Mathematical Magic With a Deck of Cards

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Abstract

This workshop proposes participants to learn several mathematical card effects. We will cover both, the mathematics, as well as the performance of said effects. It is our aim to give participants the tools and insight to be able to perform this mathematical magic in accordance of their environment making obtaining the best reactions possible.

Introduction

Magic is an age-old human activity, it exists since the dawn of mankind. It is present from the moment one person tells a story and captivates the attention of another, captivating her attention. Records of magical practices, albeit ritualistic ones, go back to pre-historic paintings from the Paleolithic age, some with more than twenty thousand years [1]. The connection to religion and ritualistic practices is evidenced by its presence in major works like the Bible, the Torah or the Koran. The confrontation between the brothers Aaron and Moses and Egyptian magicians, for example, as they call for the freedom of the Hebrews from Egypt, is present in all three books. They transmute a staff into a snake (and back) as proof of divine intervention. In this context magic has been joined with the supernatural.

In modern History, authors like Reginald Scot, with his *The Discoverie of Witchcraft* [2], or Luca Pacioli, mathematician, pedagogue and Franciscan monk friend to Leonardo, in his *De Viribus Quantitatis*, unveiled some of the "tricks" charlatans played on their victims feigning superpowers [3]. This approach to magic paved the way for magicians like Alexander Hermann, Jean Eugene Robert-Houdin or Howard Thurston, who brought magic to the stage. More recently, David Blaine, David Copperfield or James Randi, among many others, popularized magic.

Thus, there is a dichotomy between esotericism and performance art that surrounds the concept of "magic". Our interpretation of the term "magic" and its derivatives, like "magician", are understood as pertaining to a body of art that aims at engaging the audience on both emotional and intellectual levels, having for basis some form of deception or surprise factor. Its unique features are an excellent tool for performance, not restricted to theaters.

The relation between magic and mathematics has been repeatedly emphasized (see for instance [4] and [5] within the context of the Bridge seminars). David Singmaster defines Recreational Mathematics [6] as follows:

> "First, recreational mathematics is mathematics that is fun and popular – that is, the problems should be understandable to the interested layman, though the solutions may be harder. (However, if the solution is too hard, this may shift the topic from recreational toward the serious – e.g. Fermat's Last Theorem, the Four Colour Theorem or the Mandelbrot Set.)
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> Secondly, recreational mathematics is mathematics that is fun and used as either as a diversion from serious mathematics or as a way of making serious mathematics understandable or palatable. These are the pedagogic uses of recreational mathematics. They are already present in the oldest known mathematics and continue to the present day."

It comes as no surprise that magic, or at least a certain kind of magic, belongs to this category. Many books attest to this, being true treasure chests filled with wonderful effects (see for instance any book by Martin Gardner—in particular, [7] is a good example). Research has been done on its benefits in a school context (see [8] or [9]) and there are several projects that actively use magic in their endeavor to engage the public in Mathematics. It is also an accessible tool for individuals, such as teachers, science communicators, or any other mathematicaly minded enthusiast.

A great deal of modern magic is dedicated to effects with playing cards. Cards have been the starting place for many magicians, following a long tradition [10]. This is also the case for researchers like Persi Diaconis and Ron Graham [11] (the former of which started as a magician, never having lost this interest in mathematical research). Others include mathematicians like Colm Mulcahy [12] and ourselves. As a consequence of this, in this workshop we would like to share some of our favorite effects with a deck of cards, exploring both the mathematics within the effects and the nuances of presentation.

Contents

The workshop is a result of over ten years of performing mathematical magic in Portugal and occasionally abroad. All of its authors are active members of the Circo Matemático, a math outreach project founded on the 11th of January of 2011, continuing the project "Os Matemáticos Silva" [13]. Our motto is to entertain by means of mathematics. A description of our mission and our work can be found in another proposal for this Bridges conference [14] as well as online [15]. In this workshop we would like to share some of the material we've gathered and created related to card effects, that has recently been published [16] and discuss it and its mathematics, as well as to give some insight into the performance aspects we deem useful. The participants will thus be invited to learn and preform the effects.

From a performers perspective, with greater insight into the mathematics comes greater freedom. While many illusionists are aware of the methods for their effects, many lack understanding of the finer points, especially when it comes to more quantitative subjects. If the performer is able to understand and master these subjects, he can quickly adapt and adopt a different approach, ultimately being able to serve the spectator better in his performance. For the interested scholar, educator or scientific communicator, the magic trick can serve as a starting point for investigation, conversation or presentation, a way to create empathy and create interest the spectator. Magic tricks using a deck of cards are particular apt for this, as the deck is an object most people are familiar with, be it because of games or probability theory. Unfortunately, both communicators—the "magician" and the "science guy"—have their pitfalls on opposite sides of the same topic: information. The first one might purposely keep a secret in defense of the scenic effect, drama and surprise, the other, drown his spectator in it. To address this problem, we propose to bring some ideas we have collected from the magic community [17] and theorists [18], which we apply to our performances. It is our aim to discuss these as well as techniques for addressing a spectator. Ultimately, however, the pedagogical maxim "do not answer any questions that has not been asked", and the performers lemma "its better to be desired than annoying", outline our take on the subject.

What follows is a sample of three examples of effects we'll teach and discuss in the workshop, inviting participants to learn, perform, and adapt them.

Four kings at the inn

This is a classical **effect** many know. The Magician places a *four* on the table, representing an inn with four rooms (the four spots), where four kings will be lodged, with their entourage: a queen, a prince and a page. These characters are represented by K, Q, J and A. Initially, every suit is placed in its own room—that is, next to each of the corners of the 4 card on the table. Suddenly, in the middle of the night, there is a great commotion, the guests leave the rooms, after which they return to any room. This is represented by the following actions: the Magician collects the cards, shuffles them, and puts them back in the rooms, this time facing down. When morning arrives, they find that the groups have been separated, but that there is still some order: the kings are all gathered in the same room, as are all the queens, the princes, and the pages.

[Figure 1]

The **method** is to keep the order of the cards, as shown in the figure, both in the shuffling and in the dealing. This is, as the Magician collects the piles, he does so without mixing them, picking up each pile of four cards and stacking them. He then cuts the deck several times in his hand (overhand shuffle). The Magician then turns the deck down and deals the cards successively to each corner of the *four*, thus ensuring that all cards of a certain value go to the same stack – that is, to the same room. Once the method is explained the **mathematics** of the effect is evident. The cutting of the deck maintains the structure of the 16 card pile: there are always 3 cards between two cards of the same value. Dealing them out one by one, each to its own pile, breaks the order of suits, but stacks them according to value. This surprises and intrigues almost every person, especially before explaining the method. The discussion and analysis of the trick can lead to great problem solving and critical observation exercises, and serve as a starting point for mathematical discussions of, parity, order, grids, or deeper consideration of topics like what is random, chaos, probability etc. just to give a few examples from our experience and implementation of the effect.

Erdős

This effect requires two people. The Magician, and an Assistant. The Assistant produces the following five cards:

[Figure 2]

A Volunteer shuffles them and spreads them on a table. The Assistant turns three cards face down without altering the order of any card. The Magician, who was absent during this process, returns and guesses each of the cards facing down in their correct order.

The **method** is as follows: The Magician already knows the five cards being used. The Volunteer may know this, but keeping it a secret heightens the effect. Looking at the five cards, the Assistant turns three cards which are in the original order, left to right. When the Magician arrives he does not have any difficulty in identifying each of the cards, as he knows they are ordered. To ensure that the ordering is well read by the Magician, the Assistant may want to influence how the cards are being placed on the table, or tell the magician on what side of the table he should stand to look at the cards.

This is a case where the effect gains from the mathematical explanation that follows, or precedes it. Paul Erdős (1913–1996), from whom this effect draws its name, is a renowned mathematician. Without a permanent address, he traveled relentlessly with light luggage and abundant ideas to share with his many collaborators. During his lifetime he published more than 1500 scientific articles, a number that has never been equaled. One theorem of his ensures that given a sequence of five different numbers, three of them (at least) are in order (increasing or decreasing). For example, given 2,4,1,7,9, numbers 2,7, and 9 appear in increasing order. To prevent the audience from spotting a too obvious ordering, magicians often resort to order alternatives, using for example an order of suits like ChaSeD (First Clubs, followed by Hearts, Spades and, finally, Diamonds) to ensure this ordering principle. So it doesn't matter how well the cards have been shuffled there is always a configuration of 3 cards to turn over.

Let's look at a possible setting for the Helper to leave to the Magician:

[Figure 3]

Note here that even though we are using four cards with the value of 5, their order is well established, due to the CHaSeD order for the suits. So the order we use for the deck is: first compare rank, and then use the CHaSeD order in cards of the same rank. This means that the fives respect the following order: 5C < 5H < 5S < 5D. Knowing that the covered cards are in ascending order the deuce will have to be the card to the left, followed by the fives of spades and diamonds:

[Figure 4]

Colorful piles

The Magician presents a deck to the Volunteer, asks him to shuffle the cards and separates them into two piles of 26 cards, one facing up and one facing down. She then proposes a game to the Volunteer: she gives him the pile facing up and asks him to separate the black cards from the red ones, one by one, into two piles. The Magician picks up the pile of cards facing down and draws the cards at the same pace as the Volunteer: every time the Volunteer puts a card on a pile, black or red, the magician puts her card on a pile in front of it, always facing down. At the end, there is a red pile and a black pile, each with a bunch of cards facing down in front of him. The Magician then says that the colors of these piles magically influenced the ones that are facing down. How? By influencing the number of cards of each color. Taking the face down piles, and counting, the Magician verifies that the number of black cards in the pile in front of the black pile is equal to the number of red cards in the pile in front of the red pile.

This time there is no other explanation of method than what was described in the effect. This is a completely self-working trick. However, the **mathematical** analysis is a bit more refined. Let b be the number of cards in the black pile and r the number of cards in the red pile. In a complete deck, we have $b + r = 26$ because the Volunteer was left with half a pack. The piles of the Magician (facing down) have $26 - b$ black cards and $26 - r$ red cards, distributed on both sides. One could now argue in this way: these numbers actually allow for the pile in front of the red pile to have only black cards, in which case the pile in front of the black pile would have only red cards. Then the number of red cards in the pile in front of the red pile and the number of black cards in the pile in front of the black pile would both be zero. Any other disposition of cards in these two downward piles can be achieved by exchanging cards between the two piles, and this operation preserves the equality.

Alternatively, one can also do some algebra. The pile that sits in front of the black pile has b cards, of mixed colors. Let b' be the number of black cards in this pile, $b' \leq b$. Similarly, the pile in front of the red pile will have r cards, of which r' will be red and $r - r'$ will be black. We want to show that $b' = r'$.

[Figure 5]

If we count the number of black cards in the piles of the magician, we have $b' + (r - r') = 26 - b$, which, after some reorganizing, gives us $b' = r'$.

Concluding Remarks

We hope the reader has gotten a more detailed idea of some magical effects done with cards, and that this serves as an appetizer and small reference for our workshop—we may incorporate other effects, depending on the participants' taste. The depths of the mathematics involved and exploration, as well as the performance, are much to the taste of the performer, but we hope their richness has become evident. Much can be added, and the analysis and reflection on how to perform these "tricks", greatly influence the effect they have on the public you are performing for. We hope the workshop will prove to be an occasion for the tailoring of these effects and their use, according to the taste and sensitivity of each individual participant. These matters are relevant if you are doing a *simple* trick for a nephew, a demonstration for a class, or if you are mesmerizing a whole theater.

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