Landmarks in Algebra Quilt

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Abstract

The Landmarks in Algebra quilt was an effort to include as many cultures that contributed significantly to the development of algebra as we know it today. Each section of the quilt illustrates a culture or a mathematician that made important advances in the development of algebra. Space did not allow more than four cultures, even though other nationalities made contributions. Babylonian mathematicians, Greek mathematicians, Indian mathematicians, and Persian mathematicians are highlighted for their important work.

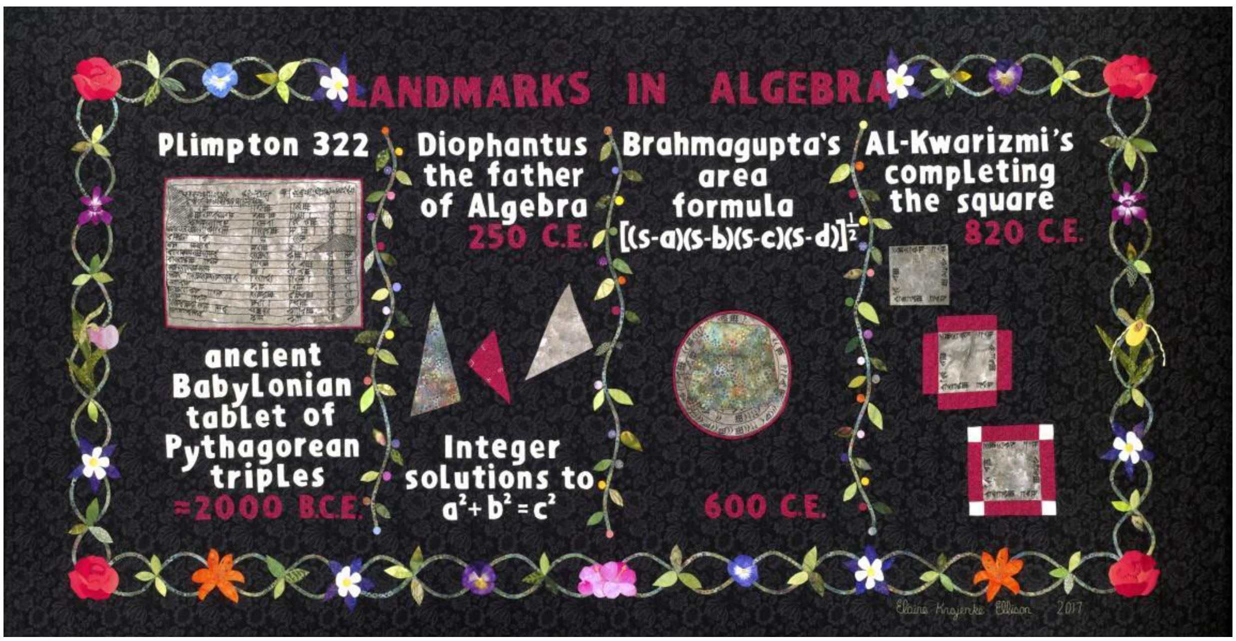


Figure 1: *The Landmarks in Algebra quilt, 82 inches by 40 inches.*

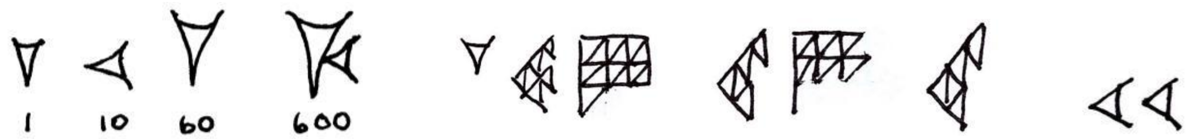
First Panel: Plimpton 322

The first panel of the quilt illustrates the Plimpton 322 tablet. The Plimpton 322 tablet inspired me to design a quilt based on algebra. Recent historical research by Eleanor Robson, an Oriental Scholar at the University of Oxford, has shed new light on the ancient work of algebra [8]. The 4,000 year old Babylonian cuneiform tablet of Pythagorean Triples was purchased by George Arthur Plimpton in 1923 from Edgar J. Banks. Banks said the tablet came from a location near the ancient city of Larsa in Iraq [4]. Our first knowledge of mankind's use of mathematics comes from the Egyptians and the Babylonians [1].

The Babylonian "texts" come to us in the form of clay tablets, usually about the size of a hand. The tablets were inscribed in cuneiform, a wedge-shaped writing owing its appearance to the stylus that was used to make it. Two types of mathematical tablets are generally found, *table-texts* and *problem-texts* [1].

The Plimpton 322 tablet gives a table of Pythagorean triples in Babylonian Cuneiform script [9]. In the Plimpton 322 tablet, column 1 shows the longest leg of a right triangle. Column 2 shows the shortest leg of a right triangle. Column 3 shows the hypotenuse. Finally, column 4 is a numbering of the item. There are 15 items in this tablet. The tablet has been recently dated to have been written between 1822-1762 BCE.

Babylonian numbers were positional and sexagesimal, meaning they were written in base 60. There was no use of zero and not all fractions were allowed. The Babylonians could extract square roots and solve linear systems. They could also solve cubic equations [1].



(a) (b) (c)

Figure 2: *(a) Cuneiform numerals, (b) a sexagesimal number sample, (c) "two birds"*

Figure 2a shows several examples of cuneiform numerals. Figure 2b is a sample of the sexagesimal number 1,57,46,40 written using the cuneiform symbols. We use base 60 to convert this to a decimal number: $1 * 60^3 + 57 * 60^2 + 46 * 60^1 + 40 * 60^0 = 424{,}000$. The symbol in Figure 2c is called "two birds." Two birds could mean $10 * 60 + 10 = 610$, or $10 * 60^2 + 10 = 36010$, or $60 + \frac{10}{60} = 60$, or just 20. The notation is ambiguous thereby making it impossible to decipher what exact right triangles are in the Plimpton 322 tablet.

Second Panel: Diophantus

The second quilt panel highlights the contributions of Diophantus. Sometimes called the Father of Algebra, Diophantus was born between 201 and 215 CE. He died between 285 and 299 CE. He was the first Greek who recognized fractions as numbers [6]. Little is known about the life of Diophantus. He lived in Alexandria, Egypt most of his life. In modern use, Diophantine equations are usually algebraic equations with integer coefficients, for which integer solutions are sought. This quilt panel shows the right triangles 3-4-5, 8-15-17, and 20-21-29. These triangles are all integer solutions to the well-known Pythagorean Theorem.

Much of our knowledge of the life of Diophantus is derived from a 5th-century Greek anthology of number games and puzzles created by Metrodorus [5]. One of the problems states:

Here lies Diophantus, the wonder behold.
Through art algebraic, the stone tells how old:
God gave him his boyhood one-sixth of his life,
One twelfth more as youth while whiskers grew rife;
And then yet one-seventh ere marriage begun
In five years there came a bouncing new son.
Alas, the dear child of master and sage
After attaining half the measure of his father's life chill fate took him.
After consoling his fate by the science of numbers for four years,
he ended his life.

Translating the puzzle into algebraic symbols we get the equation $x = \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4$, where x Diophantus' age. The solution, x = 84, implies that Diophantus was 84 when he died. However, the accuracy of the information cannot be independently confirmed.

While reading Claude Gaspard Bachet de Méziriac's edition of Diophantus' *Arithmetica*, Pierre de Fermat concluded that a certain equation considered by Diophantus had no solutions. He noted (in French) in the margin, "If an integer n is greater than 2, then $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c. I have a truly marvelous proof of this proposition which this margin is too narrow to contain." A proof of what is now called "Fermat's Last Theorem" was found in 1994 by Andrew Wiles after working on the theorem for 7 years [5].

Third Panel: Brahmagupta

The third panel in this quilt highlights Brahmagupta's contribution to algebra. Brahmagupta lived from 598 CE to roughly 665 CE. In 628 he wrote the *Brahmasphuta-siddhanta*, where zero is clearly explained, and where the modern place-value Indian numeral system is fully developed [9]. It also gives rules for manipulating both negative and positive numbers, rules for summing series, Brahmagupta's identity, and the Brahmagupta theorem. He expounded on rules for dealing with negative numbers [2].

His work on cyclic quadrilaterals was a significant contribution to mathematics. A cyclic quadrilateral is a quadrilateral whose vertices all touch the circumference of the circle. Given the length of four sides of a cyclic quadrilateral, *a*, *b*, *c*, and *d*, the area of the quadrilateral is $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where *s* is the semiperimeter of the quadrilateral.

One can find the proof Brahmagupta's formula for the area of a cyclic quadrilateral at [3]. Using the given diagram for reference and the law of sines, we can formulate the area of the cyclic quadrilateral by adding the areas of Δ POQ and Δ QOR.

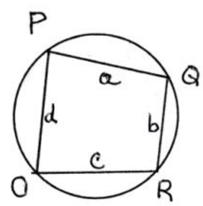


Figure 3: *A cyclic quadrilateral*

Brahmagupta's formula is related to Heron's formula. Heron's formula for the area of a triangle comes from Hero of Alexandria 10 - 70 CE. Heron's formula for the area of a triangle is $\sqrt{(s-a)(s-b)(s-c)}$, with *a*, *b*, and *c* the length of the sides of the triangle. All triangles are cyclic. Heron's formula was found years before Brahmagupta's. However, Brahmagupta's formula can be used to find the area of a triangle by setting one of the sides of the quadrilateral equal to zero.

There are other theorems that relate to Brahmagupta's and Heron's. Some of these include Ptolemy's theorem, the Japanese theorem, and Parameshvara's theorem on circumradii. The reader is encouraged to investigate these and other related theorems.

Fourth Panel: Al-Khwarizmi

The fourth panel of this quilt highlights the contributions to algebra of the Persian mathematician, Muhammad ibn Musa Al-Khwarizmi. Al-Khwarizmi was born in 780 CE and died in 850 CE. The word algebra is derived from operations described in the treatise written by this Persian mathematician. The treatise was titled Al-Kitab al- Jabr-wa-l-Muqabala meaning "The Compendious Book on Calculation by Completion of Balancing" [9]. The Arabic word al-ğabr ("forcing", "restoring") is a process of moving a deficient quantity from one side of the equation to the other side.

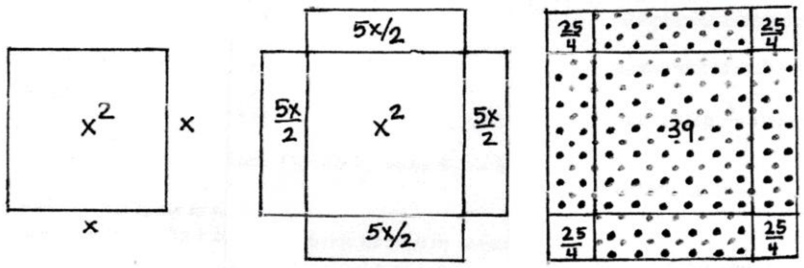


Figure 4: *Al-Khwarizmi's Completing the Square*

Figure 4 shows how to solve the equation $x^2 + 10x = 39$ by Al-Khwarizmi's "completing the square" method [7]. Start with a square of side x. The area of this square is x^2. Add four small rectangles of length x, and width $\frac{5}{2}$. Each small rectangle has an area of $\frac{5x}{2}$, hence four of these rectangles is a total area of $10x$. We know this second figure has a total area of 39. Complete the square by adding 4 small squares with side length $\frac{5}{2}$. The area of each small square is $\frac{25}{4}$. The outside square has an area of 39 + 4 $(\frac{25}{4})$ or 64. Hence each side of the square has length $8 = x + \frac{5}{2} + \frac{5}{2} = x + 5$ giving $x = 3$ as the solution. This technique is similar to the method we use today to complete the square.

Conclusion

The Landmarks in Algebra quilt highlights four significant areas of contribution to the development of algebra. During the last 4,000 years of recorded mathematics, other mathematics and mathematicians have contributed to algebra as we know it today. Research showed the four contributions illustrated in this quilt as being of significant value. The space constraints of a quilt limited what mathematics could be honored.

Acknowledgements

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References

[1] Babylonian Mathematics. http://www.math.tamu.edu/~dallen/masters/egypt_babylon/babylon.pdf.

[2] Brahmagupta. http://www.storyofmathematics.com/indian_brahmagupta.html.

[3] T.Brooks and K. Fischbein, http://jwilson.coe.uga.edu/emt725/Class/Brooks/Brahmagupta/Brahmagupta.html.

[4] B. Cipra. "Rewriting History." *What's Happening in the Mathematical Sciences*, vol. 5, AMS, 2002, pp 54–59, http://www.ams.org/publicoutreach/happ5-history.pdf.

[5] Diophantus. https://en.wikipedia.org/wiki/Diophantus.

[6] Diophantus. http://www.storyofmathematics.com/hellenistic_diophantus.html.

[7] Islamic Mathematics–Al-Khwarizmi. http://www.storyofmathematics.com/islamic_alkhwarizmi.html.

[8] E. Robson. "Drawing of Mathematical Cuneiform Tablet Plimpton 322, photograph by Otto Newgebauer and Abraham Sachs, 1945." Courtesy of the American Oriental Society.

[9] Timeline of Algebra. https://en.wikipedia.org/wiki/Timeline_of_algebra.