The Pentagram and the Golden Angle in Almada Negreiros' Mural "Começar"

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Abstract

José de Almada Negreiros (1893-1970) is one of the most eminent representatives of Portuguese modernism in both literature and visual arts. From the 1940s, Almada started developing a taste for geometric abstraction, which invaded more and more his visual work. His last piece, a large mural called "Começar" ("To begin," 1969) is the summit of this geometric quest: it contains only geometric constructions and annotations, with very little explanation to the public. In the past four years, the two authors of this article have studied Almada's geometric work extensively, making it possible to understand both content and intention of this monumental work. In this paper we present some of his own constructions for the golden angle, a geometric element very dear to Almada,which provide very good approximations to this angle.

Almada Negreiros and the Golden Angle

José de Almada Negreiros (1893-1970), who liked to refer to himself simply as "Almada," was one of the most renowned Portuguese visual artists of the 20th century. Having started as a cartoonist in newspapers, he quickly got interested in the avant-garde movements of the first half of the 20th century, and came to include futurist, cubist and, later on, abstractionist ideas in his work (for more, see article [2]). Along with his visual work, he also wrote one novel, poetry, some poetic prose and especially manifestos.

Since the 1920s, Almada grew an interest in the compositional analysis of art, and its use of geometry. He started collecting references of authors who worked on this context, such as Matila Ghyka (1881-1965) or Jay Hambidge (1867-1924). There is a clear sense of a slow assimilation of geometrical concerns in his art. In the late 1940s, 50s and 60s he published some of his views on the great importance he gave to Geometry as a set of universal rules which permeated cultures all over the world, with different manifestations in different eras.

He posited that these geometric elements were part of a greater body of knowledge, passed on from generation to generation, common to all humankind and revealing itself in all cultures, especially through art, and he called this the *Canon*. As time passed, he became more and more enthralled with this idea, to the point that, in an interview given to a newspaper in 1960, he said, "To encounter the Canon: this is the reason for all my work." For more on this, see [2].

Because of these ideas, he started to develop an abstract geometric line in his work. In 1957 Almada presented, for the first time, four works of depurated geometric abstraction, that caused some controversy in the national cultural establishment. Privately, he spent hours and hours drawing with ruler and compass, trying to find relations between the geometric elements of the Canon (such as regular polygons, division of the circle in equal parts, rectangles with given proportions, the golden ratio, etc.). A study of about 30 of these drawings is done in [1], Almada made more than two hundred drawings of this type. Figure 1 shows an example of one of these drawings.
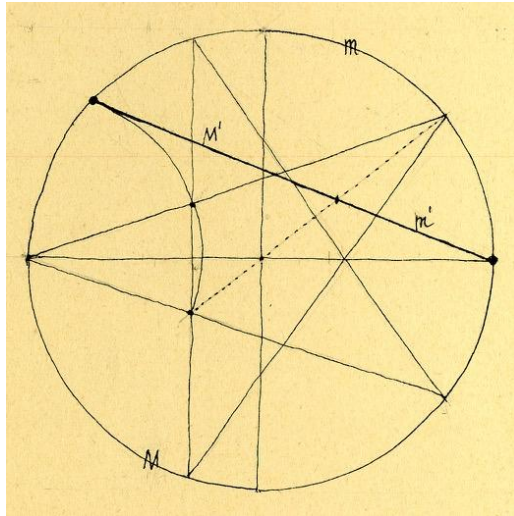
Figure 1: *A pentagram with some references to the golden ratio. Detail of a geometric study by Almada.*

In this drawing, we start from a regular star pentagon, or pentagram, and draw an arc of circle, determining a point from which a line is drawn, a chord in a dark line, having the letters *M'* and *m'* on it. The same letters appear on the circle. These letters refer to the *golden section*, both on the line and on the circle: Almada uses these two letters consistently when he wants to refer to the golden section (see [2] or [4], for example). The golden section of the circle defines two angles, a larger one of $360°/\varphi \approx 222.49°$ and a smaller one of $360° - 360°/\varphi \approx 137.51°$—the letter φ refers to the golden number, 1.618034.... The small angle is sometimes called the golden angle. The authors do not know of any exact construction for the golden angle, using straightedge and compass, and suspect no such construction is possible. Almada produced quite a few constructions for this angle, which he probably verified using a ruler and a protractor, no computations of his have been found so far. Even though no construction is exact, most of them provide very good approximations, and all of them are, according to the author, and to the best of our knowledge, original to Almada.

We determine the error of the approximation of a golden angle by dividing the angular distance between the exact and the approximated points by the exact measure of the angle. In the case of this drawing, the darker line, with the letters *M'* and *m'* determines an angle on the circle. Using an elementary, yet laborious computation, it is possible to determine exactly the value of this angle, which measures 137.40 degrees, which has an amazing error of 0.08%:

$$\frac{137.51 - 137.40}{137.51} = 0.00080 \ldots$$

Moreover, Almada states that the dotted line divides the darker line in the golden ratio—he does so by writing the letters *M'* and *m'* on each segment, which is his consistent notation for the golden section. It is well known that two intersecting sides of the pentagram divide each other in the golden ratio (exactly), but Almada was looking for his own constructions, even having to pay the price of only getting approximations (and we believe he was aware of this, at least in some cases). This is the case here. We drew Almada's construction, using a unit circle (the percent error is independent of this choice), obtaining *m'* = 0.69848 and *M'+m'* = 1.86337. If the section were exact, the smaller segment would measure $(1 - 1/\varphi) \times 1.86337 = 0.711744\ldots$. So, the error for *m'* is $(0.711744 - 0.699848)/0.711744 = 0.0186359\ldots$, or approximately 1.9%.

This drawing is an example of Almada's program: to find original relations between canonical elements—in this case, the star pentagon, the golden section and the golden angle.

The Mural *Começar*

In 1968, Almada was commissioned to create a mural for the main office of the Calouste Gulbenkian Foundation, in Lisbon. He was given total freedom to choose the theme, and he decided to make it a summary of his geometric experimentation, throughout several decades. Figure 2 shows a picture of this mural, which is about 13 meters wide and 2.30 meters high.

Figure 2: *Mural "Começar," 1969*

In the paper [3], the authors analyzed the approximation of the golden angle that can be found on the 16-pointed star (on the left part of the mural), shown in Figure 3. The golden angle is approximated by a fraction of the whole circle, 49/128 (the number 49 in Figure 3 refers to this). The error is 0.2%.
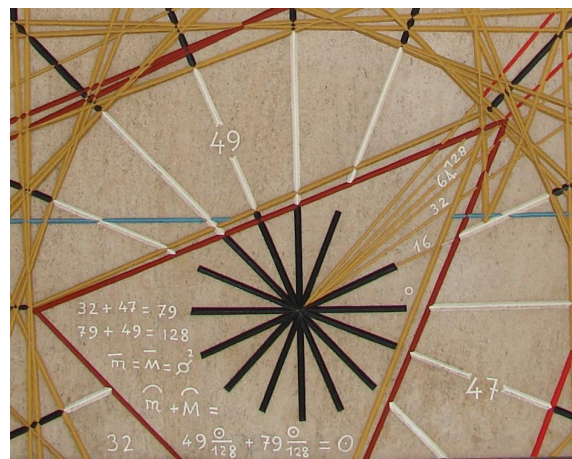

Figure 3: *The center of the 16-point star*

Just like in the previous drawing, the aim is not just to produce a method that affords a good approximation of the golden angle, but to connect several aspects and periods of art history through geometry. In the present case, the 16-pointed star is a lesser-known illustration made by Leonardo da Vinci for Luca Pacioli's book "De divina proportione" (itself, a book about the golden ratio).

At the center of the mural, we find a large pentagram, superimposed on a mesh of lines, see Figure 4. Behind the 5-pointed star, we see a sort of a 4 by 4 grill, in golden yellow, which is then rotated by 45° and superimposed on the first. This grill is a reference to a pre-Columbian civilization of Peru, which left some artifacts in the Marcahuasi plateau, near Lima, mostly sculptures and stone engravings (still shrouded in some mystery). Figure 5 shows a photo of the original grill, taken from a brochure in the

author's library, entitled "A ciência misteriosa dos antigos" ("The mysterious science of the ancients"), which refers explicitly to the Marcahuasi civilization.
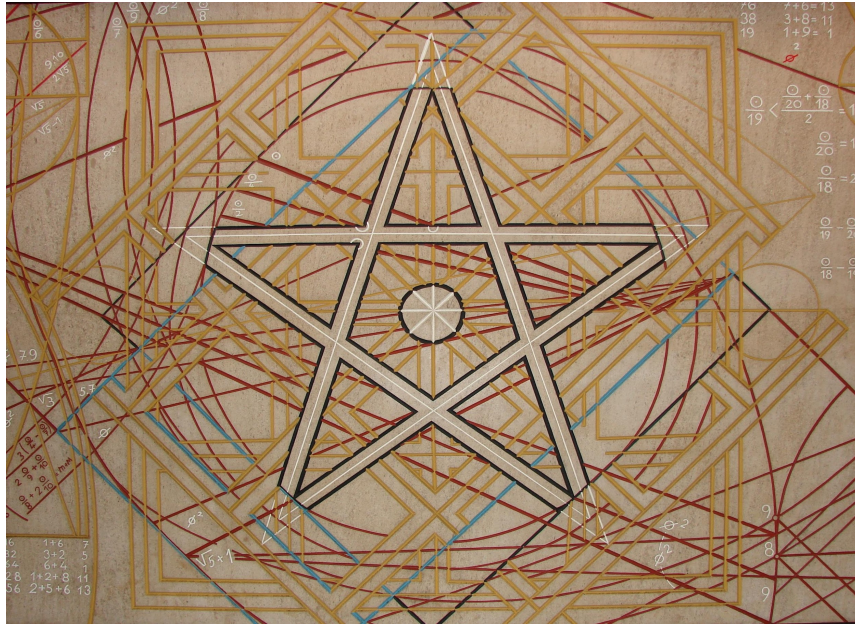
Figure 4: *Central pentagram in Começar*

Figure 5: *The Marcahuasi grill*

Before we analyze this central part of the mural, we examine another geometric drawing that also uses this Marcahuasi grill, and has a simpler interpretation, see Figure 6. The drawing is made on grid paper, the circle has radius 8 (using the grid as unit length) and there are several slanted lines that intersect the circle. Almada makes some remarks on these intersection points, see Figure 7.

In this figure, we also find the following notes:

$$ac': \quad \frac{m}{M} = \frac{M}{\frac{\odot}{4}} \qquad bb': \quad \frac{m}{M} = \frac{M}{\frac{\odot}{2}} \qquad cc': \quad \frac{m}{M} = \frac{M}{\odot}$$

These are references to the golden sections of a quarter circle, a semicircle and a circle. The letters *M* refer to the larger angle and the letters *m* to the smaller one in each of the sections.

In the figure, the arcs that are divided in the golden ratio are *a'c'*, *b'a'*, and the full circle. The notation is not completely consistent: *ac'* and *bb'* refer to the larger angles of the respective golden sections, whereas *cc'* refers to the smaller.
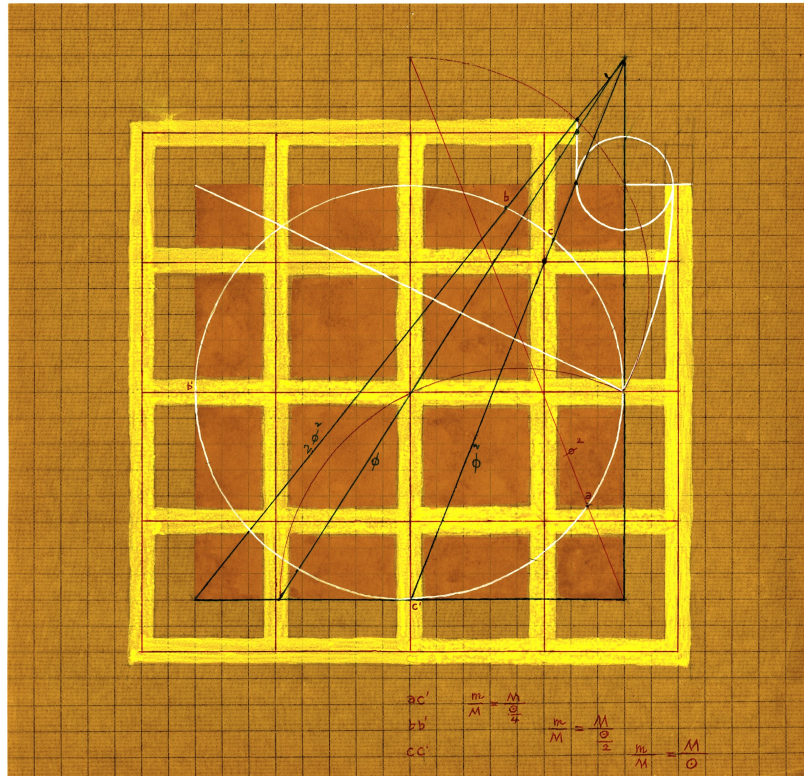

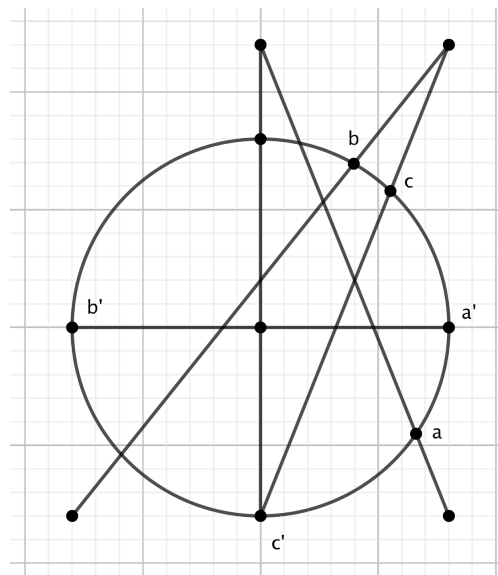**Figure 6:** *A drawing by Almada Negreiros on kraft paper, 33.5 × 33.5 cm, mixed media.*

**Figure 7:** *Explanation of Figure 6*

Considering the smallest angles in each section as the golden angles, the errors are as follows in Table 1 (measures in degrees).

Table 1: *Errors in the constructions of Figures 6 and 7.*

	circle	half circle	quarter circle
exact golden angles	137.51	68.75	34.38
angles obtained in the drawing	136.40	60.33	34.34
errors	0.8%	12.2%	0.1%

Note that the error for the golden section of the half circle is quite larger than the others, even with a different order of magnitude.

We turn again to the center of the mural, referring to Figure 4. As you intersect the vertical grill (in the mural) with its rotated image, the intersection points allow you to draw an approximately regular pentagram. We consider the whole figure drawn on grid paper, as in Figure 6, which is the case in some preliminary drawings of the mural. From this grid, Almada draws three circles in red (shown only partially in the mural) with three inscribed pentagrams. Considering the width of the grid squares as unit of length, the circles have radii 8, 9 and 10, see Figure 8 (note that the circle in Figures 6 and 7 also have radius 8).
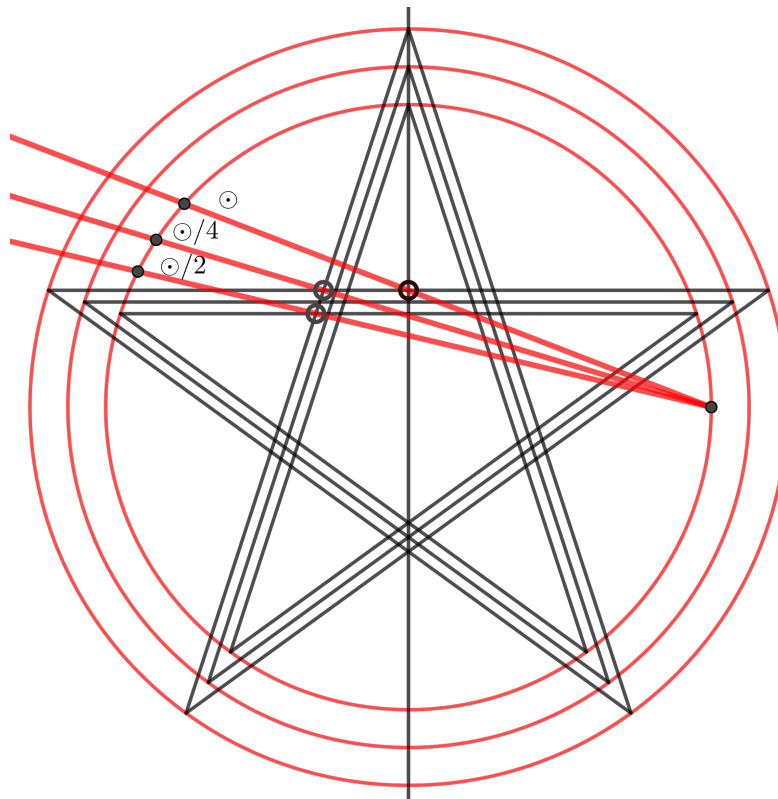

Figure 8: *A reproduction of some elements of the central pentagram*

Before going into the mathematics of the construction, we should note that this figure is a representation of one of the coins minted by the first Portuguese king, Afonso Henriques (12th century). The pentagram in the coin has its vertices cut and a small circle at the center, just like in the mural, see Figure 9.

Using the three points marked with an open dot and the rightmost point of the inner circle, Almada draws three red lines, obtaining the three dots on the left, located on the inner circle. With this, Almada draws one of the most extraordinary connections of the whole mural: by writing $\odot$, $\odot/4$ and $\odot/2$ on the three dots on the left, he claims that these points establish the golden sections of the circle, the quarter circle and the half circle. Figure 10 shows the positions of the half circle and the quarter circle considered.

Figure 9: *Portuguese 12th century coin*
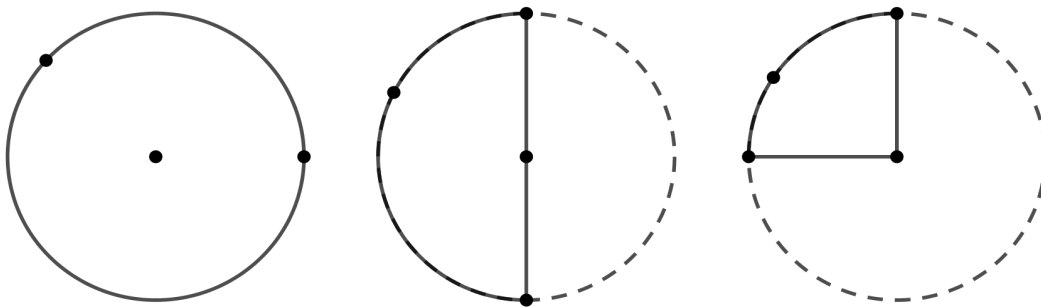
Figure 10: *Golden sections as obtained in the mural*

In Table 2 we find the approximations obtained in the mural (to the hundredth of a degree).

Table 2: *Approximations to golden angles in the center of the mural "Começar"*

	circle	half circle	quarter circle
exact golden angles	137.51	68.75	34.38
angles obtained in the mural	137.76	63.37	33.57
errors	0.2%	7.8%	2.4%

We note again that the error for the golden section of the half circle is larger than the others — and, again, the approximation is smaller than the exact value. It may be that Almada had a wrong measurement for the golden section of the half circle (smaller that the real one), and used it for both constructions, without realizing it was wrong (further research would be needed to support this claim).

Even though the aim of the constructions described in Figures 7 and 8 have is the same, they are quite different in nature. For instance, the lines that give us the points in Figure 7 are drawn from points in the grid, whereas in Figure 8, the lines depend on the pentagram. At any rate, both constructions were meant to derive these sections from elements coming from several backgrounds, showing unexpected connections between different cultures and illustrating Almada's claim of a universal geometric Canon, underlying all art.

Summary and Conclusions

Almada's interest in geometry stemmed from studies in composition, but took a much more philosophic turn, as he strived to relate many artistic and cultural artifacts within the same Canon. In the process, he came up with several geometric constructions, some simpler than others, to obtain the division of the circle in equal parts, or the golden section of the circle.

One should always bear in mind that he did not have any deep mathematical training, nor did he define himself as a mathematician: as he said in the interview of 1960, "I am a person of the arts, and I do not live of anything else." Therefore, neither the processes nor the analysis he did of his work had the spirit of a mathematician. This makes his work, in our opinion, even more fascinating from a mathematical viewpoint.

Finally, we would like to call the reader's attention to a recent website [4], created by the two authors of the paper, that briefly describes, with short texts, images and animations, all the contents of the mural "Começar."

Acknowledgements

References

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[2] S. P. Costa and P. Freitas. "Almada Negreiros and the Geometric Canon." *Journal of Mathematics and the Arts*, vol. 9 nos. 1-2, pp. 27-36 (2015). doi: 10.1080/17513472.2015.1012699

[3] P. Freitas and S. P. Costa. "The Golden Angle and How to Construct It." *Proceedings of the Recreational Mathematics Colloquium*, Ludus Associacion, 2017.

[4] *Começar*, the panel of Almada Negreiros. http://gulbenkian.pt/almada-comecar/en/