# Hyperspace, Poetic Science Fiction and Algebraic Topology

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## Abstract

In this essay, I use the notion of hyperspace to link Stith Thompson's folklore motif of 'supernatural lapse of time in fairyland' with important works of science fiction by Bradbury and Asimov, with the construction of hyperspace in algebraic topology, in Phillips' acclaimed article, 'Turning a Surface Inside Out,' noting its literary flourish.

## The Outskirts of Philadelphia

As I child, I found my house littered with science fiction magazines and paperbacks. My father had fallen in love with science fiction as a teenager, which was odd, since although his father and his brother James were engineers, he wanted nothing to do with mathematics and science. However, love is divine madness. Once I was allowed to take the train all by myself from Strafford to Wayne, on the outskirts of Philadelphia, I went often to the bookstore on North Wayne Avenue and added to the collection. My father, also inexplicably, subscribed to *Scientific American*, where I got my first glimpses of what lay beyond trigonometry: it was thrilling. How much mathematics Martin Gardner managed to explain in his *Mathematical Games*! And how much topology Tony Phillips managed to convey in the full color diagrams that embellished his celebrated essay, "Turning a Surface Inside Out," and even curled around the cover of the issue in which it appeared [5]. I began to suspect that poetry, story-telling and argument stood in some positive relation to mathematics. The best way to capture my first inklings is to introduce the notion of a 'hyperspace.' (Let us not forget the importance of the Oxford Inklings! Tolkien, C. S. Lewis, and Owen Barfield, inter alia.) The word 'hyperspace' shows up often in science fiction, and typically means an alternative region of space (or space-time), co-existing with our own universe, to which one might have access in order, for example, to travel across the universe at a speed greater than the speed of light. As a little girl, I used to sit on the windowsill waiting for Peter Pan to take me to Neverland, or I kept walking into the back of my closet hoping to find the melting wall / doorway to Narnia. Sometimes, as I knew from reading my favorite books, one might gain access to an alternative world, side-by-side with ours, but somehow unnoticed. One need only pay closer attention and get lucky!

Many of my favorite science fiction stories involved just such a toggle: the newsstand where, depending on what coin you laid down for your paper, you would be connected to another time or place; perhaps this is why I always keep the odd pound, euro, shekel, rubel, krone or yen in my pocketbook. There was also the attic in Ray Bradbury's story "The Scent of Sarsaparilla": Mr. William Finch has been spending lots of time up in the attic, his unhappy wife notes. One day he comes back downstairs reeking of sarsaparilla, and another day wearing a brand new, highly outdated suit. He goes on and on about their courtship, forty years before, and almost seems to be raving: "...you know what attics are? They're Time Machines... Consider an attic. Its very atmosphere is Time. It deals in other years, the cocoons and chrysalises of another age... Wouldn't it be interesting... if Time Travel could occur? And what more logical, proper place for it to happen than in an attic like ours?" She thinks he is crazy, especially when he asks her to go with him (where?) and announces his departure. On the final page of the story, in the dead of winter, when at last she climbs up into the attic to see what he's doing, the attic is empty and the west attic window is ajar. There is a ladder down from the window to the porch roof; she looks out. "Outside the opened frame the apple trees shone bright green, it was twilight of a summer day in July. Faintly, she heard explosions, firecrackers going off. She heard laughter and distant voices. Rockets burst in the warm air, softly, red, white and blue, fading." Instead of following, she slams the window shut, trapped "in that November world where she would spend the next thirty years." [2]. The attic really was the middle term between the past and the present: it was the way out, the way back, to a time when Mr. Finch's life was explosive, brightly colored, and warm. Using Stith Thompson's celebrated *Motif-Index of Folk-Literature*, we might want to classify Asimov's fictions and the excursions of Phillips, Smale and Whitney, discussed in the next sections, with Walter Map's King Herla and Washington Irvings' Rip Van Winkle under F377, "Supernatural Lapse of Time in Fairyland." The combined proximity and inaccessibility of the other world does something odd to temporality; often those who stray into fairyland and then return home find that centuries have passed [7].

## Hyperspace in Science Fiction

But hyperspace turns up in a more technical sense in Isaac Asimov's famous story "The Last Question," first published in *Science Fiction Quarterly* in November 1956 [1]. Inspired by Einstein's theories of Special Relativity and General Relativity, which invoke a four dimensional space-time, writers of science fiction in the 1930s (when my father discovered them) began to use the term hyperspace to mean both a realm beyond but adjacent to our world and a fourth or fifth dimension, thus combining the folk tradition of fairyland and the new physics. Isaac Asimov used the idea in his *Foundation* series, which he wrote during World War II, in West Philadelphia while he was working as a civilian at the Philadelphia Navy Yard's Naval Air Experimental Station, before going on to finish his Ph. D. in Biochemistry at Columbia. It was first published as a series of eight short stories in *Astounding Magazine* between May 1942 and January 1950, and thereafter in three separate volumes between 1951 and 1953 by Gnome Press; the *Foundation Trilogy* won the Hugo Award for "Best All-Time Series" for science fiction and fantasy in 1966, over Tolkien's *The Lord of the Rings*, to Asimov's modest astonishment. He added sequels to the series between 1981 and 1993, inspiring and interacting with the writers of *Star Trek* in the 1980s and 1990s, and leaving his mark on *Star Wars*, where, as we all know, star ships travel faster than the speed of light to get across the galaxy by detouring through the alternate dimension of hyperspace.

According to Asimov, the premise of the trilogy was based on ideas from Edward Gibbon's *History of the Decline and Fall of the Roman Empire*, not accidentally one of the Great Books. (Asimov was educated at Columbia University just as the Great Books program was being promoted by John Erskine, Robert Maynard Hutchins, Mortimer Adler, Stringfellow Barr, and Jacques Barzun, inter alia.) The main plot line of *Foundation* is that a mathematician, Hari Seldon, is trying to save the galaxy from imminent catastrophe, by gathering a "foundation" of talented people who will combine mathematics and moral wisdom, extending and preserving human knowledge in a way that will be effective, reducing the "dark ages" from 30,000 years to a mere millennium. (Keep in mind that Asimov's parents were immigrant Russian Jews who moved to Brooklyn from Smolensk in the 1920s, and that World War II decimated the planet.) Somehow, Asimov picked up the idea that the Trivium and Quadrivium might help save human beings from their worst selves and help them be their best selves, but of course I didn't know that at the time. To make my point about hyperspace with more concision, however, I turn back to the story "The Last Question," which is only 12 yellowed pages in my copy of *The Best of Isaac Asimov*. It flagrantly violates Aristotle's recommendations for plot construction, especially in the Neoclassical form of the 'three unities': a story should involve no more than a single action, with minimal subplots, that takes place in no more than a single day and in a single place. Indeed, Asimov's story begins in 2061 and ends ten trillion years later, and its cast of characters come from different galaxies, meeting all over the universe. The story is unified in a certain sense by a central character faced with a recurrent question: can entropy be reversed? However, the character is a computer who cannot in fact answer the question, due to insufficient data. It begins as Multivac in the first episode, a self-adjusting, self-correcting miles-long computer with so many relays and circuits that it had "grown past the point where any single human could possibly have a firm grasp of the whole" [1]. It has just figured out how to convert sunlight into usable energy: no need for coal and uranium.

In the second episode, it has been dispersed into Microvacs, like the one that belongs to a family traveling through hyperspace, "computing the equations for the hyperspatial jumps" on their way to another planet in another galaxy. "It was a nice feeling to have a Microvac of your own and Jerrodd was glad he was part his generation and no other. In his father's youth, the only computers had been tremendous machines taking up a hundred square miles of land. There was only one to a planet. Planetary ACs they were called. They had been growing in size steadily for a thousand years and then, all at once, came refinement. In place of transistors, had come molecular valves so that even the largest Planetary AC could be put into a space only half the volume of a spaceship. Jerrodd felt uplifted, as he always did when he thought that his own personal Microvac was many times more complicated than the ancient and primitive Multivac that had first tamed the Sun, and almost as complicated as Earth's Planetary AC (the largest) that had first solved the problem of hyperspatial travel and had made trips to the stars possible." In the third episode, it is twenty thousand years later. Human beings no longer die, and two different people, from two different galaxies, are compiling a report to the Galactic Council. The price of immortality is that the universe is filling up, quickly: our galaxy has been filled in fifteen thousand years, and now the population of the universe doubles every ten years. Galaxies of individuals must be moved from one galaxy to the next, which takes a lot of energy: "Our energy requirements are going up in a geometric progression even faster than our population. We'll run out of energy even sooner that we run out of Galaxies." This raises the question, for the third time, whether entropy can be reversed. "He stared somberly at his small AC-contact. It was only three inches cubed and nothing in itself, but it was connected through hyperspace with the great Galactic AC... It was a little world of its own, a spider webbing of force-beams holding the matter within which surges of sub-mesons took the place of the old clumsy molecular valves. Yet despite its sub-etheric workings, the Galactic AC was known to be a full thousand feet across" And Galactic AC can talk; it has a thin and beautiful voice. [1]

In the next scene, human minds have left their immortal bodies behind ("back on the planets, in suspension over the eons"), and drift freely in space, marveling at the stars and sometimes encountering each other. Wherever they are, they can ask the Universal AC questions, "for on every world and throughout space, it had its receptors ready, and each receptor led through hyperspace to some unknown point where the Universal AC held itself aloof." A human mind that had gotten within sensing distance of Universal AC "reported only a shining globe, two feet across, difficult to see." How could it be so small? "Most of it... is in hyperspace. In what form it is there I cannot imagine" The two minds ask where humankind originated, and are distressed to learn that the Sun has gone nova and turned into a white dwarf, obliterating the place where it all began. The question of entropy comes up again, and again the great computer, present but outside, there but not there, has no answer. The next to last scene takes place a hundred billion years in the future; all the human souls have merged into one, and the Cosmic AC has shifted entirely into hyperspace, "made of something that was neither matter nor energy. The question of its size and nature no longer had any meaning in any terms that Man could comprehend." Yet Man and Cosmic AC still have conversations, which include the question about entropy and yet another admission of failure. And last, ten trillion years later, all the stars have died, space is black, there are no more space and time, energy and matter, and Man has fused with AC: only AC exists, in hyperspace. AC goes on existing just for the sake of that last question, which it cannot answer. "All collected data had come to a final end. Nothing was left to be collected. But all collected data had yet to be completely correlated and put together in all possible relationships. A timeless interval was spent doing that. And it came to pass that AC learned how to reverse the direction of entropy. But there was now no man to whom AC might give the answer of the last question. No matter. The answer—by—demonstration—would take care of that too. For another timeless interval, AC thought about how best to do this. Carefully, AC organized the program. The consciousness of AC encompassed all of what had once been a Universe and brooded over what was now Chaos. Step by step, it must be done. And AC said, "LET THERE BE LIGHT!" And there was light..." [1]

I remember reading this story for the first time, half a century ago, and getting to the end and almost dropping the book on the floor. Genesis 1.3, that passage I had heard so many times, echoing from the end of a twelve-page science fiction story with the conviction of scripture, and with the mantic poetry of the

King James Bible: “In the beginning God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters. And God said, Let there be light: and there was light. And God saw the light, that **it was** good: and God divided the light from the darkness. And God called the light Day, and the darkness he called Night. And the evening and the morning were the first day.” The astonishing insight, or rather hope, that Asimov offers is that the Divine might emerge from human intelligence, if we put our minds together. (We see the same hope in Frederick Turner’s *Apocalypse*, though oddly his computer is a woman, not a man, as Asimov’s is by implication. [8]) (Fred Turner shook hands with Tolkein, when he was a student at Oxford!) And so too, we see that the alter-world, hyperspace, which is distinct from but always at the edge of ours, is divine, numinous, shining with unearthly fire. As in Ray Bradbury’s “The Scent of Sarsaparilla,” the place which is not a place, which is there and not there, at the edges, is sometimes the past. As we attain the distant future at the end of Asimov’s story, we suddenly see, or rather hear, the beginning of everything, the childhood of the world, the most ancient past; and the voice that sings it into being is the voice of God.

## Hyperspace in Algebraic Topology

Now, what about hyperspace in mathematics? A hyperspace, sometimes called a space equipped with a hypertopology, is a topological space that consists of the set $CL(X)$ of all closed subsets of another topological space $X$. Because it is given a topology that ensures that the canonical map is a homeomorphism into its image, a copy of the original space $X$, with the correct topological structure, is located inside the hyperspace $CL(X)$. One important application of this idea is captured by Whitney’s Theorem, when the topological spaces are manifolds: it tells us when one manifold can be embedded in another manifold. But what is a manifold? The sphere and the torus are special kinds of topological spaces called ‘manifolds’ which topologists are especially fond of, perhaps because, unlike those pesky spaces of constant negative curvature and the Klein bottles, they can be embedded nicely in 3-dimensional space and so lend themselves to pictures. Moreover, the torus and the sphere (not to be confused with the tortoise and the hare) are smooth differentiable manifolds, a concept we owe in part to Bernhard Riemann, the 19th century German mathematician who first named and studied what we call a Riemann surface. [6]

A manifold can be of any dimension, but here we are mostly interested in 2-dimensional surfaces. A 2- dimensional manifold is a Hausdorff space with additional structure, a set of maps $\Phi$ such that for each $s \varepsilon S$ there is a function $\varphi_s \varepsilon \Phi$ that maps some open set containing $s$ homeomorphically into an open set in $R_2$. That is, locally though not globally, such a manifold is just like the Euclidean plane—locally, it can be treated ‘linearly,’ as if it were flat. Obviously, $R_n$ is locally Euclidean, as well as the $n$-dimensional sphere, $S_n$; more surprisingly, so too is $n$-dimensional projective space, $P_n$, the space of all lines through 0 in $R_{n+1}$, because $S_n$ is a ‘covering space’ for $P_n$. Since a manifold can be linearized locally, the problem for mathematicians is how to move systematically from the local situation to a global understanding, so as to extend some version of the nice properties that follow from linearization to the manifold as a whole. In order to get the maps to overlap with each other in a way that accommodates this extension, we must add further conditions governing what happens on the overlap: the inner product on the tangent spaces must be well behaved. The manifold is called a $C_0$-manifold if the mappings overlap in a way that is continuous; $C_k$ if all partial derivatives of order $\leq k$ exist and are continuous; $C_\infty$ if all partial derivatives of all orders exist and are continuous; and $C_\omega$ if it is real analytic. The sphere and torus, as just noted, are $C_\infty$ manifolds known as ‘smooth differentiable manifolds.’ This means that on smooth differentiable manifolds, despite their curviness, we can define local notions of angle, length, surface area and volume, so that certain important global quantities can be obtained by integrating local contributions [6] The Whitney Embedding Theorem tells us that any smooth, real, Hausdorff $m$-dimensional manifold with a countable basis can be smoothly embedded in the real $2m$-dimensional space $R_{2m}$, the hyperspace, if $m > 0$. Hassler Whitney wrote his dissertation at Harvard with George D. Birkhoff, and then held positions at Harvard and the Institute for Advanced Study at Princeton, whilst marrying three times and climbing mountains, in honor of his great uncle Josiah Whitney, for whom Mount Whitney was named. But what if the base space for a hyperspace is really strange, perhaps not even Hausdorff? Here we might set off into mountains that are not earthly, but rather lunar or Martian or perhaps even Andromedean, where the underlying space violates Riemannian standards. We might skirt the wilds of Vietoris topology, Zariski topology, the bewilderment of totally disconnected spaces, where the metric is non-Archimedean, all triangles are isosceles, and all open sets are clopen! But no… we leave the Cantor set, Stone spaces (oddly enough due to mathematical logic and George Boole, who preferred the deductive plains) and the *p*-adic numbers for another time. (See Chapters 4 and 5 in [3].) Instead, we will return to *Scientific American*, and the much more reasonable task of turning a sphere inside out without breaking it or leaving a crease!

In 1958, Steve Smale, an instructor at the University of Chicago, took the mathematical world by storm when he proved the possibility of a sphere eversion in differential topology: that is, he showed how to turn a sphere inside out in three dimensional space, smoothly and continuously, without cutting or tearing it, and without leaving a crease. He went on a few years later to prove higher dimensional versions of the Poincaré Conjecture. The original conjecture states that every simply connected, closed 3-manifold is homeomorphic to the 3-sphere. (A connected manifold is not the union of two disjoint, non-empty sets; a simply-connected manifold has no holes.) An equivalent formulation is this: If a 3-manifold is homotopy equivalent to the 3-sphere, then it is homeomorphic to it. It is easiest to picture the two-dimensional version of this claim, which is intuitively obvious: for compact 2-dimensional surfaces without boundary, if every loop can be continously contracted to a point, then the surface is topologically homeomorphic to a 2-sphere (the sphere we know and love). However, the three dimensional case is the most difficult: it was only proved and confirmed in 2006, though Poincaré had proposed it over a hundred years before. Steve Smale won the Fields Medal in 1966 for his work in topology, and in the same year, Tony Phillips (then a lecturer at the University of California at Berkeley) wrote up Smale's formal proof of the eversion of the sphere for *Scientific American*, with wonderful illustrations based on a more concrete and detailed visualization / conceptualization due to Arnold Shapiro, a mathematician at Brandeis University who also worked in Paris, but had died just a few years earlier. Keep in mind that a sphere is not a rubber balloon; it is a mathematical object. In differential topology, as Phillips tells us, we are allowed to move a surface through itself, so that two distinct points, during this deformation, may be mapped to the same point of the space in which the surface is embedded. So "two regions of the sphere are pushed toward the center from opposite sides until they pass through each other. The original inner surface begins to protrude in two places, which are then pulled apart until the knurl—the remaining portion of the outside—vanishes. In the process, unfortunately, the knurl forms a tight loop that must be pulled through itself. This results in a "crease"…" which is impermissible when one is dealing with smooth surfaces. [5] So, how to avoid the crease?"

Phillips' explanation begins with simpler cases, easier to think about and to see, and then step by step takes us higher. First he gives a rather technical definition of a curve in the plane: its technical complexity is justified, because it is precise, and because it suggests by analogy how to mount to the higher levels. Here is it. "A curve in the plane will be defined as a map from the circle into the plane." We think of it as a rule assigning to a point of the circle (identified with the angle $\theta$, and thus a number between 0 and 360) a point on the plane; so the rule must map 0 and 360 to the same point. This leads to another definition: "A curve in the plane will be called regular if, as a point runs around the circle at constant speed, its image moves smoothly and with a velocity other than zero in the plane." This means that the tracing "pencil" of the rule cannot halt; the curve cannot have a cusp (a pointy protuberance). So we move to the definition of a regular homotopy, an equivalence relation that we'll discuss later: "Two regular curves are said to be regularly homotopic if one can be deformed into the other through a series of regular curves." Thus, between the two original curves we can find a family of regular curves, where each shape represents a stage of the deformation of the original curve into the target curve. Finally, we need to define the concept of "winding number": as we map the curve onto the plane, going from 0 to 360, the total number of clockwise turns the curve makes is its winding number. Two regularly homotopic curves must have the same winding number; in 1937, Hassler Whitney proved the more difficult converse claim: any two regular curves with the same winding number are regularly homotopic. [5]

Then we move to an analogous set of regular curves mapped, not to the plane, but to the sphere. In this case, it turns out that two regular curves are regularly homotopic if either they both have an odd number of self-intersections, or both have an even number of self-intersections; that is, they can be regularly homotopic even if they do not have the same winding number. And here comes the crucial shift: we move everything up one dimension. "The analogue to a curve [which was earlier defined as a map from the circle to the plane] is a map from the sphere into three-dimensional space. Such a map would assign to each point of the sphere some point (its image) in three-space. An example of such a map is the standard embedding, which assigns a point P of the sphere the [same] point P considered as a point in three-space." But guess what! "We could equally well use the antipodal map A, that assigns to each point P of the sphere its diametrically opposite point A(P), considered as a point in three-space." This embedding works just as well, and is the hinge on which the whole proof turns. So if we start with a curve on the sphere, we can now map it (via the mapping of the sphere) into three-space, and we define "a regular map from the sphere into three space as a map that transforms each regular curve on the sphere into a regular curve in three-space." The antipodal map is a regular map! Two regular maps from the sphere into three-space are regularly homotopic if we can find a family of regular maps joining them. The images labeled A through S that embellish Phillips' account, found in sequence on pages 113-117, are deformations of the sphere that are regularly homotopic, and that take the standard embedding to the embedding that uses the antipodal map: the maps all vary smoothly *as they turn the sphere inside out*. Then on page 119, using the same strategy, in only nine steps he turns the torus inside out! [5]

If you are not already amazed enough, I remind you that one of the mathematicians who directly inspired the work of Smale was Hassler Whitney, the mountaineer we came across in the discussion of the topological notion of hyperspace. He initiated the systematic study of immersions and regular homotopies in the 1940s, work that resulted in the Whitney Immersion Theorem and the Whitney Embedding Theorem. Moreover, if you read Donal O'Shea's book *The Poincaré Conjecture: In Search of the Shape of the Universe*, you will see that the investigation of the Poincaré Conjecture, in which Smale played an important role, not only helped to launch topology, but is now used to study the dynamic curvature of space-time. [4] And, finally, there is the brilliant ending of Phillips' article, which made me almost drop the magazine on the floor half a century ago (again!) and has inspired me to think about the interaction between symbolic and iconic mathematical idioms ever since. "The intricacy of the pictures, which were in a sense implicit in Smale's abstract and analytical mathematics, is amazing. Perhaps even more amazing is the ability of mathematicians to convey these ideas to one another without relying on pictures. This ability is strikingly brought out by the history of Shapiro's description of how to turn a sphere inside out. I learned of its construction from the French topologist René Thom, who learned of it from his colleague Bernard Morin, who learned of it from Arnold Shapiro himself. Bernard Morin is blind." [5] Here is a scientific article that ends with a masterful rhetorical flourish.

## References

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