‘Dreamlinks’: Link Theory Meets Music Composition. An Introduction to Compositional Methods Related to Primary Links

Saverio Tesolato

Dept. of Mathematics, Liceo Celio-Roccati, Rovigo, Italy; autunna@libero.it

Abstract

New methods of music composition are illustrated, focusing on the relationships between music and topology. Composition techniques of a vocal piece are shortly described, adopting new procedures related to modern dodecaphonic music and able to translate some topological notions of link theory - primary links and their three-dimensional shapes - into music.

S’engouffrer Dans Ces Nœuds… (for Soprano and Mezzosoprano)

Over the years, many artists have explored the intersections of mathematics and art, focusing in most cases on visual arts, rather than on music. Here, though, it will be shown how music composition can draw on topology.

In knot theory, a link is called *primary* if it is not a connected sum of non-trivial links. According to Rolfsen [1] classification of links, the simplest primary links formed by two threads are designated by the notations 0^2_1, 2^2_1, 4^2_1, 5^2_1, 6^2_1, 7^2_1, 8^2_1, where numbers, superscripts and subscripts indicate respectively the number of crossings, the number of threads, and the link order.

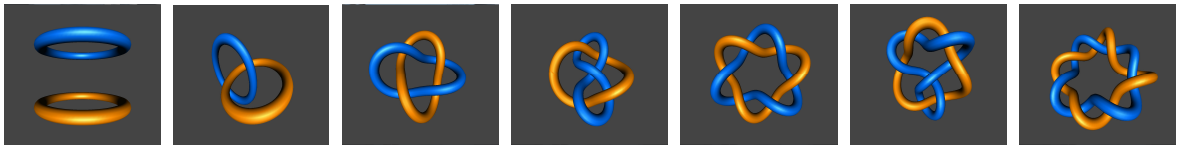

Figure 1: *The first seven primary links of first order: 0^2_1, 2^2_1, 4^2_1, 5^2_1, 6^2_1, 7^2_1, 8^2_1 respectively.*

The project for the score starts from the idea of “translating” these links into an equal number of musical phrases: by assigning a single thread to each vocal part (soprano/mezzosoprano), two series of notes for 0^2_1 are chosen. Afterwards, in order to create the phrases related to the other links, these series have been manipulated following several hereby described criteria. Since the number of crossings is increasing, time signatures of consecutive phrases will have an increasing number of beats per bar.

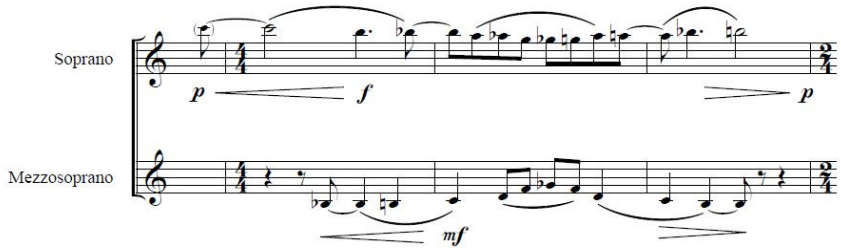

Figure 2: *S. Tesolato, ‘S’engouffrer Dans Ces Nœuds…’ (2016), bars 1-3 of the score.*

The first 3 bars of the composition represent 0^2_1, the so-called *unlink*: here the voices do not actually intersect. The textures of the soprano and the mezzosoprano are in fact narrow enough, with the first vocalist singing only high notes, and the second relatively low ones. I used a single chromatic series of notes that I cut in two parts assigning them to each voice: the soprano sings from C5 down until G♭4 and

then up again, closing the loop, and forming a sort of "wave" on her staff, due to the phrase ending with B4. Likewise, the mezzosoprano sings from B♭2 up until G♭3 and then down again until B2; the two central G-flat notes are synchronic. If the beginning of bar 1 were sequenced after the end of bar 3, the soprano part would become a sort of (musical) "ring". As for the mezzo part, symmetrical pauses interfere in a way that apparently prevents the loop to close**.** As a matter of fact, the parts of soprano and mezzo have not the same length, because the entering voice is always represented by the upper thread in the diagram, which is the first one to be visually perceived, appearing therefore longer. Likewise, the reason why the notes duration decreases from the edges to the centres of each phrase depends upon how threads are perspectivally perceived: longer notes represent the points of the threads that are closer to our sight, and vice-versa. Symmetries in each phrase reproduce intrinsic symmetries of links.

2^2_1: here the meter turns to 7/8 and once again the block occupies 3 bars. It relates to the *Hopf link*. This time, the mezzo starts first, while the soprano has symmetrical pauses of a quarter note (slightly shorter than mezzo's ones in 0^2_1). From now on, we will have (two, in this case) *glissando* notes representing the acoustic equivalent of the link crossings. The soprano has longer glissandos, which incorporate the ones sung by the mezzo: in any case, both voices glide for a duration of a subdivision of an irregular group. This is the case of *nintuplets* (afterwards, in a symmetrical form, the latter will be just at the edges, while in the middle we will have sextuplets).

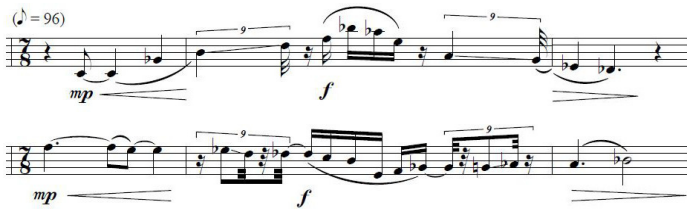

Figure 3: *S. Tesolato, 'S'engouffrer Dans Ces Nœuds…' (2016), bars 5-7 of the score.*

The series used here for the soprano part is a 12-note series (denoted as *C-series*): C-G♭-B-D-F-B♭-A♭-E-A-G-E♭-D♭. The first six notes are ascending while the other ones are descending. *D-series* (14 notes), which has been created for the mezzosoprano, is a chromatic series that has been cut in two in order to reverse the second half. (see Figure 3). Hence, the mezzo sings the first half in descending notes and the second half in ascending notes: in this case the "ring" structure of the full phrase is not as smooth as the one sung by the soprano, but there is a fifth interval between the end of bar 7 and the beginning of bar 5 (B♭/F). Note that ascending and descending segments *intersect each other* during the glissandos only (for instance, the soprano enters the glissando with B3 and comes out of it with D4, while the mezzo begins with E♭4 and ends with D♭4, just as the soprano does with D4). This means that if the beginning and the end of the phrase were sequenced, the structure would take the form of a Hopf "musical" link. The choice of glissandos as acoustic devices was due to the need to simulate that instant of bewilderment when, to the viewer's perspectival perception, one thread is partially hidden by the other. Two notes occupy the lateral bars for both vocal parts, whereas the glissandos are placed at the borders of the central bar.

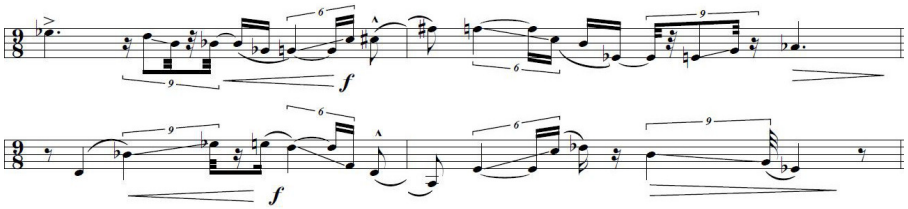

Figure 4: *S. Tesolato, 'S'engouffrer Dans Ces Nœuds…' (2016), bars 9-10 of the score.*

The remaining links have been translated into music following similar procedures. In **4^2_1** (*Solomon*'s link, bars 9-10) the roles between the two voices exchange, compared to 2^2_1, and the rhythmical situation

develops identically, except for the announced variation, due to the introduction of new irregular groups, such as two sextuplets, while glissandos represent again the acoustic equivalent of crossings.

In the same way, the series have been swapped, with the soprano taking the D-, and the mezzo taking the C-series. Additionally, both series were transposed, down a tone for the soprano, and up a tone for the mezzo. Furthermore, to both series two notes were added according to the schemes shown in Figures 5-6. Therefore, the soprano series has now 16 notes and the mezzo 14. In the soprano series, a further subdivision has been made by grouping the three middle sets of 4 notes: the very central one has been transposed up a tone, the other two down a tone (see Figure 5 - right). Finally, the two added notes are chosen to be one semitone lower than the neighbouring notes occupying the central block.

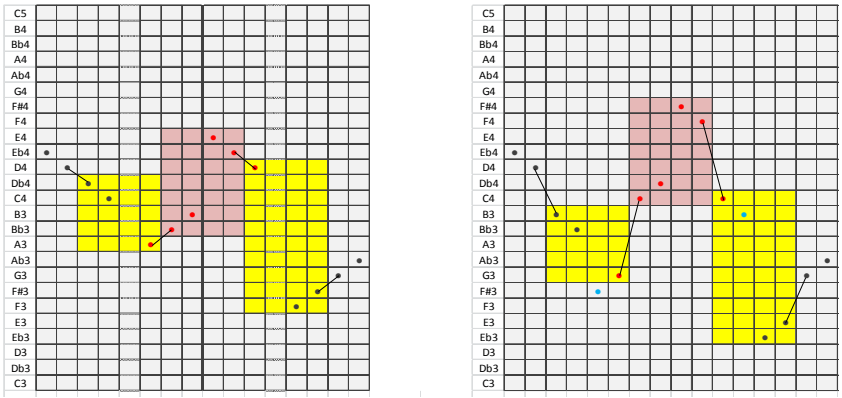

Figure 5: *Scheme of the manipulations applied to D-series to obtain the soprano series for $4^2{}_1$. On the left the first version, directly from $2^2{}_1$ (-2 semitones); on the right the actual series.*

As for the mezzo series, similar ideas were applied, starting from the C-series. The full procedure has been adopted to expand the series in order to allow vocal crossings at every glissando, keeping the wavy course of the two parts (see also Figure 6).

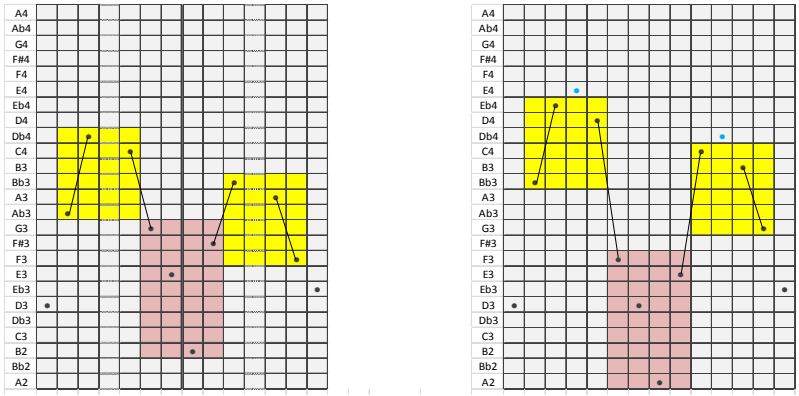

Figure 6: *Scheme of the manipulations applied to C-series to obtain the mezzosoprano series for $4^2{}_1$. On the left the first version, directly from $2^2{}_1$ (+2 semitones); on the right the actual series.*

As for $\mathbf{5^2_1}$ (*Whitehead* link), I have applied similar ideas as in $4^2{}_1$. In this link, however, one thread has a self-intertwinement, and I have chosen the soprano to represent it (darker colour in Figure 1). This special feature of $5^2{}_1$ forced me to add *two* more glissandos, not associated to any tuplet. In fact, if you move along the soprano thread, you meet its self-intersection twice, seeing it the first time from underneath, and the second time from above.

When the soprano sings these regular glissandos, the mezzo sings long notes. The mezzo glissando in bar 12, overlapping those sung by the soprano, has to be ascending because the points it represents in the thread are perspectivally perceived *above* the soprano self-intertwinement (see Figures 1 and 7).

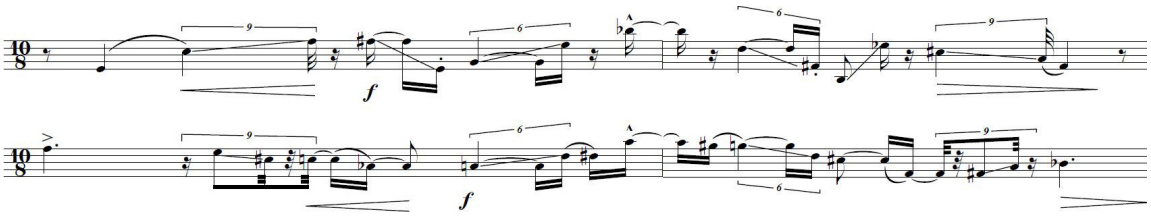

Figure 7: *S. Tesolato, 'S'engouffrer Dans Ces Nœuds…' (2016), bars 12-13 of the score.*

As regards the utilized series, these are directly related to 4^2_1 and subject to similar manipulations. About $\mathbf{6^2_1}$ (*Star of David*) the situation and the roles return to their setting in 4^2_1, with two more sextuplets for two symmetrical glissandos. Figure 8 shows the creation of the new glissando sextuplets for each voice, due to the broadening of 4^2_1 series to 19 notes for the soprano and 17 for the mezzo. Analogous procedures used for 4^2_1 are applied here for determining the series to utilize.

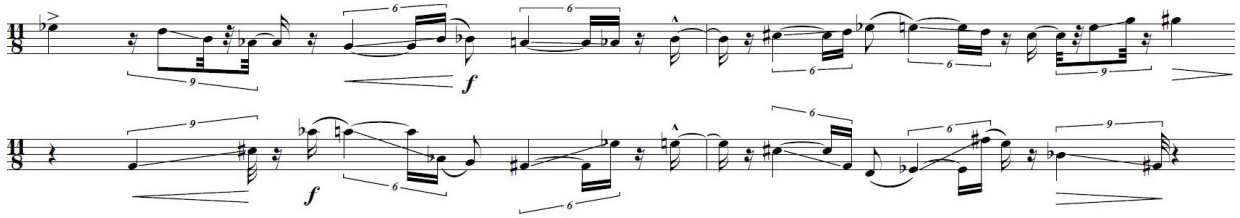

Figure 8: *S. Tesolato, 'S'engouffrer Dans Ces Nœuds…' (2016), bars 15-16 of the score.*

Figure 9 shows the parts of the score resulting for $\mathbf{7^2_1}$ (bars 18-19) and $\mathbf{8^2_1}$ (bars 21-22), along to similar procedures as shown above (7^2_1 obtained from 5^2_1, 8^2_1 from 6^2_1).

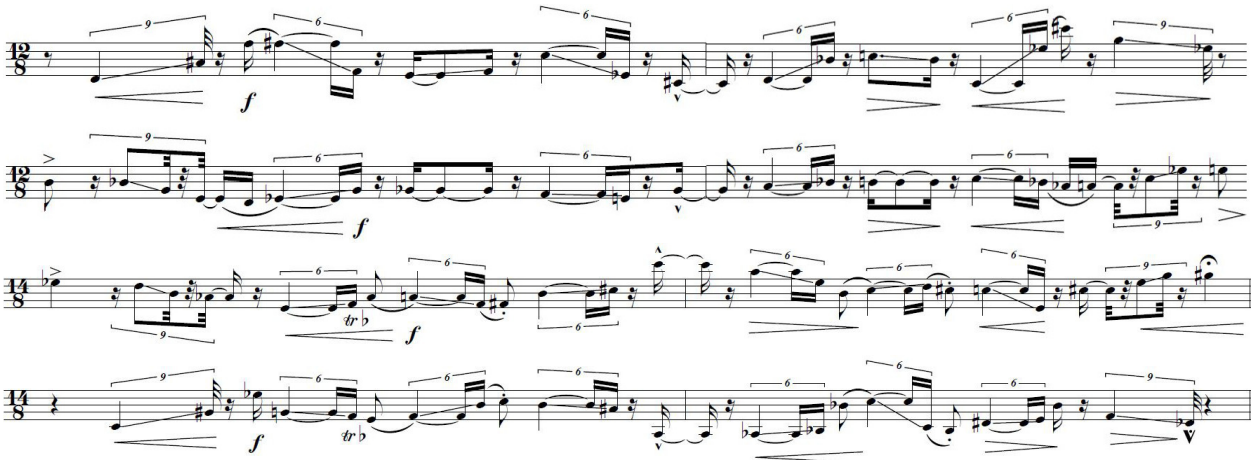

Figure 9: *S. Tesolato, 'S'engouffrer Dans Ces Nœuds…' (2016), bars 18-19 and 21-22 of the score.*

Summary and Conclusions

This compositional method has led to a work where the visual aspect of links has been transformed into a corresponding acoustic version [2]. This result can be a challenge for other composers to "translate" more primary links into music, not necessarily for vocals.

References

[1] D. Rolfsen. *Knots and Links*, AMS/Chelsea Publishing - American Mathematical Soc., 1976.

[2] S. Tesolato (Autunna et sa Rose). *S'engouffrer Dans Ces Nœuds…* on Spotify. https://open.spotify.com/album/58Rhc1gais2itQFmRkEeku.