

Method for Designing a Hinged Cube Puzzle

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Abstract

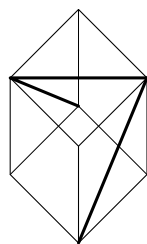
The paper describes a method for creating a hinged cube puzzle and its geometrical analysis. A hinged cube puzzle is a puzzle whereby a cube is dissected into a number of identical units and hinged together along its edges to form a long chain of units. This chain when folded in the correct sequence along the various hinges forms a cube, thus solving the puzzle.

Introduction

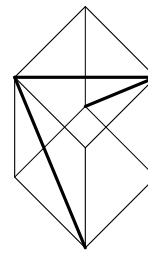
This hinged cube puzzle dissects a cube into twelve polyhedral pieces. Some of these units are mirror image units of the other. The units are hinged together along their edges to form a continuous chain of linked units. The puzzle is solved by folding units in their correct order to form a cube. This paper explores one type of dissection where a cube is divided into 12 congruent units, 8 of one handedness and 4 of the other. Geometry and configuration of units thus formed is analyzed and classified.

Cube Dissection

A unit cube is selected and diagonals are drawn on three of its faces such that they form a continuous line. This can be done in two different ways as shown in Figure 1 and 2. These are named as S-division (clockwise division) and Z-division (anti-clockwise division). This paper explores S-division only.



Clockwise



Anti-clockwise

Figure 1: *Division of cube - 'S division'* **Figure 2:** *Division of cube - 'Z-division'*

Similar diagonal lines are drawn on the remaining three faces. Additional two parallel lines straddling along the diagonals and passing through midpoints of edges are constructed (Figure 3). This network of lines divides the surface of the cube into 12 trapezoids and 12 triangles. Each trapezoid is one face on a 5-sided, wedge-shaped polyhedron unit. Eight of these polyhedra (called the alpha-units), together with four mirror images of them (called the beta-units) form the whole cube. Beta-units are depicted in light grey hatch in this paper. The "unit" in the decomposition of the cube is the cone from the centre of the cube. That is, it is a convex polyhedron with six vertices, one on the centre of the cube and the other five being the vertices of trapezoid and triangle formed on the surface of the cube.

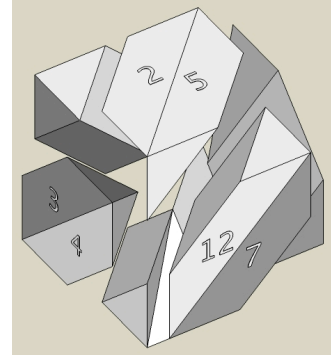
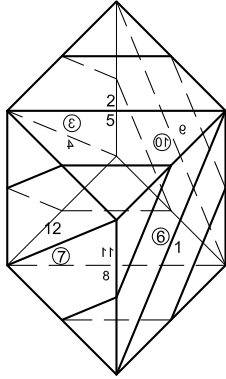


Figure 3: Slicing edges of the cube and unit numbering **Figure 4:** Exploded view of units forming a cube. (numbers with circle denote beta-unit)

Geometry of Units

Figure 5 shows development of the surface of a unit. The five faces are called R,Q,T,I and P face; for Right-angle triangle, Quadrilateral, Trapezium, Isosceles triangle and Parallelogram respectively. Figure 6 shows the geometry of the unit cut out of a prism of dimensions $\sqrt{2}/4 \times 1/2$ with a length of $\sqrt{2}$.

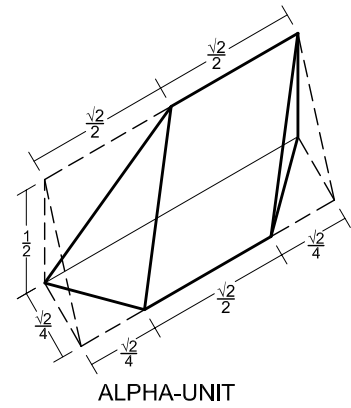
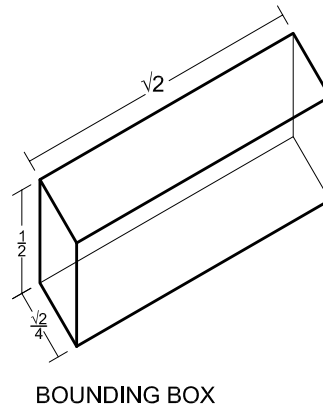
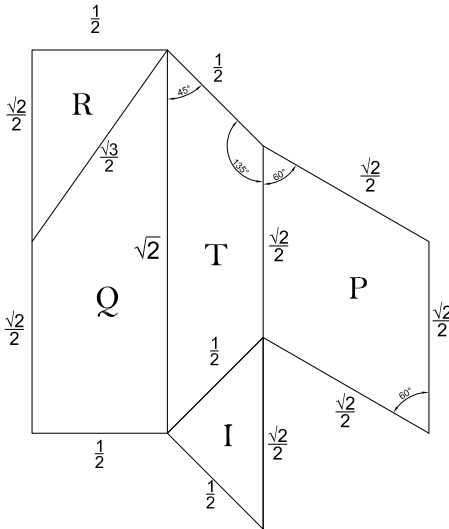


Figure 5: Development of surface of the unit. **Figure 6:** Geometry of alpha-unit.

Possible Configurations of Units

The position of units are identified with its number from 1-12 (Figure 3). The position of a unit in relation to one of its adjacent units gives rise to six different possible configurations. They are named according to common connecting faces between them. Eg: If the Quadrilateral face of alpha-unit connects with Quadrilateral face of alpha-unit, it is called Q-Q. If the Quadrilateral face of alpha-unit connects with Quadrilateral face of beta-unit, it is called Q-Q'. (A prime symbol is used to denote the presence of beta-unit). For simplicity these geometric configurations are color coded. (Figure 7)

TOP VIEW OF CONFIGURATIONS						
FACE CONNECTIONS	Q-Q	Q-Q'	P-P	P'-P'	R-R'	R-Q
COLOUR CODE	brown	red	yellow	orange	green	cyan
HINGE TYPES AT EXTERNAL COMMON EDGES	Q	I, Q'	A,A	A',A'	L	R
HINGE TYPES AT INTERNAL COMMON EDGES	C,C	Y,V	T,T	T',T'	V,C'	T,C

Figure 7: Different configurations of two adjacent units.

Possible Hinges Between Configurations

Each configuration of a pair of units has one common intersecting face. The two units could be hinged along each edge of the face. This gives rise to the number of ways the units can be hinged. It should be noted that in the case of configuration Q-Q and R-Q, the faces don't entirely meet. There is only a partial overlap between two faces, as a result of which the hinges are formed between an edge of one unit and along a line within the face of the other unit. The configuration Q-Q' has a common intersecting face that is a quadrilateral; thus these two pieces can be hinged in four different ways. A study of all six different configurations reveals that there are in total 13 types of hinges possible. Hinge types are classified as an external or an internal hinge depending on whether a hinge lies on the surface of cube, or within the cube. These hinge types are shown in Figure 8. The hinge along an edge is depicted by sign '=' in the diagram. Shaded units indicate that it's a beta-unit. Solid lines denote that faces are parallel to the viewing plane, while fine lines indicate that they are not. A hinge type name is given the suffix of prime (') , when one or both units are replaced by beta-unit.

							EXTERNAL HINGES
Q	Q'	A	A'	I	L	R	
							INTERNAL HINGES
T	T'	Y	V	C	C'		

Figure 8: Possible hinge types.

Graph Diagram of Configurations of Units

Figure 9 shows a graph diagram of the adjacencies and the configurations between units. The numbers in the graph are unit numbers according to Figure 3. Numbers with double circle around it denote a beta-unit. The coloured line and text denote the configuration type between two units.

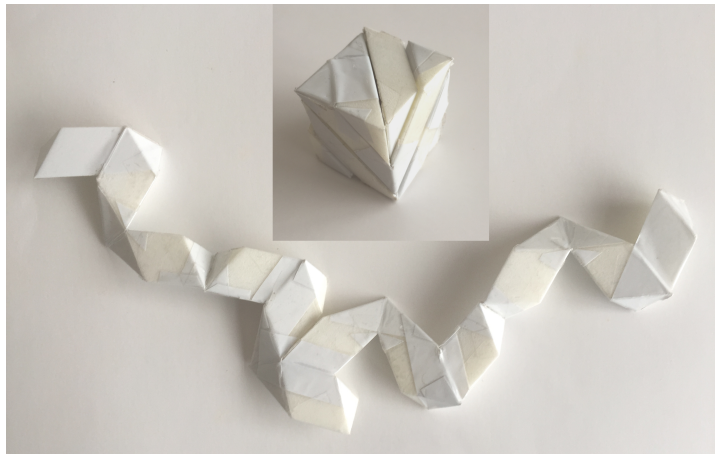
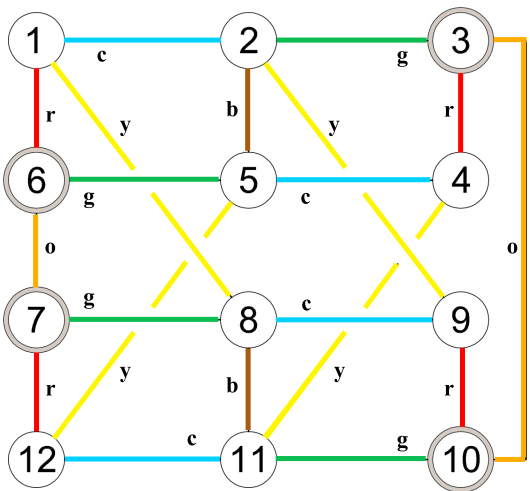


Figure 9: Graph diagram of units.

Figure 10: Photograph of prototype of the puzzle.

Constructing the Puzzle

The puzzle is designed by choosing one of the many possible Hamiltonian paths. As an example the Hamiltonian path 1-2-3-4-5-6-7-8-9-10-11-12 is chosen. A hinge type is chosen for each configuration. Depending on the configuration, there are 3 or 4 hinge types to choose from. An example of possible hinge combination is as follows: T-L-I-T-L-A'-L-T-I-L-T. The puzzle is complete when the units are hinged together in this configuration and a chain is formed. Multiple variations of the puzzle can be formed by selecting a different Hamiltonian path and a different hinge type between two units. A physical prototype of this puzzle with the above mentioned Hamiltonian path and hinge combination is constructed (Figure 10) and is shown to have a physically workable solution. It remains a question to know how many of all possible Hamiltonian paths and their possible hinge combinations lead to a physically workable solution.

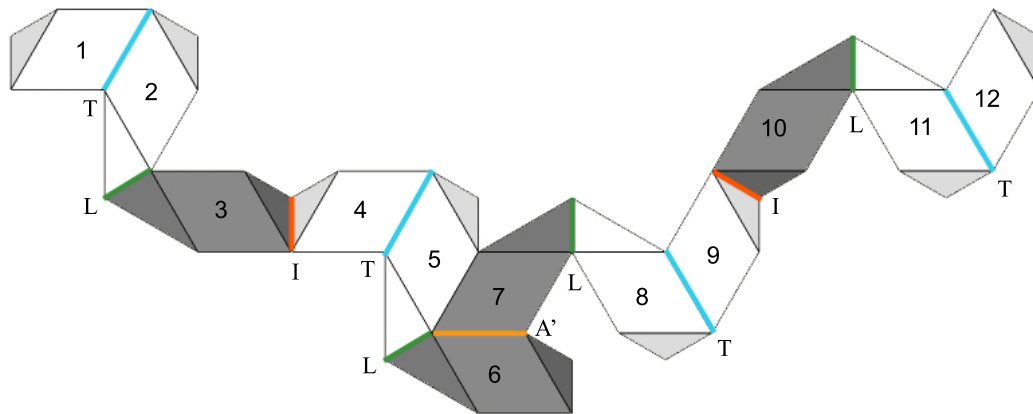


Figure 11: An opened up version of hinge sequence . T-L-I-T-L-A'-L-T-I-L-T