Loop-Forms: From Construction to Composition

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Abstract

The author introduces loop-forms and the variables of their construction. Six possible types of loop-form are defined, and three of these types—the 3-orbit, the 2+1-orbit, and the 1+1+1-orbit loop-forms—are examined in detail. The complete form-set for each is given, and various features and subsets are discussed. Recent artworks that employ loop-forms are illustrated, and the author explains the color and composition strategies employed.

Early Developments

The aims of this paper are to describe *loop-forms* and their features, to examine the various loop-form sets and their subsets, and to introduce some artworks based upon them. Loop-forms have precedents in my early studio work as *circuit-forms*—closed, continuous, colored lines that have occupied my abstract paintings from 1995 onwards. The compositions were organized by golden-ratio, or Φ (*phi*), geometry, and the circuit-forms were made by connecting the diagonals of squares and Φ rectangles in the compositions [1]. The earliest circuit-forms were made of straight lines; by 2005 they were made of curved lines, yielding more sinuous and smoothly coiling shapes (Figure 1). Circuit-forms were not developed by any systematic combinatorics, but instead were ad-hoc compositional solutions to explorations of color. Although not the subject of this paper, it should be noted here that my ongoing interest in closed paths has been, and continues to be, to explore simultaneous color contrast—the capacity for a single color to change its appearance on differently colored backgrounds. Closed paths, whether straight or curved, are ideally suited to this task, since a continuous colored line makes it visually clear that the color is both physically constant and illusorily changing as it crosses onto the colored grounds. In 2015, I began working in earnest towards a more systematic development of these circuits. In doing so, I set aside both Φ geometry and diagonals of squares and rectangles as base conditions, and I established a more simplified template on which to systematically vary the curves.

Figure 1: *"Circuitous (Red)" 2006. Example of a circuit-form.*

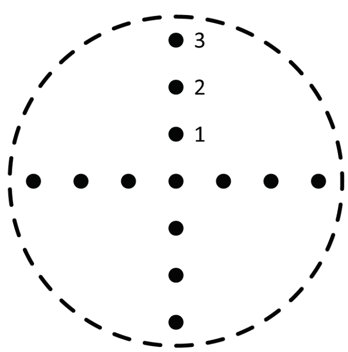


Figure 2: *Circle-template for loop-form construction.*

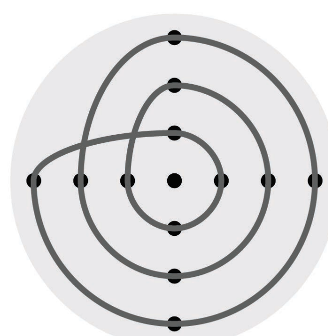


Figure 3: *A simple 3-orbit loop-form, 1111,2222,3333.*

The Circle-Template and Loop-Form Variables

Loop-forms are constructed on a circular template, which possesses a center point and 3 points on each of 4 vertical and horizontal radii (Figure 2). These will be referred to as *pass-through points*, since they are the locations at which the loop-form lines pass, and will be numbered *1,2,3* in sequence from inner to outer positions. The curves connecting two pass-through points are either quarter-circles or quarter-ellipses. Loop-forms utilize some or all pass-through points, but curves never contact the center point; thus, there are 12 available pass-through points. Importantly, loop-forms may contact a pass-through point only once. Although loop-forms may use pass-through points 1, 2, or 3 in any order on any given radius (this is the basis for the permutations in each set), loop-forms must contact one pass-through point on each of the 4 radii in clockwise/counter-clockwise succession (that is, must always orbit around the circle's center-point).

Loop-forms are always comprised of closed paths. The minimum loop-form, then, is a single 360-degree closed curve, or a *1-orbit* loop-form, which employs 4 of the 12 available pass-through points. Since it is a single closed path, we shall refer to this as a *simple* loop-form. The other simple loop-forms are the *2-orbit* loop-form (employing 8 of 12 pass-through points) and the *3-orbit* loop-form (employing all 12 pass-through points). The 3-orbit loop-form (example, Figure 3) is the maximum simple loop-form possible in the circle-template set forth here. Loop-forms may also include multiple closed paths, such as 3 independent 1-orbit loops in the same form, and these we shall call *compound* loop-forms. Consequently, it is possible to envisage 6 possible types of loop-form, 3 simple and 3 compound:

- 1-orbit simple loop-form (4 used, 8 unused points)
- 2-orbit simple loop-form (8 used, 4 unused points)
- 3-orbit simple loop-form (all points used)
- 1-orbit + 1-orbit compound loop-form (8 used, 4 unused points)
- 1-orbit + 1-orbit + 1-orbit compound loop-form (all points used)
- 2-orbit + 1-orbit compound loop-form (all points used)

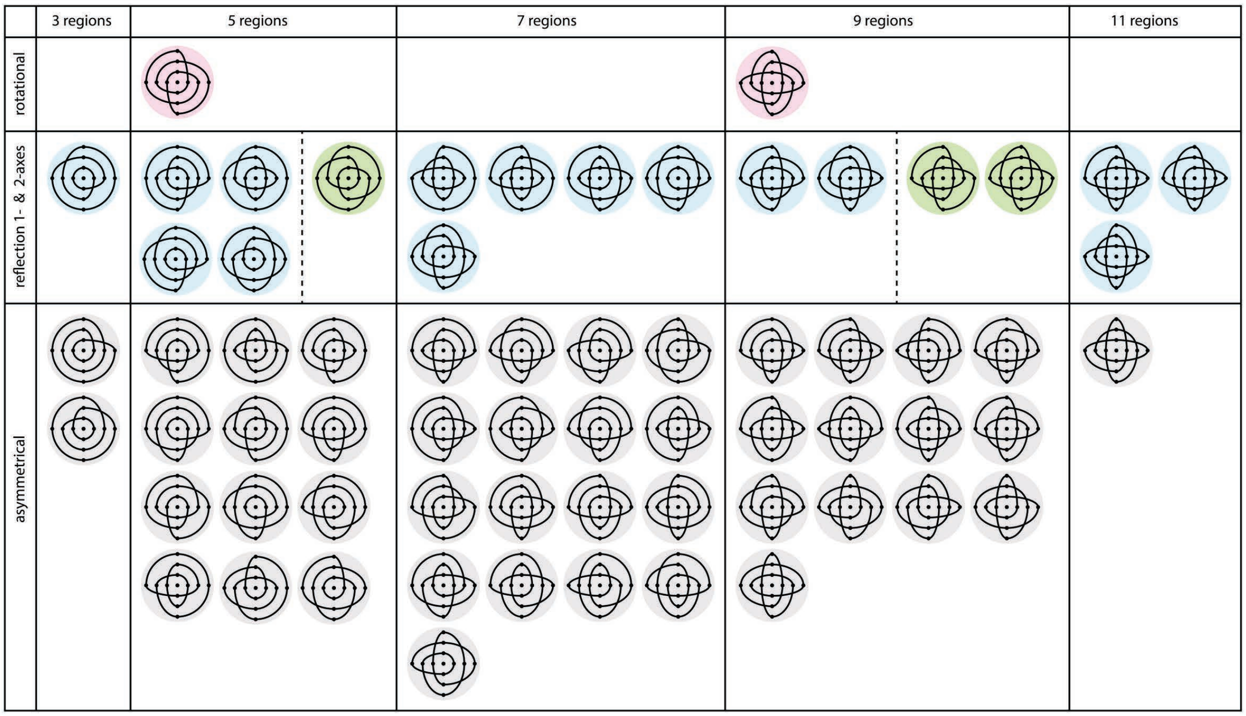
Creating Loop-Form Sets

While it is not the aim of this paper to explain in detail the combinatorial processes leading to a set of all possible loop-forms of a given type, I offer a brief description here of my particularly visual approach to both the production of loop-forms and the organization of their sets. I do not employ computer programming or mathematical formulae; instead, I draw the loop-form curves digitally in a vector-based design program, creating the combinatorial variations by copying, rotating, reflecting, and joining the curves into the completed loop-forms. Working with combinatorial trees, I systematically generate the full array of loop-forms, and then I reduce this larger set by discarding any loop-forms that repeat another after rotation or reflection. In the end, I am seeking the *minimum-complete form-set*, which is both <u>non-redundant</u>, in that no form repeats another after rotation or reflection, and <u>complete</u>, in that all forms permissible by the combinatorial parameters are present. As an artist, I consider these to be aesthetic values for my work as much as they are mathematical qualities.

The Set of 3-Orbit Loop-Forms

The minimum-complete set of 3-orbit loop-forms consists of 65 distinct forms (Table 1), which is a very large set to include in an artwork. Without some basis for dividing and grouping the many loop-forms in an artwork, it becomes difficult to visually discern the subtle similarities and differences among the forms and especially to recognize the completeness of the form-set. I place a high priority on making the structure of both the individual forms and the set of forms visually recognizable, with minimal or no dependence upon verbal or mathematical explanations. This typically requires precise use of colors, sizes, and positions to stimulate recognition of the mathematical and shape relationships among the forms. The 3-orbit form-set exhibits two kinds of structure that interested me for artworks: symmetry characteristics and division into regions. The human visual system is exquisitely adapted to recognize symmetry [2], so symmetry characteristics are often the most important basis for my establishing subsets within a form-set. Of the 65 3-orbit loop-forms, 20 are symmetrical: Eighteen possess reflective symmetry (15 of them about one axis, and three of them about two axes), and two have strictly rotational symmetry (one with 2-fold and one with 4-fold). The remaining 45 asymmetrical loop-forms, while visually differentiable from the symmetrical forms, are difficult to differentiate from each other. Although not shown in the table or the artworks discussed below, I have created subsets based upon what we can call the *core-shape*—the smallest closed shape surrounding the central point. There are six different core-shapes possible, defined by the shape of the lines (curves or corners) connecting the four pass-through points closest to the center: four curves, three curves/one corner, two adjacent curves/two adjacent corners, two opposing curves/two opposing corners, one curve/three corners, and four corners. Core-shapes can provide a means for compositionally ordering both asymmetrical and symmetrical loop-forms in future artworks.

Table 1: *The Set of 65 3-Orbit Loop-Forms.*



Additionally, the 3-orbit loop-forms may be organized by the number of regions and intersections resulting from the crossings of the curved line upon itself. There are five categories: loop-forms with three regions/two intersections (3 of them), loop-forms with five regions/four intersections (18 of them), loop-forms with seven regions/six intersections (22 of them), loop-forms with nine regions/eight intersections (18 of them), and loop-forms with 11 regions/10 intersections (four of them). The symmetry and region characteristics are grouped in rows and columns respectively in Table 1, and these subsets have provided the compositional bases for multiple artworks. For example, *Bouquet (3-orbit loop-forms)* (Figure 4) includes all 65 forms, whose symmetry characteristics are indicated by scale, color, and location. The symmetrical loop-forms are larger, and the asymmetrical loop-forms are smaller; the scales of the circles are determined by the inscribed circles of squares (larger) and triangles (smaller) in a semi-regular tessellation (3,3,4,3,4). Among the larger, symmetrical loop-forms, the one-axis symmetries are blue, the two-axis symmetries are yellow, and the strictly rotational symmetries are magenta/pink. The smaller, asymmetrical forms are darker red/red-orange, but are not further differentiated into sub-groups. The 20 forms in *Collected (for Alfred Russel Wallace)* (Figure 6) are the symmetrical 3-orbit loop-forms only, arranged in 5 groups according to the number of regions. As in *Bouquet*, the symmetries are color-coded: one-axis symmetries are blue, two-axis symmetries are yellow, and strictly rotational symmetries are red. The symmetries, textures, colors, and shadows of the loop-forms suggest the subject matter of collected insect specimens, but the metaphoric levels at work here pertain to the deep currents of symmetry and order that run between nature, art, and mathematics.

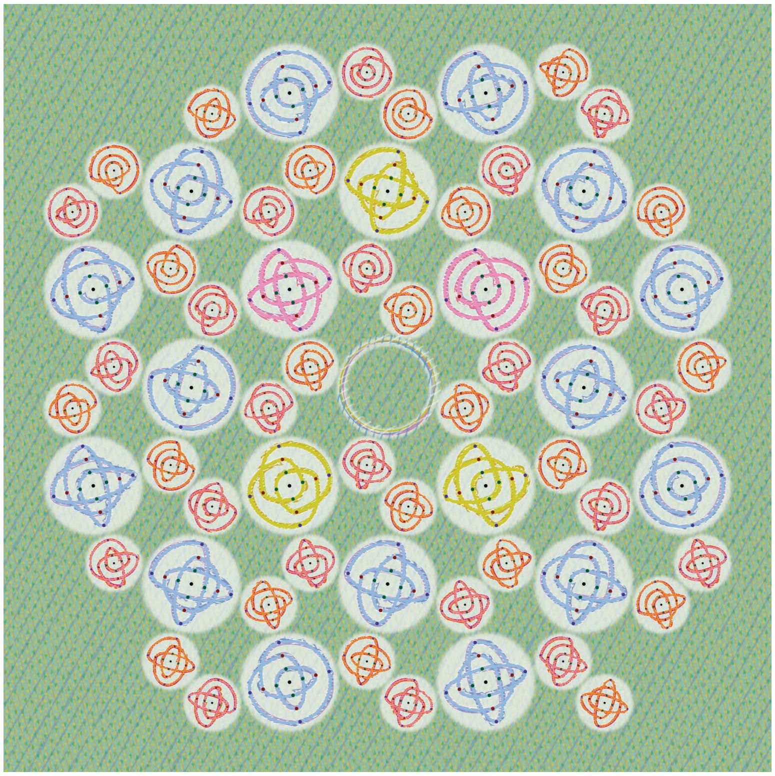


Figure 4: *"Bouquet (loop-forms, 3-orbit)", digital print, 2017.*

Figure 5: *Detail of Figure 4.*

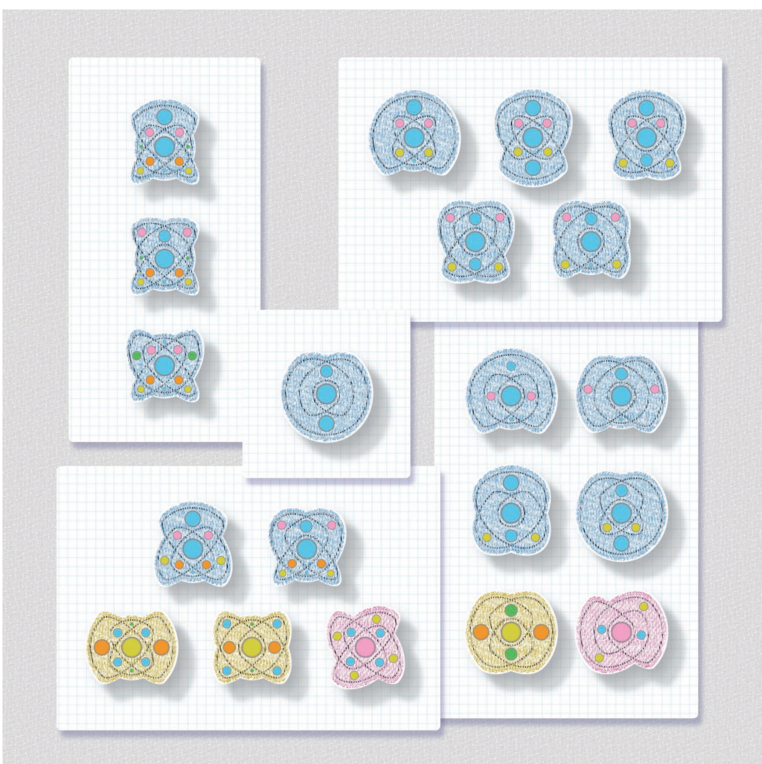


Figure 6: *"Collected (for Alfred Russel Wallace)," digital print, 2017.*

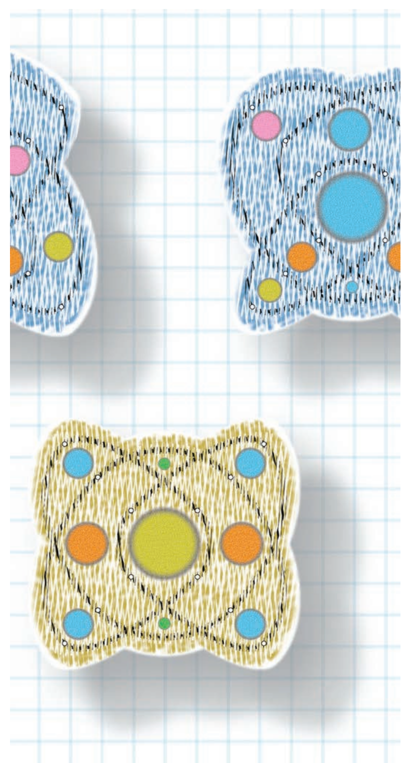
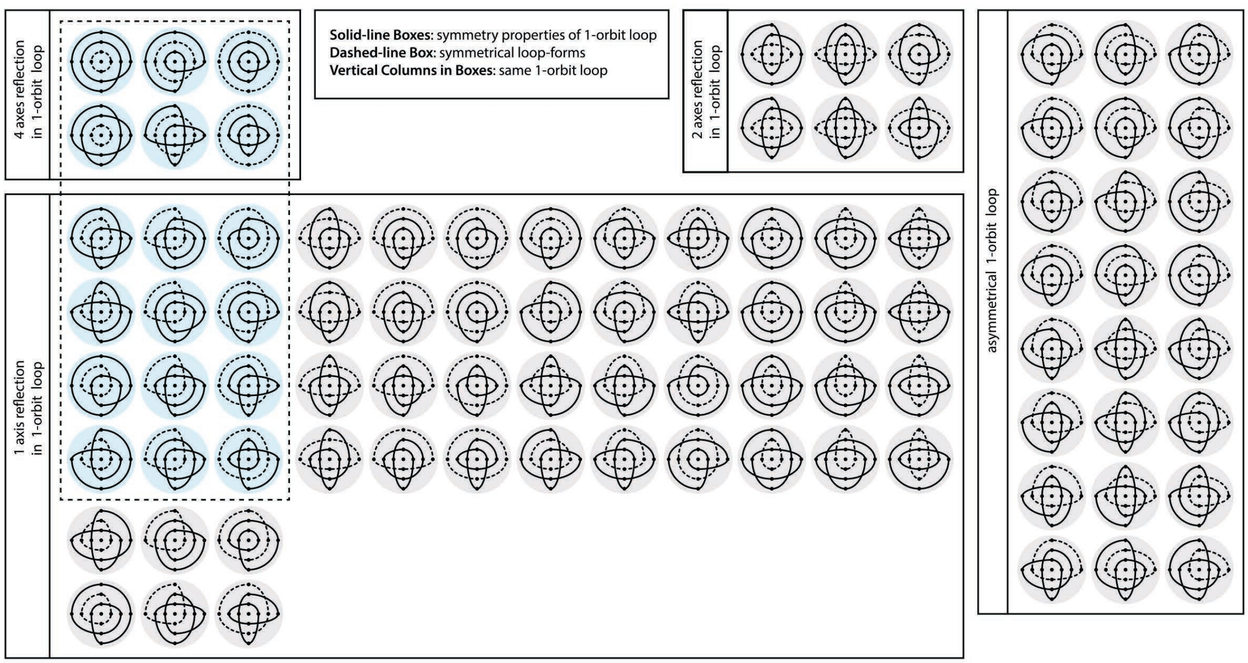


Figure 7: *Detail of Figure 6.*

The Set of 2+1-Orbit Loop-Forms

The set of compound 2+1-orbit loop-forms, each possessing two independent loops, totals 90 distinct forms (Table 2). This large set contains some interesting subsets, offering corresponding possibilities for compositional and color groupings in artworks. There are two levels of symmetry discernible in the 2+1-orbit loop-forms: on the level of the whole loop-form and on the level of the 1-orbit loop. The former consists of 18 with one-axis symmetry (Table 2, dashed-outline box, upper left) and 72 which are asymmetrical. The latter consists of six forms with circular loops, six forms with elliptical loops, 54 forms whose 1-orbit loops have one-axis symmetry, and 24 forms whose 1-orbit loops are asymmetrical. Further division of these 1-orbit symmetry subsets (solid-line boxes) is possible: there are 21 distinct 1-orbit loop shapes, and these are grouped by columns within each box in Table 2. Also notable is that each 1-orbit loop shape (i.e., each column) has an even number of loop-forms, from a minimum of two to a maximum of eight. This presents the possibility of making paired relationships in the composition of an artwork.

Table 2: *The Set of 90 2+1-Orbit Loop-Forms.*



I employed all of the above characteristics—symmetry/asymmetry of loop-forms, symmetry/asymmetry of 1-orbit loops, and pairing of same 1-orbit loop shapes—in organizing the ninety 2+1-orbit loop-forms in *Far-flung* (Figure 8). The circular loop-forms in *Far-flung* are distributed on the same tessellation pattern (3,3,4,3,4) as used in *Bouquet* (Figure 4), here organized in 180-degree rotational symmetry. Each 1-orbit loop is paired with a like-shaped 1-orbit loop in the rotationally corresponding location across the composition center. The loop-forms appear in three sizes and colors. The largest, blue circles are the 18 loop-forms with reflection symmetry (Table 2, dashed-line box, upper left): the six forms with circular loops are left and right of center, and the rest extend above-right and below-left of center. The middle-size, red circles are the 48 asymmetrical loop-forms possessing 1-orbit loops which are elliptical or have one-axis symmetry. The smallest, yellow circles are the 24 asymmetrical loop-forms with asymmetrical 1-orbit loops. In all loop-forms, the 1-orbit loops are colored according to symmetry: circular loops are orange, elliptical loops are violet, loops with one-axis symmetry are green. Asymmetrical loops are light grey (Figure 9).

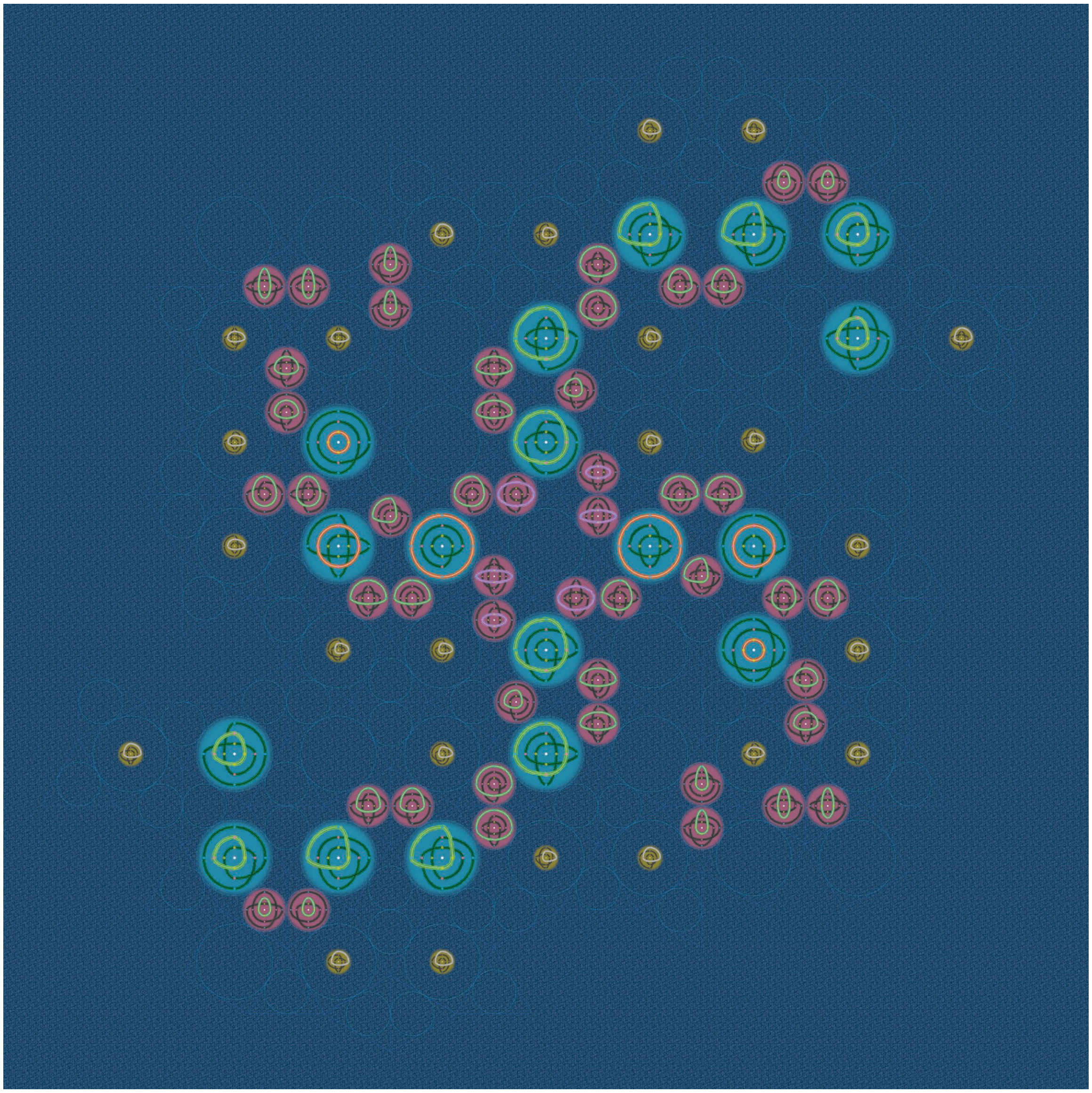


Figure 8: *"Far-flung (2+1-orbit loop-forms)", digital print, 2017.*

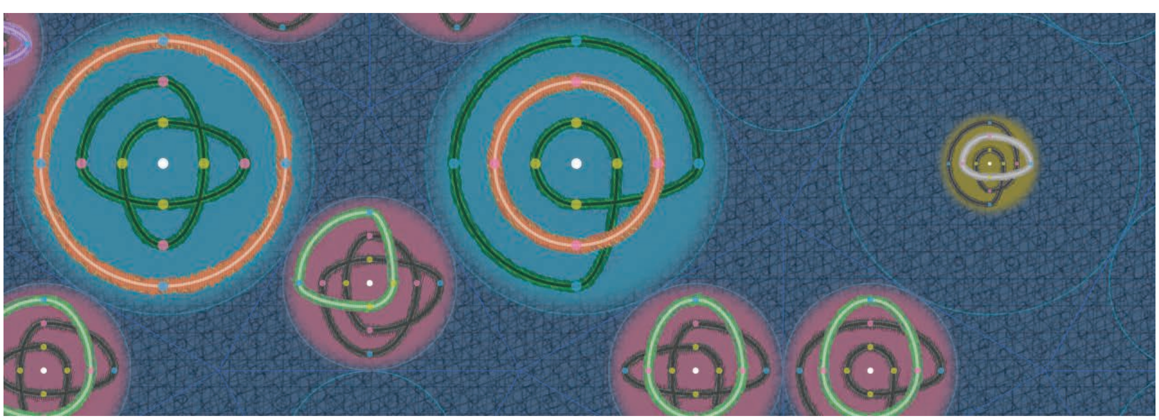
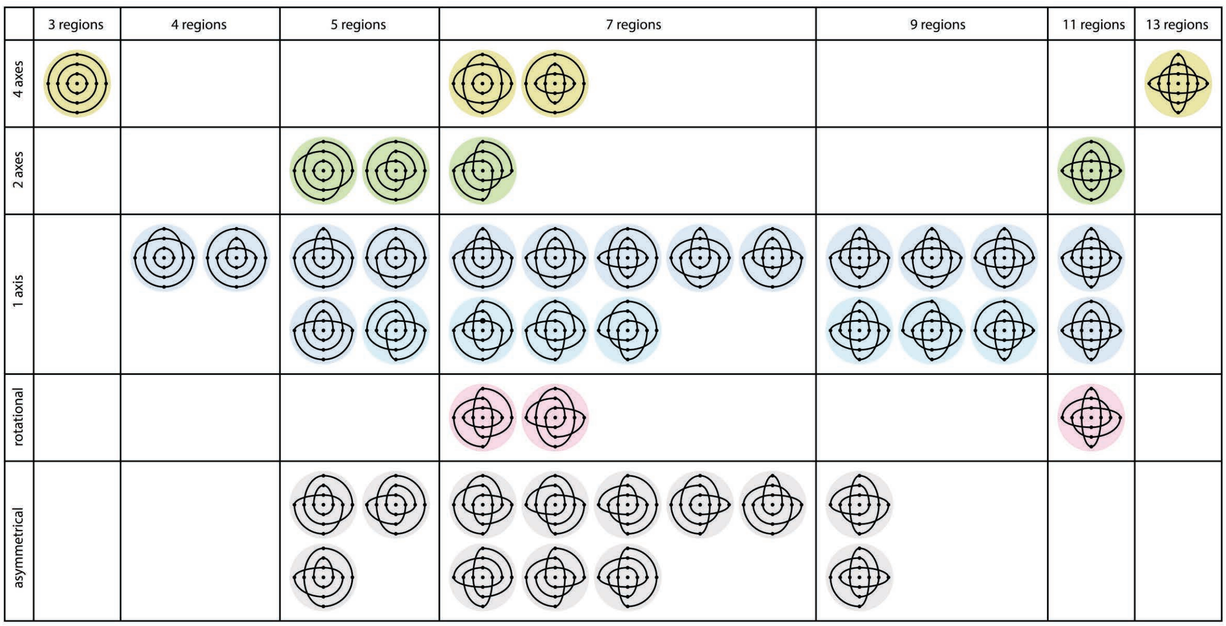


Figure 9: *Detail of Figure 8.*

The Set of 1+1+1-Orbit Loop-Forms

There are 21 distinct simple 1-orbit loops possible in the circle-template; these are the same 21 1-orbit loops in the 2+1-orbit loop-forms (columns in Table 2). If these simple 1-orbit loops are combined three at a time, such that each independent loop employs a different set of 4 pass-through points, then a compound 1+1+1-orbit loop-form is created. There are 46 distinct combinations of the 21 loops taken three at a time (with no restriction on using the same 1-orbit loop shape more than once), comprising the full 1+1+1-orbit loop-form set (Table 3). The inventory of symmetry characteristics is: 4 forms have four-axis symmetry; 4 forms have two-axis symmetry (3 with diagonal axes, and 1 with vertical and horizontal axes); 22 forms have one-axis symmetry (15 with a vertical or horizontal axis, and 7 with a diagonal axis); 3 forms have strictly rotational symmetry; and 13 forms are asymmetrical. Interestingly, the forms in this set determine as few as three and as many as 13 regions, which is the broadest range among all the form-sets.

Table 3: *The Set of 46 1+1+1-Orbit Loop-Forms.*



Although I have not yet made artworks that include the full set of 1+1+1-orbit loop-forms, I have produced compositions with individual and small groups of loop-forms from the form-set. Since loop-forms retain many of the characteristics of the earlier circuit-forms, I am now employing loop-forms to continue my work with simultaneous color contrast. In *Sojourns* (Figure 10), for example, each colored line crosses into different colored regions on the circular ground and is changed in its appearance along the way. (For this and other color-dependent features of the figures and tables, the reader is encouraged to consult the full-color electronic version of this paper.) Many similar artworks are in process.

Conclusion and Future Directions

The loop-forms introduced in this paper have been designed on a circle-template with clear limitations: equidistant pass-through points; 3 pass-through points on each of 4 radii; circular and elliptical arcs; and pass-through points that may be contacted only once by a loop curve. Future work lies ahead for altering these conditions individually or in combination. The original circuit-forms, which led the way to loop-forms, were built on Φ proportions, and I am interested to see the proportional changes in loop-forms whose pass-through points are distributed by Φ ratio or some other unequal distances. Also, I have begun work on forms with 4 pass-through points on each radius, as well as forms with 3 and 5 radii; these are still in

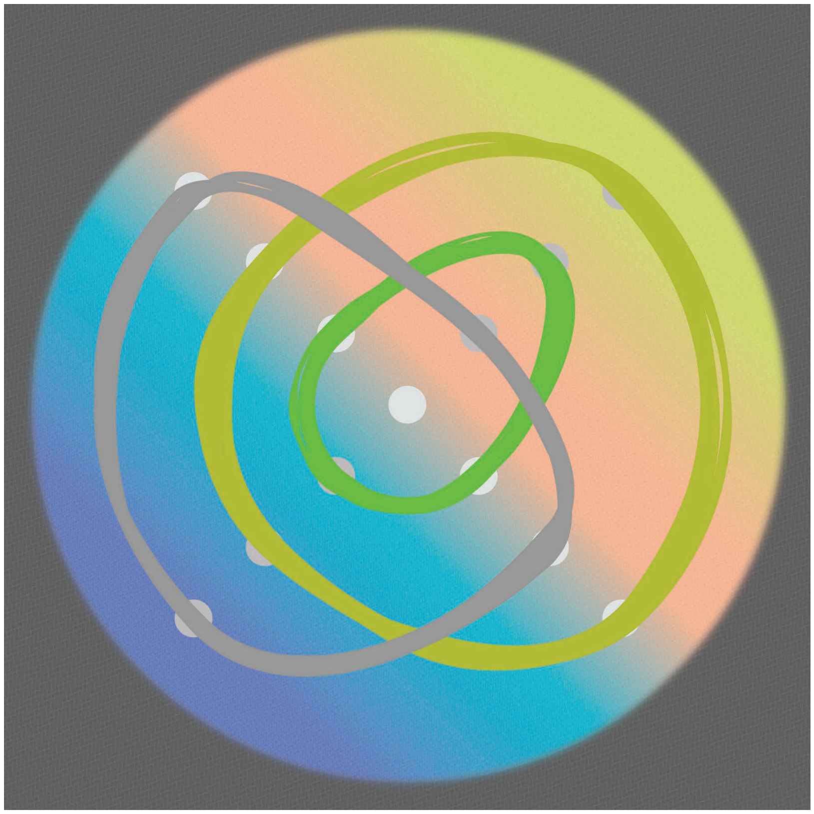


Figure 10: *"Sojourns," digital print, 2017.*

development and have not yet yielded artworks. Although this brief paper does not permit detailed examination, I have produced complete sets of 1+1-orbit and 1+1+1-orbit *tangent-loop-forms*, where compound 1-orbit loops are permitted to share 1 or 2 pass-through points. These compound tangent-loop-forms produce the visual appearance of simple 2- and 3-orbit loop-forms. For example, the point of tangency of 2 independent 1-orbit loops is strongly perceived as the crossover intersection of a single 2-orbit loop—but unlike the previously-discussed 2-orbit forms, the point of crossover is located on a pass-through point.

My ongoing dedication to mathematics in my studio work, and particularly to developing form-sets from combinatorial processes, is rooted in the pleasure of finding new and unanticipated shapes for composition. Equally interesting, and perhaps more surprising, to my composition processes are the discoveries of higher-level features and structures that emerge from the sets—the symmetry subsets, region subsets, and the subtle differences and similarities among the individual forms. For my creative work, mathematical form and visual art not merely compatible, but mutually reinforcing.

Acknowledgements

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References

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