Reimagining the Mathematical Paper

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Abstract

This workshop is aimed at encouraging creative engagement with the form of the mathematical paper, to encourage both paying serious attentions to such forms, and a more empowered engagement with mathematical texts. Participants are encouraged to cut up and reconfigure existing mathematical texts, to create poetry, to organise them according to new logics, and to use colour and space in a creative re-presentation of the material. The aim is, through experimentation, both to see creative new possibilities, and to better understand how communication is achieved and what is communicated in the paper's existing form.

Introduction

The particulars of mathematical writing is sometimes undervalued in the mathematical world, as evidenced by the popular view of mathematics as happening primarily in the 'heads' of mathematicians:

> "People can be thinking about mathematics all the time if they want to. They can carry it all around in their heads…" Prof. Elmer Rees [13].

This picture implicitly subscribes to a view that writing is auxiliary to the doing of mathematics, or as Danielle Macbeth explains, 'the view that language, whether written or spoken, is invariably after the fact in mathematics, serving only to document or report results obtained independently,' [11]. Macbeth refers to Brian Rotman's influential book exploring the semiotics of mathematics, in which he notes one of the unwanted by-products of such a view.

> "One consequence of what we might call the documentist view of mathematical writing, whether Husserl's or the standard Platonic version, is that the intricate interplay of imagining and symbolising, familiar on an everyday basis to mathematician in their practice, goes unseen" [15].

Even more commonly, we see this understanding of mathematics being imposed on the work of mathematicians in the way they are interviewed:

> "Interviewer: When you shut your eyes, do you see something mathematical?
>
> "Don Zagier: No. […] I need not only open eyes, but paper and blackboard before I can think sensibly about anything mathematical." [2]
>
> "Interviewer: what does it look like in your head, when you're doing mathematics?
>
> "Terence Tao: um… [long pause] It's always a combination of working in your head, and speaking out loud, and working on the board…" [12]

These quotes, in which the mathematicians themselves resist the unrealistic questions of the interviewers and shift the focus back to practical, real-world processes, are indicative of the move that I want to make with this research. This workshop is about creatively working with the material form of the paper, understanding the form, organisation and aesthetic aspects as contributing in an essential way to understanding, experience and development in mathematical work. Initially participants will work to deconstruct a mathematical paper, exploring possible logics that can be employed in engaging with the content of mathematical work, and then to creatively reconfigure it using resources like colour, space, physicality and interconnectedness.

What Can We Gain from Thinking Harder about Writing?

An image of mathematics that deprioritises its material manifestations in the world could be understood as contributing to a public perception of mathematics as inaccessible and mysterious to all but the initiated few. What's more, de-emphasising the exacting practices that produce mathematical work may be harmful for progress in the discipline itself. As well as the threat of potentially intimidating readers through austere presentation, there is a real danger of actually hampering understanding by overly refining an argument, as Lorenzo Lane notes in his fascinating ethnographic study of mathematicians in two large institutions.

> "Oftentimes mathematicians begin by working with pictures, building up intuitions about objects through manipulating such pictures. It is only later on in the proof construction process that these physical intuitions are erased, as the pictures are abstracted away, in the formal process of "writing up". In writing up much of the informal processes of experimentation, as well as the physical intuitions on which arguments are based, get written out of the end publication. However, in erasing pictures and examples from the published proofs, it can often become difficult for other mathematicians to follow a proof." [10]

It is potentially quite inhibiting to mathematical progress if its practitioners don't pay attention to communication, one another's cognition, and mathematical writing. In a famous paper that discusses questions of the nature of proof, Richard De Millo, Richard Lipton and Alan Perlis see *social knowing* as central and essential to mathematics. 'First of all, the proof of a theorem is a message. A proof is not a beautiful abstract object with an independent existence. No mathematician grasps a proof, sits back, and sighs happily at the knowledge that he can now be certain of the truth of his theorem. He runs out into the hall and looks for someone to listen to it' [4].

In *Toward a Semiotics of Mathematics,* Brian Rotman analysed mathematical papers using a semiotics-based framework and put forward a philosophically interesting and rich picture of the entities invoked in mathematical writing [15]. This influential essay, populated by fictional homunculi imagined to be carrying out infinite tasks such as the summing of a series, introduces a fascinating and exciting drama into the subtleties of phrasing in mathematical texts, one that cannot help but be fascinating to mathematicians and non-mathematicians alike. This surprising and imaginative work had implications for effective communication and even philosophical understandings of what exactly mathematics consists in, demonstrating that closer scrutiny of and experimentation with the forms that mathematics takes in the world has much to offer.

Colour and Space

Mathematical papers are very plain and formal in appearance, but there are features of their conventions that do a lot to orient readers and give them a sense of the purpose of a particular piece of text. Section headings can give an enormous amount of detail about how sections relate, for example if a section is labelled 'proof' or 'corollary'. The section heading 'lemma' lets a reader know that the content is particularly interesting in itself but will later prove instrumental; the label 'proposition' suggests something more interesting than a lemma but less so than a proof, and 'conjecture' signals something that is thought likely but remains unproven, indicating degrees of certainty in a way that will help to preserve trust in the author.

What's more, the large amount of cross-referencing within the paper provides the reader with an important sense of the connections within the argument. Explicit reference is made to any previous sections whose content is useful to the present section, with the number of the section referenced given to allow a reader to understand quickly how the argument hangs together (see Figure 1). This transforms the paper from a purely linear form into something far more articulated and interconnected.

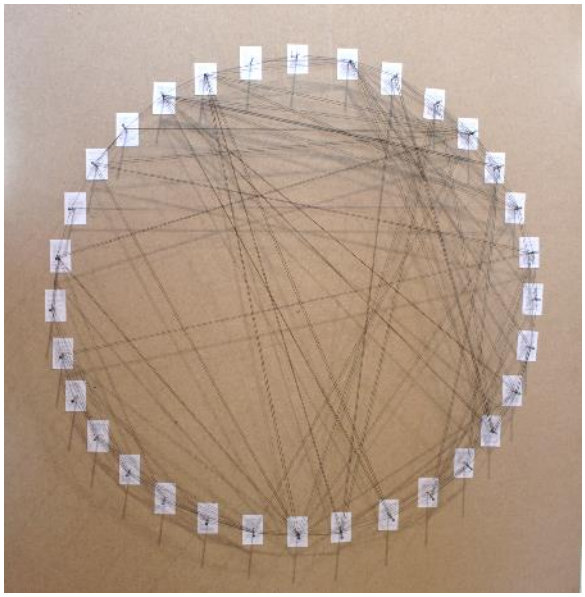

Figure 1: *Map of the internal references in a 32-page paper*

Seeing how much depth is added by such simple means, it is interesting to consider how much could be done using space and colour, if that plain, formal format were relaxed and the colourful, spatial formats seen in other aspects of mathematical communication such as, for example, live presentations, were used.

Blackboard communication allows substantial extra communication to take place over and above the content of the written material, as has been observed in the ethnographic work of Christian Greiffenhagen and f Michael J. Barany and Donald Mackenzie [8, 9]. Board use for talks has a tendency to follow certain structures, which give an audience member clues about the function of a piece of writing. For example, this diagram from Greiffenhagen (Figure 2) shows how the role of the lemma is indicated by placing it to one side, indicating its separation from, but relevance to, the rest of the text.

Figure 2: *Figure VI: 'Architecture' [reconstructed]* [5]

The placement of a piece of text can thus give the audience member clues about its importance and role, just as the section headings do in a paper structure. In *Chalk: Materials and Concepts in Mathematics Research*, Barany and Mackenzie expand upon this point, describing a number of placement and writing strategies as follows:

> "Specific board locations can carry mathematical significance. Parts of an expression can be separated visually, and corresponding terms are often aligned or written over each other, even

> when this requires the writer to sacrifice some of the marks' legibility. For instance, when a new bound is introduced for an analytic expression many speakers simply erased the bounded expression and contorted their writing so that the new bound would fit in its place. Similarly, when a proof hinged on the proper grouping or regrouping of terms in an expression speakers exaggerated the physical spacing between certain terms when writing them. Thus spatialized, statements can be mobilized or demobilized by emphatic or obfuscatory gestures. Multiple speakers, for example, mimed erasing an expression or simply blocked it with their hands in order temporarily to exclude it from a consideration or to show that an explanation strategically ignores it." [1]

There is evidence here of alignment and grouping strategies, giving cues about the structure of an argument.

> "These boxes visually highlight which sentences and symbols 'go together'… Lines and boxes … are part of what Coleman (1988, 1990) calls 'paragraphy' and make visible that the proof is not a series of linear, homogeneous steps, but rather consists of 'chunks' (major steps), which can take different shapes (e.g., just one or several lines)." [5]

Another underused resource is colour, used when available to distinguish elements in an intuitive way among other things, as explored in a workshop at Bridges by Knoll *et al.* [9]. For the most part, such extra resources tend to be abandoned in the austere form of published papers except in the occasional diagram.

Art and Intuition

This workshop is intended as a creative exploration of the potential to use extra resources in constructing a mathematical paper and so to increase communicative bandwidth.

Art is an area of human life in which we are accustomed to dealing in variable and non-determined aspects of experience. James Hellings, in an exegesis of the work of Theodor Adorno, provides the image of "a message in a bottle" that "is not necessarily about the message, one message above all others. Rather, messages in bottles are like art's impulse, which openly addresses itself to the unknown, to the impossible, to the future." [8]. Art by this view is concerned with everything but determinate explicit content, is an exploration of all of the less distinct avenues of human interaction.

Though conceived with an interest in the unexpected resources that might help communication, this workshop is intended as more of an exploratory than a design exercise. Participants are to be encouraged to adopt a variety of exploratory approaches to the content, testing out really innovative strategies. The intention is to encourage the adoption of some logic that really resists and rethinks the logics normally applied to mathematical writing, just as Rotman's work does, in a way that approaches what art theorist Jacques Rancière might call break from a regime of sense, a break from the expected that allows a view from the outside of a particular established regime for seeing the world [14]. A reconfiguration of the sensible (the staging of a conflict of two sensory regimes, which is the system of what is visible, sayable and therefore doable in a certain group consensus) makes it possible to both perceive what the assumptions and norms of a particular system of seeing are, and to see the possibility of an alternative, an achievement that Rancière considers one of the primary things that art can aim for. The aim of this workshop is to reimagine the form of the paper through creative exploration, and by so doing, to also better understand the ways in which it functions in its current form.

Workshop Plan

This workshop is intended as a creative exploration of the potential to use extra resources in constructing a mathematical paper. The intention is to explore the possibility of increasing communicative bandwidth by using these extra resources, and to develop a broader idea of what exactly is communicated

in a mathematical paper, from the determinate content of mathematical statements to the hazier social functions played by mathematical publication.

The experiment may inform an exhibit at a future exhibition, with participants of this workshop credited, for which permission will be sought at the beginning of the workshop. Participants who decline will still be able to participate, and their ideas will not be in any way referenced in future work.

The workshop will begin with a 10-minute introduction, bringing up some of the ideas about the resources used in mathematical presentations, and the functions of mathematical publishing. After that, participants are invited to take part in three tasks.

Warm-up

This section begins with a short, playful experiment in which each participant is given a page from a mathematical paper and they spend 5 minutes constructing a short poem from the content, such as that shown in Figure 3.

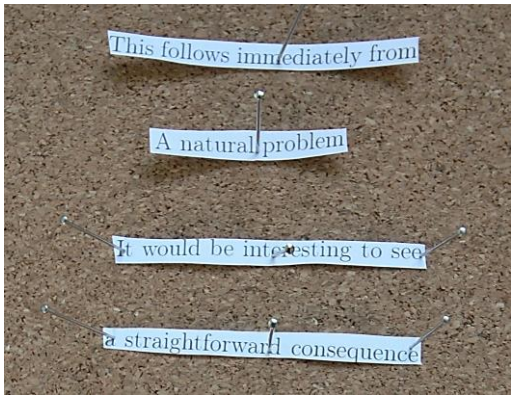

Figure 3: *Poetry fragment from an earlier experiment*

This section is intended to open up playful and creative engagement with the text. Participants are invited to share their creations once they are written.

Deconstruction task

Secondly, participants are invited to deconstruct and reconsider the content of a few pages from a mathematical paper. In groups of three, participants will be invited to cut up and classify the content of a paper according to innovative classification systems. Participants are to be encouraged to find a system that is unlike that used by any other group.

The paper to be used is *Families of knots for which Morton's inequality is strict* by Mark Brittenham and Jacqueline Jensen [3], a nine-page paper that uses a range of diagrams, notations and argumentation strategies. Participants have 10 minutes to agree upon and enact a particular system, using scissors to cut up the paper and arrange it on a tabletop; it is not necessary to classify the entire paper, and will probably be best to work on one or two pages, but a paper with rich content has been chosen to offer participants plenty of interesting material to work with. As an indication of the possibilities available, suggested possible classification systems include classification according to:

- Grammatical structure, such as text in the passive voice, the authorial 'we', or the imperative
- Illocutionary force of the content, to assert, define, exemplify, contradict, contextualise, and so forth
- Content and orienting markers, whether in formatting or in the text.

- Mode, such as diagram, notation or text, modes that operate by convention or by resemblance, and nuances within these categories
- Text and negative space
- Relationship or absence thereof to external resources

After this, each group will briefly present their classification and explain the logic that they adopted, and there will be a period of discussion in which any surprising discoveries or potential alternative approaches are noted (10 minutes total).

The purpose of this task is to begin to engage with the content in disruptive and innovative ways, and to help free up participants to play and experiment with the content. The discussion after the activity is intended to encourage playfulness as participants are confronted with one another's ideas, presenting a forum for speculative ideas that participants do not feel bound to enact.

Reconstruction task

The second task will be to put forward a reimagining of a particular six-page paper, *An Elementary Proof of "the Most Elementary Theorem" of Euclidean Geometry*, by Mowaffaq Hajja [6]. This paper, as the title indicates, uses relatively elementary mathematics and has an argument whose gist the majority of participants should be able to follow.

Participants, again in groups of three, are provided with large cork boards, pins, coloured pens and string, and have 30 minutes to work on reimagining the paper. They are invited to lay out and alter the paper in whatever way they see fit but this time working in a constructive way, adding in colour and string connections to give articulation to the content. They may choose to work in such a way as to aid understanding of the argument, or may take a more experimental approach.

After this activity, each group will talk briefly about the approach that they adopted and the decisions that they took. After 10 minutes, this will move on to a more general discussion of the experience of engaging with mathematical text in this way, the possibilities that this opens up and any ideas that participants came up with but were not able to enact. In particular participants will be encouraged to reflect on the text and any discoveries that the process helped them to make within it.

Precursor to this Workshop

Figure 4: *Participants at a precursor to this workshop*

A precursor to this workshop was held at the University of Brighton Postgraduate Research Forum on the 21st February 2018. Three groups of participants were invited to spatially reorganise a very short paper, entitled *A Very Short and Simple Proof of "The Most Elementary Theorem" of Euclidean Geometry*, also

by Mowaffaq Hajja [7]. The participants on this occasion were not mathematicians and had limited mathematical backgrounds, but nonetheless engaged enthusiastically with the task, reporting a feeling of empowerment in relation to the text. The approaches taken by the participants were excitingly diverse.

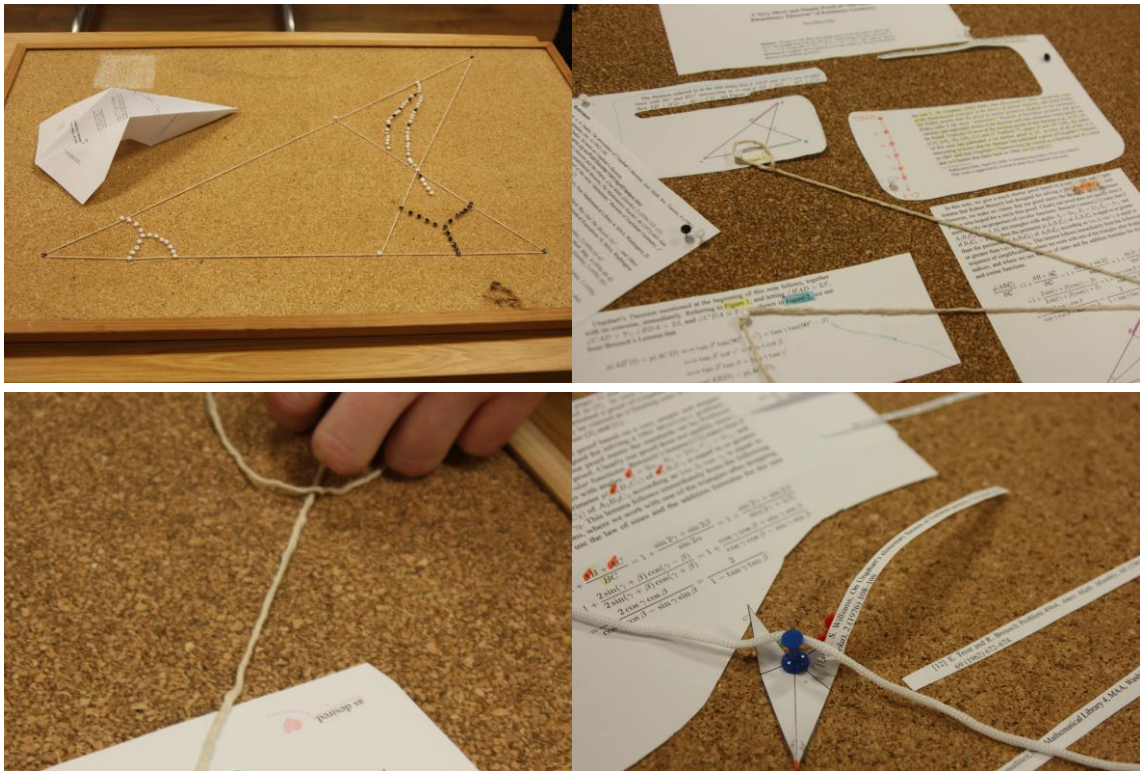

Figure 5: *Images from the previous workshop—the reconfigurations from group 1, 2, 2 and 3*

The first group opted to simply construct the main diagram from the paper in physical form, experimenting with a method of construction using only one piece of string, and playing with the points of the triangle to explore the theorem. Then, interested by the arrowhead shape, they folded a page of the paper itself to match, transforming the paper itself from a transparent vehicle for content back to a physical material with sculptural properties.

The second group set about organising the paper to make important links within the text more intuitive, laying it out such that references to diagrams were placed next to the diagrams themselves, and using colour-coding and string to make such links more explicit. The string links themselves made another triangle, referencing the forms of the diagrams in the text. They also highlighted an interestingly emotive moment in the text, surprised by the phrase "as desired" appearing in a mathematical text.

The third group were excited to tell a story in their presentation and so took control of the narrative of the paper, reorganising the text to give it the trajectory that they wanted. They placed the text so that the history of the problem was given first, and the narrative progressed from there in the form of a story. They also placed the references projecting outward as they occurred in the text, links pointing outward to extra information.

This smaller-scale experiment had very exciting results; in particular the narrative experimentation with the text was a surprise, as was the returning of paper to its often-forgotten sculptural nature. The audience at Bridges is both intensely creative and mathematically sophisticated, and the ideas brought by

such an audience seem certain to have exciting results; it will be particularly interesting to see how more familiarity with mathematics will affect the approaches taken.

Summary and Conclusions

This workshop hopes to leverage the creativity and expertise of the Bridges audience in a creative experiment that engages with material from contemporary mathematical research. An experimental engagement with such material may serve to help participants to feel more empowered in relation to those texts, and encourage them to bring new and alternative ways of seeing and perceiving into play in forming those texts. Breaking with the usual ways of engaging with such texts is a way to see them anew and better perceive the ways that they function. Understanding the forms that mathematics currently takes and the reasons why they have developed in that way is as important as finding ways to rethink them, and such experimental engagement is a means to achieve both of these aims.

Acknowledgements

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