The Art and Mathematics of Cycling: Using Old Bicycles to Draw Spirograph Patterns

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Abstract

This paper describes a series of drawing machines made from scrap bicycles by the author. The characteristics of these machines are discussed, as are the maths and geometry of the curves that can be drawn with them. These patterns include: *hypotrochoid* (*"Spirograph"*), *harmonograph* (aka *Lissajous curve* or *"pintograph")*, and *epitrochoid* ("reverse *Spirograph*"). Some context is provided: science-inspired art made by the author from everyday materials, geometric drawing toys, and other artists investigating similar themes.

Introduction

Like many children in the 1970-80s, I had a *Spirograph* set (first marketed in 1965 by Denys Fisher). These were comprised of several plastic cogs and toothed wheels, which could be used to draw geometric patterns. Other geometric drawing toys marketed around the same era include *Hoot Nanny 'The Magic Designer'*, *Whirly Wizard*, *Design-o-Graph*, *Sketch-a-Graph*, *Harmonograph* and *Etch-a-Sketch*. My drawing machines were also inspired by my love of cycling, and recycling.

Some of my inspiration also comes from kinetic sculptures made by artists including Robert Howsare, Conrad Shawcross, Rowland Emett, and Stephen Pippin.

The bicycle drawing machines I have made fit into my wider body of work, in which I explore the connections between maths, science and art. In my work I relate the abstract world of maths to the everyday materials of human life. I make sculptures, functional items and photographs – often from everyday or recycled materials. My works include drawing machines, pinhole cameras, shelter domes and spherical sculptures.

Bicycle Spirograph 1 (Sand)

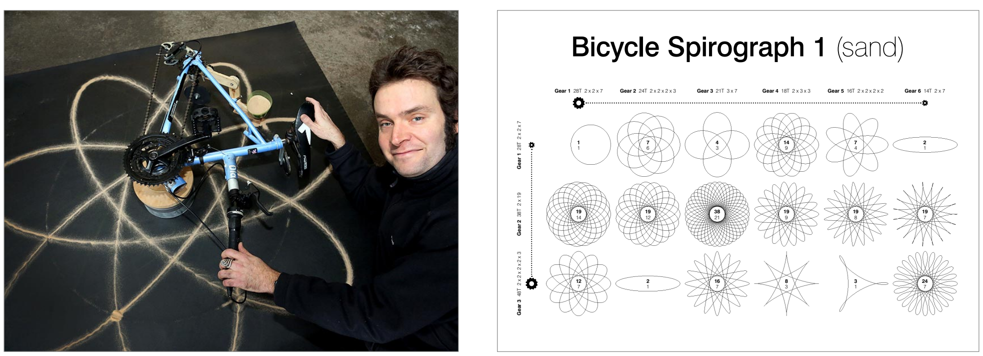

Figure 1: *Bicycle Spirograph 1 (Sand), 2012, and the 18 patterns that can be drawn with its 18 gears.*

I made the first of my bicycle drawing machines, *Bicycle Spirograph 1 (Sand)*, in 2012 (Figure 1). This came about partly because someone saw my *Giant Pantograph* (2011) body-drawing machine, and referred to it as a "Giant Spirograph"! I wanted to see if the differential in speed between the crank and back wheel of a bike might produce Spirograph (*hypotrochoid*) patterns.

The machine consists of the sawn-off back half of a bicycle, laid on its side, with stabiliser wheels beneath. I replaced the back wheel with a wooden drawing arm, with a plastic bottle for sand or paint attached at the end. I also made an optional pen/pencil attachment. Rotating the bicycle frame around the stationary crank produces 1.5-metre diameter patterns. Shifting between the bike's 18 gears (3 front chain-rings x 6 rear sprockets) produces 18 patterns (two of these patterns are the same – simple ellipses – as their cog teeth ratios, 48:24 and 28:14, simplify to 2:1). A pen can be attached, to draw on large pieces of paper. However, I discovered it could also be used with a bottle of sand, to draw temporary patterns on a mat. These can be quickly cleared away and another pattern drawn in its place, which is useful for demonstrating them at schools and science events. I also like the fact it can be used with water-based paint or pigment to draw on roads – a bicycle making a beautiful mark!

Construction of the machine involved finding a scrap bicycle from a local bike shop. The one I found had a cracked seat post, from where the previous owner had raised the saddle past the minimum insertion line. Despite cutting the bike in half, I wanted to keep its functional appearance, so I kept the saddle, pedal and even its bell! I made the drawing arm from wood and plastic, for contrast.

Bike Cog Spirograph Drawing Activity

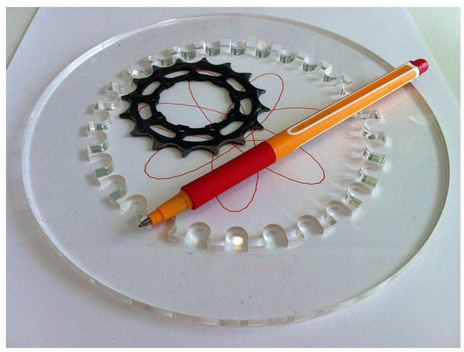

Figure 2: *Bike Cog Spirograph drawing activity, 2012.*

I created this drawing activity (Figure 2) as a companion piece to *Bicycle Spirograph 1 (Sand)*. It consists of a set of separate bicycle gear sprockets (cogs) and specially laser-cut, internally-toothed acrylic wheels that I designed with Adobe Illustrator. The teeth of these wheels mesh with the bicycle gears, which have a standard 1-inch pitch, allowing Spirograph patterns to be drawn.

Bicycle Harmonograph

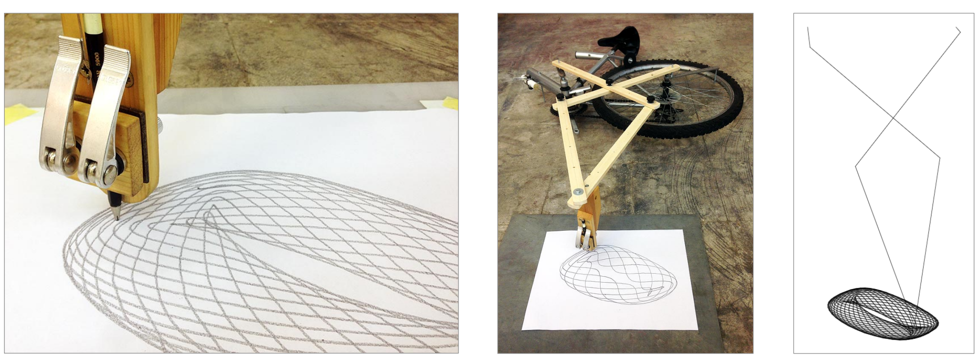

Figure 3: *Bicycle Harmonograph (2016), and simulated pattern using Processing language.*

Developing the theme set by *Bicycle Spirograph 1* – and inspired by pendulum- and turntable-based drawing machines I'd seen – I built *Bicycle Harmonograph* in 2016 (Figure 3).

With this machine, the bicycle frame stays stationary, and the user turns the crank arm. A wooden *linkage* is pivoted at the crank and back wheel, and scissors back and forth as the crank and wheel spin at different speeds. The resulting patterns are similar to a rectangular-based *Lissajous curve* (the phase pattern between two sine waves of different wavelength) or *harmonograph* (the phase pattern between two pendulums swinging at different speeds). This sort of curve has been dubbed a "*pintograph*" due to its use of a *pantograph*-like linkage to produce curvaceous distortion. The patterns resemble Spirographs that have been warped out of shape, or doughnuts with fishnet stockings on!

There are various parameters that affect the patterns drawn with this machine. I have locked a few of these parameters down for practical purposes (so that pattern switching is quick at science events, and so patterns fit on A3 art paper). However, it has been interesting to investigate how these factors affect pattern size, complexity, width and "sway". I experimented with different paramaters by writing a simulation app using the *Processing* language:

Crank and back wheel radii (variable) – affects pattern width/sway

Gear ratio (variable, but kept at 34:21 for complex patterns) – affects pattern complexity

Distance to linkage cross-over pivot (variable) – affects "scissoring" and pattern height

Lengths of the four linkage struts (fixed at 50cm each) – affects pattern width and height

Bicycle Spirograph 2 (Paper)

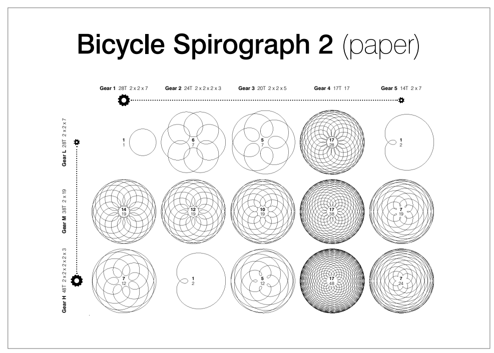

Figure 4: *Bicycle Spirograph 2 (Paper), 2017, and the 15 patterns that can be drawn with its 15 gears.*

I made *Bicycle Spirograph 2 (Paper)* (Figure 4) following on from my previous Spirograph machine, to allow users to take more manageable-sized drawings away on paper. This would be particularly nice for science festival participants! I also discovered that the pattern-drawing is quite hypnotic in action.

The back wheel of the machine is used as a rotating drawing surface, to which 30cm square art paper is attached. Turning the crank creates the patterns – the crank powers both the back wheel (which turns at a different speed), and the circular drawing motion, which is scaled down by 50% via a *pantograph* linkage. This scaling allows the patterns to fit within the paper edges.

The machine draws Spirograph-like *epitrochoid* patterns. Unlike conventional Spirograph *hypotrochoids*, however, these curves have their "petals" or lobes on the inside of the circle. Similar curves can in fact be drawn with Spirograph, but only if the small cog wheel is rolled around the outside of the larger wheel.

This came as a surprise when making the machine – I expected the patterns to be hypotrochoids. Hypotrochoids are formed in classic Spirographs when a small cog moves slowly clockwise around the inside of a larger, toothed wheel. In doing so, the smaller cog turns anti-clockwise at a higher speed (being smaller). I expected the same of this new machine, as the "inner wheel" (crank) is turning anti-clockwise while the drawing is being made clockwise (on the anti-clockwise spinning drawing surface).

The reason the machine draws epitrochoids is because of the relative *speeds* of the inner and outer rotations: this is a slow-turning inner circle moving fast around an outer circle.

Common Theme: Mathematics and Geometry of Phase Patterns

All of these machines and drawing activities share a common theme: two wheels rotating at different speeds to create phase patterns. The number of petals/lobes of these patterns, and the number of turns of the wheel to create them, can be calculated from the numbers of teeth on the two cogs:

For example, for cogs with 48 and 28 teeth: Their prime factors are $48 = 2^4 \times 3$ and $28 = 2^2 \times 7$. Their lowest common multiple is $336 = 2^4 \times 3 \times 7$. The 48-tooth cog will thus have to turn 7 times (and make 7 petals), and the 28-tooth cog will turn 12 times ($2^2 \times 3$ – the number of times the machine crank will have to be turned), to complete the pattern.

How these phases translate into patterns is different for each of the machines, but the underlying principle is the same.

Summary and Conclusions

All of the machines I've made have been an education for me in both mathematics and engineering. It has given me a better understanding of prime number factorisation, introduced me to a world of mechanical bearings (thrust bearings for pivots, axle bearings for wheels, etc) and improved my engineering skills (working with component weighting, tolerances, counterbalancing, etc).

More importantly, I have taken them to school and art/science events (including Maker Faire, Abu Dhabi Science Festival, London Science Museum, TEDxBrighton and WOMAD), where I have used them in an educational context to demonstrate mathematical and engineering concepts in a fun, quirky and entertaining way. People enjoy making patterns without the use of computer, and can be quite hypnotised by the patterns being drawn.

I'm constantly making tweaks and improvements to these machines, mostly to make them child-safe when I take them to different events.

I plan to make more machines, including a traditional pendulum harmonograph, camera obscura, giant *Pinscreen* board, etc.