoretical relevance, in this context, we are mostly interested in the visualization of KdTrees. The following section will elaborate on the connections between the work of Piet Mondrian and KdTree visualizations.



**Figure 3:** A Mondrian-like structure created from a two-dimensional point set utilizing KdTrees.

### Two-dimensional Reproduction of Mondrian Pieces

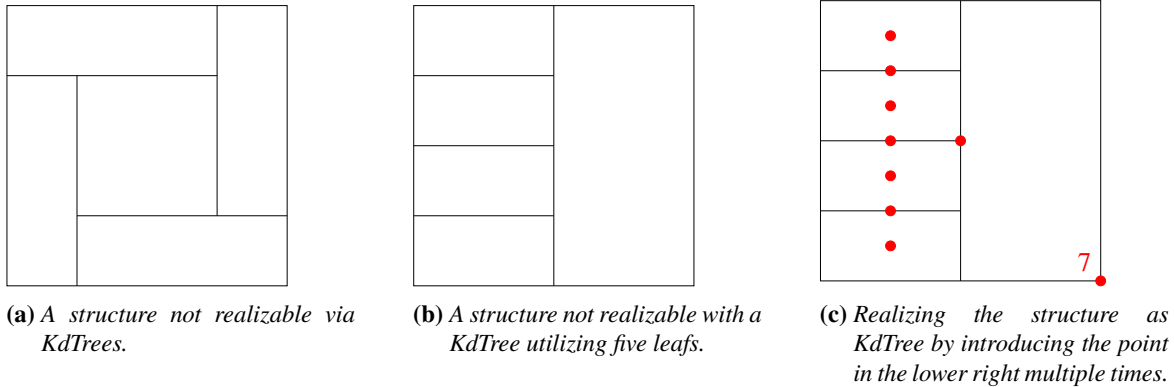
In this section, we will discuss how to reproduce some of the paintings of Piet Mondrian utilizing the KdTree data structure presented in the previous section. The general idea is to pick a set of  $n$  points  $p_1, \dots, p_n \in \mathbb{R}^2$  and to build a KdTree on them. As a complete binary tree with  $n$  nodes (both internal and leafs) has  $\ell(n) := \lceil (n+1)/2 \rceil$  leafs, the corresponding KdTree visualization will contain  $n - \ell(n)$  hyperplanes and  $\ell(n)$  rectangles. We choose a probability  $\rho \in [0, 1]$  and color each of the rectangles with probability  $\rho$ . If a rectangle is to be colored, it is colored red, blue, or yellow with probability  $1/3$  respectively. This yields a *Mondrian-like* structure on the bounding box of the points  $p_i$ . For example, Figure 3 shows the essential steps of obtaining a Mondrian-like structure from a point set consisting of 15 points (Figure 3a), with their KdTree (Figure 3b), colored with probability  $\rho = 0.5$  (Figure 3c).

#### Restrictions in the Reproducibility

Given the process outlined above, a first question naturally arises: What structures can be obtained this way? Here, two main limitations<sup>6</sup> have to be taken into account when trying to reproduce work of Piet Mondrian with the presented approach:

1. The lines need to follow the recursive building strategy of the KdTree data structure. That is, a structure as given in Figure 4a cannot be realized utilizing KdTrees. Each line has to have an ancestor in the hierarchy, that is incident to exactly two lines of the current bounding box at this level. In the example of Figure 4a, this is not the case, as all lines are incident to only one side of the surrounding square. Indeed, Piet Mondrian included a corresponding substructure into some of his paintings. For example, the piece “Tableau II”, dating 1921–1925 [5, p.110], cannot be reproduced utilizing KdTrees (assuming the occurring black patches to be single connected rectangles).
2. As the hyperplanes introduced by KdTrees always split the given point set at the median, there is at most a difference of one in the number of rectangles lying on either side of the hyperplane. This property is violated in some pictures of Mondrian, e.g. in “Composition with Yellow”, 1930, two rectangles on one side oppose six on the other. Unlike the first restriction, this problem can be overcome, though. As the number of points on each side of the hyperplane can vary at most by 1, we have to introduce auxiliary points on the border of the structure. These create rectangles without area, i.e. lines, that lie on the frame of the structure and can thus not be noted, see Figures 4b and 4c.

<sup>6</sup>There might be more limitation, i.e. this is not a complete list.



**Figure 4:** A structure that cannot be realized utilizing KdTrees and a solution for non-balanced structures.

### Mondrian-Like Reproduction of Tableau I

In this section we will describe how to reproduce Mondrian’s “Tableau I” utilizing KdTrees as introduced above. Given the dimensions of the original piece, we pick points from a rectangular region of width 43 and height 41. For the initial hyperplane, there are two possibilities. One can choose the horizontal line in the middle of the image or the vertical line towards the right of the image. As these are the only two lines connecting the borders of the painting, there are no more possibilities. As the horizontal line has six rectangles above and eight below, we pick it as first line and introduce the first point 1 here. Choosing the vertical line as first hyperplane would also work, but create a tree with many more auxiliary points, as it has four rectangles to its right, but ten to its left.

We proceed recursively by first introducing points above the line 1 and then below. For the second point, two vertical lines are splitting the rectangle above line 1 into two halves. Once more, we decide for the right vertical line as it has three rectangles both to the left and to the right. This process is continued until, above of line 4 (see Figure 5a) only one rectangle is left. We place a final point, 4’, in this rectangle and continue on the other side of line 4. Having finished this process, we introduced 27 points, shown green in Figure 5a, with corresponding relation shown in the upper tree of Figure 5b.

As a KdTree has to be balanced, we introduce several auxiliary points on the top left corner of a rectangle to ensure the correct placing of hyperplanes when following the KdTree procedure. In Figure 5a, we show the auxiliary points in magenta with the number of points at this location noted next to the point.

### Generalization to Three-dimensional Mondrian Sculptures and Beyond

Having introduced the technique of creating Mondrian-like images from KdTrees in the previous section, we will now generalize to higher dimension. Observe that the data structure of KdTrees is not limited to two-dimensional data sets, but can be applied in arbitrary (finite) dimension. Thus, a point set  $P = \{p_1, \dots, p_n\} \subset \mathbb{R}^d$  in a  $d$ -dimensional space gives rise to a  $d$ -dimensional Mondrian-like structure.

Several artists have created three-dimensional sculptures that are inspired by Piet Mondrian. Some are rendered<sup>7</sup> or animated<sup>8</sup> utilizing graphics software, others are built physically<sup>9</sup>. However, none of them is created in a systematic way as presented in this paper.

<sup>7</sup>e.g. [mb-neo.deviantart.com/art/An-Piet-Mondrian-in-3D-59414078](http://mb-neo.deviantart.com/art/An-Piet-Mondrian-in-3D-59414078) or [renderosity.com/mod/gallery/mondrian-3d-/1293340/](http://renderosity.com/mod/gallery/mondrian-3d-/1293340/)  
?p

<sup>8</sup>e.g. [youtube.com/watch?v=Yh1H-YMrA2A](http://youtube.com/watch?v=Yh1H-YMrA2A)

<sup>9</sup>e.g. [flickr.com/photos/pi-p/1537989163](http://flickr.com/photos/pi-p/1537989163) or [moc.bricklink.com/pages/moc/mocitem.page?idmocitem=158](http://moc.bricklink.com/pages/moc/mocitem.page?idmocitem=158)

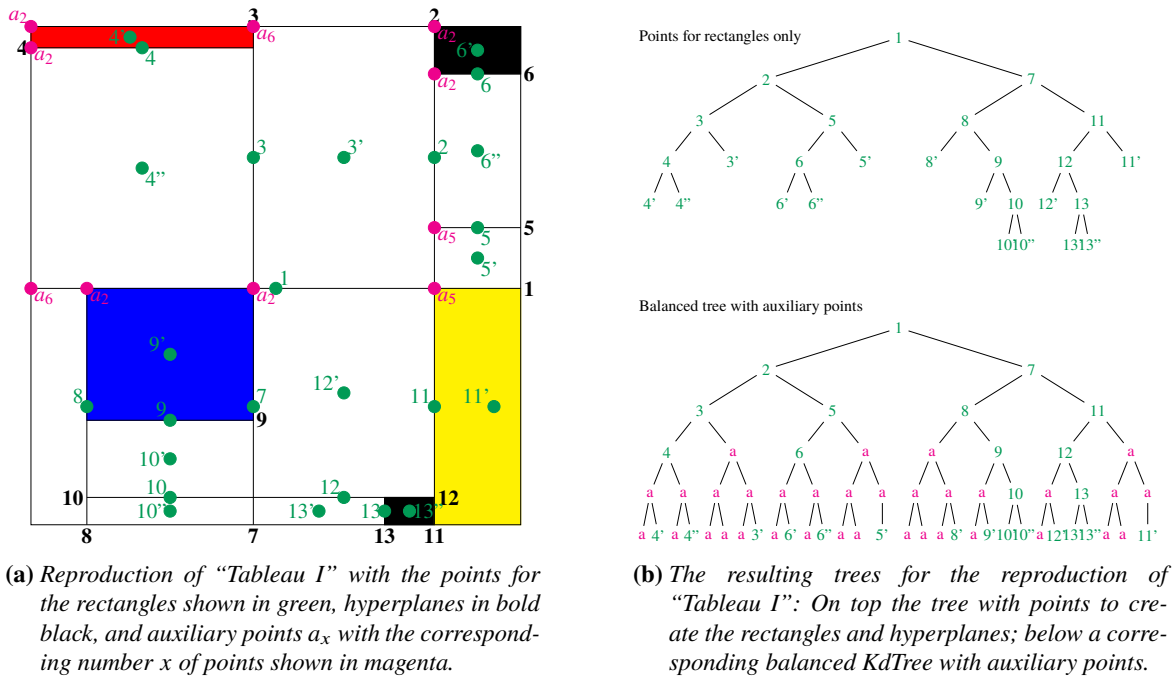


Figure 5: Reproduction of "Tableau I" as a KdTree with the corresponding tree.

We start our exploration of three-dimensional Mondrian-like sculptures by picking 15 points randomly from the unit-cube in  $\mathbb{R}^3$ . Then, we follow the KdTree procedure as introduced above and draw a frame around the bounding box of the points and the boxes created as KdTree regions. The resulting frame with the points is shown in Figure 6a. As reasoned above, a KdTree on 15 points has eight leaves, i.e. eight boxes. We color each box with a probability of  $\rho = 0.2$  and obtain a sculpture shown in two perspectives in Figure 6b.

Note that, by our design process, any axes-parallel two-dimensional view of a three-dimensional Mondrian-like sculpture gives rise to a two-dimensional Mondrian-like image. This is exemplified in the front-, right-, and top-view of the created sculpture show in Figure 6c, 6d, and 6e. Because the points have been drawn from the unit-cube randomly, the resulting images are not squares, but rectangles.

We continue our exploration of the created shape space by systematically varying the two possible parameters. First, we work on a point set consisting of 15 points in  $\mathbb{R}^3$ . On these, we build the corresponding KdTree and visualize it with different probabilities of its boxes to be colored. A corresponding series of images is shown in Figure 7.

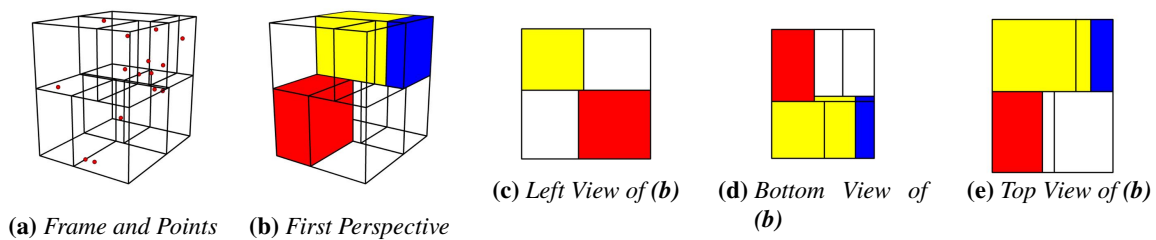
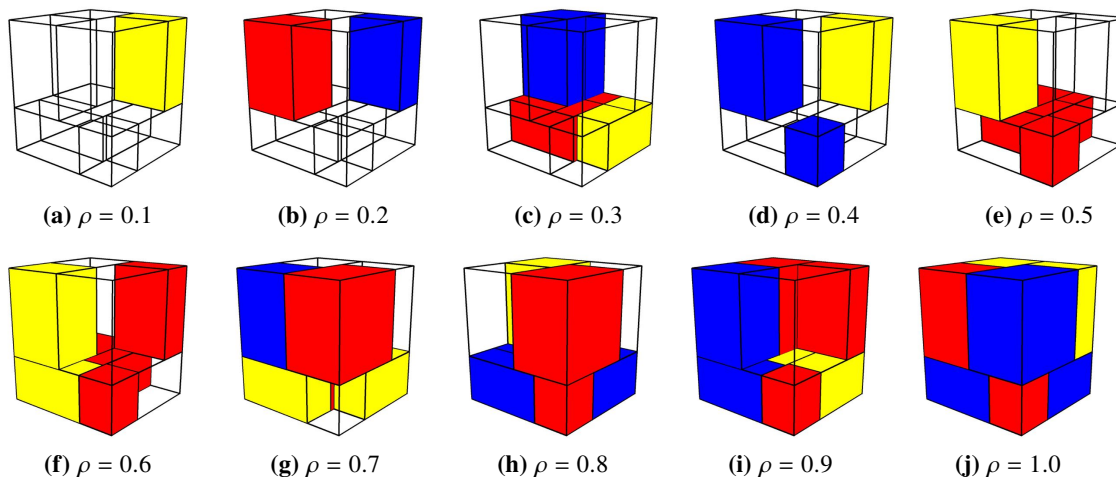


Figure 6: Three-dimensional Mondrian-like sculpture on 15 points colored with probability  $\rho = 0.2$ .



**Figure 7:** Visualization of the same KdTree on 15 points in  $\mathbb{R}^3$ . The coloring probability  $\rho$  grows from left to right in both rows. Starting at  $\rho = 0.1$ , it grows with step size 0.1 until it reaches  $\rho = 1.0$  in the lower right picture.

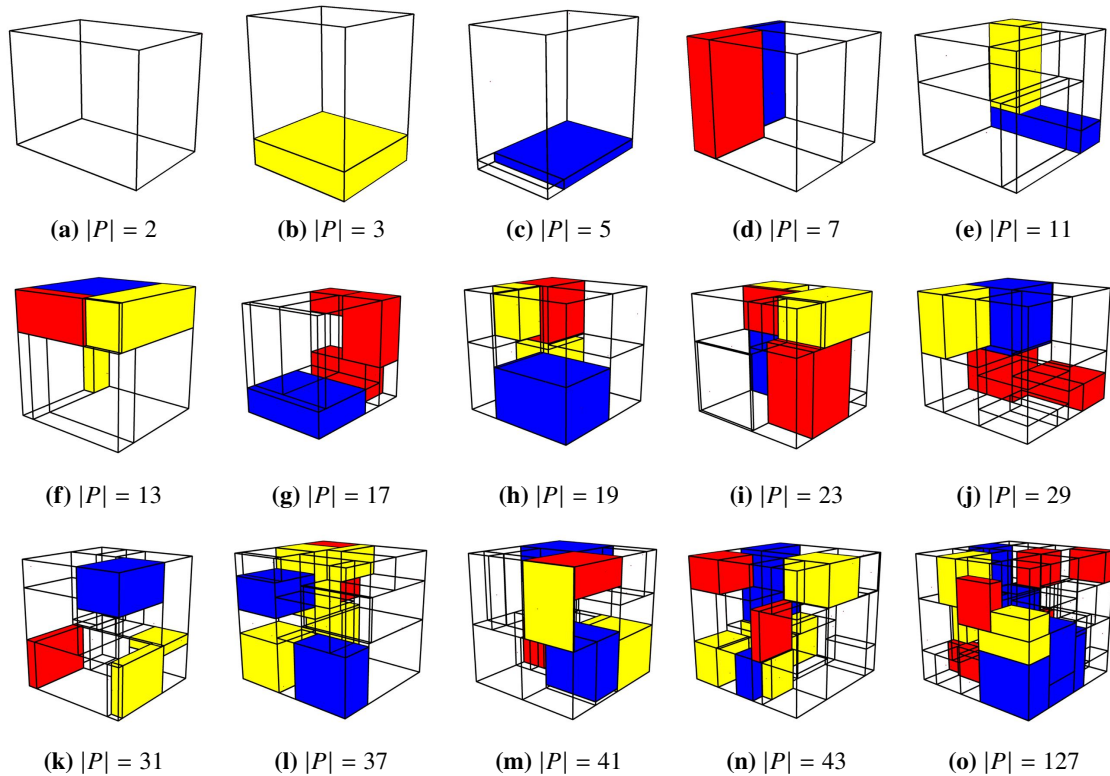
A second parameter to alter is the number of points used. Therefore, we fix a coloring probability  $\rho = 0.3$ . Then, we randomly create point sets of different sizes from the unit-cube in  $\mathbb{R}^3$  and visualize the corresponding KdTree. A series of images depicting this variation is shown in Figure 8.

Technically, the structure of KdTrees is not bound to dimensions 2 or 3. Thus, sampling a unit-cube randomly in  $\mathbb{R}^d$  and building a KdTree on the sampling gives rise to a  $d$ -dimensional Mondrian-like structure. In order to visualize these structures for  $d = 4$ , a possibility is to visualize three-dimensional sections of the four-dimensional Mondrian-like structure. The exploration of this direction is left as future work.

Even though we call our sculptures “Mondrian-like”, not all of them directly remind of pieces by Piet Mondrian. In fact, the proportions of the rectangles in the works of Mondrian seem all carefully chosen. Our approach however uses uniformly distributed random point clouds which can only create proportions resulting from their distribution. These are not similar to those of Piet Mondrian. Further research is necessary in order to tailor a probability distribution for the points that faithfully reconstructs the proportions utilized by Mondrian. Also, we did not take into account the existence of black rectangles, see Figure 1. As for the distribution of points, also the probability of coloring has to be carefully tailored to mimic Piet Mondrian better. This is also left as further research.

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**Figure 8:** Visualization of different KdTrees for growing point sets in  $\mathbb{R}^3$ . The number of points in the set grows from left to right in all rows.

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