Making Math Visible

George Hart
Stony Brook University
Stony Brook, NY 11794, USA
george@georgehart.com

Elisabeth Heathfield
Bluewater School Board
Hepworth, Ontario, CANADA
elisabetheathfield@gmail.com

Abstract

We believe there is a cultural need to normalize mathematics and have spent three years developing art/math activities and documenting them with detailed lesson plans. By testing these activities in many classrooms ranging from first grade through university, we have been able to refine them and provide instructions on how to adapt them to different levels. The result is a free online resource, MakingMathVisible.com, that teachers can access to enrich their practice and anyone can use for fun inspiration. We propose that displaying beautiful physical 3D manifestations of mathematics will incite people to look beyond arithmetic and rote learning and help them see the delights that other math topics have to offer. The positive feedback we have received from students and teachers supports our view that making math visible is a useful strategy for improving education and creating a wider culture of mathematics.

Introduction

We are developing and documenting a series of hands-on activities [3] which address several cultural problems connected to the public perception of mathematics. A fundamental difference between math and language in our culture is that language is manifested everywhere in many different ways. No one can doubt that language is essential and meaningful and touches people's lives on a daily basis. Inspiring quotes are carved in stone; classic plays are repeatedly performed both by amateurs and by national art centers; great books are found in libraries, bookstores, and private homes in every town. Language has been “normalized” in our culture and as a result, every child internalizes the importance of learning language. In contrast, the public does not see and celebrate mathematics in a similar way. Typically, children do not encounter formal mathematical ideas until they enter school, where math may be introduced as a subject to be studied rather than enjoyed. Sadly, mathematics is often presented as something that will be useful later in adulthood, rather than as a beautiful part of what it means to be human and a language that connects people across time and civilizations.

Figure 1: *Catenary arch made of laser-cut wood pieces joined with cable ties. (6.5 feet tall)*

Figure 2: *Catenary arches of paper (16 inches tall) and of cardboard (4 foot tall).*

This is a systematic problem in our culture which we believe can be remedied through small changes that slowly normalize math for everyone, in a way similar to language. This paper presents a few ideas which will hopefully contribute to this effort. By making the beauty of math visible in physical forms, we hope it will penetrate the consciousness of the general public. We believe it is essential that mathematics be part of the everyday landscape and we hope to achieve this with the project described here.

A second motivating observation is the following: Most people are comfortable with the idea that language and art blend in a continuum. Their deep associations are celebrated in poetry, music, theater, cinema, opera, and a myriad of other ways. Yet many people are not aware of the range of connections–both modern and historical–between mathematics and art. The activities of this project lie at various points on the art/math spectrum and are designed to display beautiful mathematical ideas on a daily basis.

Thirdly, math is often perceived as a field that is only accessible to experts. Even recreational mathematics is not widely enough enjoyed. While a professional mathematician can walk down the street and see mathematical structures underlying everything, the public remains sadly unaware. In designing our activities, we have incorporated nontrivial math content that can be accessed at different levels. The hands-on component appeals to all and can be injected with as much mathematical substance as is appropriate for the audience. We have trialed these workshops with participants ranging from elementary to university students and have found that every group extracts something positive from the experience.

In addition, there is a lack of resources for cultural engagement surrounding mathematics and often a state of math anxiety is passed on from generation to generation. We hope this project will help address these issues as well.

Communities

One of our goals is to build communities around engaging mathematical objects that are non-threatening to people who would traditionally shy away from anything labeled "mathematical". We want to spark people's interest in math with inspiring artifacts and activities that intrigue them and encourage them to ask questions. We have seen that informal conversations will naturally arise around objects that blur the line between math and art.

The participatory nature of these activities results in a deep engagement with the content. We have observed students to take proud ownership of the work. Although we haven't conducted any formal assessment, we have noticed that participants are eager to share their learning with family and peers.

In the same way that the fine arts lead to amateur communities—book clubs, art classes, poetry slams, or musical jam sessions—that informally celebrate the joy of making art together, we hope to foster enthusiasm around the joys of mathematical thinking. The collaborative aspects of group projects in math/art bring people together around a common goal and therefore are powerful agents that bind communities.

The parallels between art and math extend to the attitudes that young people develop about the subject [5]. Many students in middle school start losing confidence in their ability to do math or art. In similar ways, they develop the idea that one must be born with a special gift for either subject. It is not uncommon to hear students or even adults say they have no "talent" for art or no "brain" for mathematics. This attitude is not felt as viscerally in other disciplines, e.g., an innate "talent" for reading isn't typically seen as a prerequisite for literacy.

We hope this project will help alleviate the pervasive problem of math anxiety, especially in young people, before it becomes a lifelong disability. We have found schools to be the most accessible and fertile ground for this research, because of our backgrounds in elementary and university teaching. For that audience, we have developed a free online resource that teachers can use to enrich their practice by incorporating art when teaching mathematics. We hope to expand the reach of this project to include more public venues with broader demographics, such as museums, libraries, math clubs, and math cafés.

A good illustration of the type of public institution that builds a community around math is a Museum of Mathematics, of which there are currently dozens around the world. These can be small and informal or curated on a large scale, such as MoMath in New York City, which one of us helped design. While large public museums are expensive and difficult to create [2], we hope that the approach promoted here will make inexpensive hands-on workshops more widely replicable.

When math is made visible, beautiful conversation pieces emerge that become the focus of mathematical discussions. In effect, schools or other public venues are transformed into mathematics museums of the students' or community's own making.

Personal Development

Another commonality that math and art share is that they both can be excellent vehicles for developing positive personal attitudes. If either subject is taught in a way that allows and encourages students to be creative, they will discover how focus and persistence lead to worthy results. Students will take ownership of their learning and seek out new challenges for the sheer pleasure of finding a solution. We have created Making Math Visible so teachers can supplement traditional lessons with activities that nourish intrinsic motivation.

Figure 3: *Paper construction of 30 squares. (8 inch)*

Figure 4: *Skewer Hyperboloids. (10 inch)*

Significant benefits of activities that involve "spatial reasoning" are well documented [1]. We have also found that many young students are comforted by the logic and structure of mathematics. Students who struggle to deal with the chaotic world around them often find a refuge in the rationality of a mathematical task. They are happy to spend hours patiently concentrating on solving a puzzle or creating a mathematical structure. There is a meditative quality to working on a complex "piece of math" analogous to the flow in other subjects. Just as art may be used as therapy, the world of mathematics provides a rational sanctuary of "math therapy" for some individuals.

It has become a mission of many math educators to encourage exploration [6]. Discussions comparing different problem-solving approaches are known to have great pedagogical value [7, 8]. This gives students an opportunity for personal expression when developing and supporting their methods, analogous to what is seen in the arts. Creativity can and should be celebrated in both math and art, as we try to illustrate in the design of each of our workshops. Ideally, each activity will act as a launching pad for deeper artistic and mathematical exploration.

Figure 5: *"Disc-O-Ball" made from CDs and cable ties in the form of a truncated icosahedron. (30 inch)*

List of Workshops

Our website is a detailed guide for anyone interested in deep mathematical exploration through hands-on construction. The site includes material lists, templates, and instructions for teachers, museum educators, home-schoolers, and individuals looking to create visually engaging, tangible math objects. Each activity blends mathematics and art with high standards of math content and of artistic sensibility. A wide range of topics are touched upon that encourage mathematical thinking, including patterns, symmetry, spatial reasoning, puzzle-solving, logic, combinatorics, isomorphisms, etc. The following paragraphs briefly summarize some of the workshops we have written up so far. Additional details for each are at `MakingMathVisible.com`.

Following a trip to Saint Louis, we were inspired by the majesty of the Gateway Arch and decided to adapt a similar **catenary arch** into a form accessible to students. We developed three versions of the workshop: paper, cardboard, and wood models. These grow successively larger and the wood version is large enough for adults to walk through, Figures 1–2. The catenary form can be experienced on a purely visual level or explained with mathematical depth.

Paper is one of our favorite materials because it is inexpensive and extremely versatile. We have designed a series of **paper polygon puzzle** assembly workshops which only require cardstock and scissors, Figure 3. Each construction is a puzzle that can challenge students at different levels. Added difficulty is introduced with a logic puzzle involving color constraints. Students enjoy taking these home to share and explain to their family.

Figure 6: *Star, from twelve playing cards. (10 inch)*

Figure 7: *72 Pencil construction. (9 inch)*

The **hyperboloid** workshop illustrates the idea of a "ruled surface"–a curved surface composed of straight lines. Using shish-kabob skewers or chop sticks, participants assemble structures that flex and contract in compelling ways, Fig. 4. This is a great exercise for getting students to visualize and create a geometric pattern, a skill we have found to be surprisingly variable. The workshop concludes with a large scale version constructed from wood dowels—an inexpensive way to make math visible on a grand scale.

Recycled CDs make an exciting, eye-catching construction medium. In this workshop, participants use cable ties to assemble 150 CDs into the shape of a truncated icosahedron (a.k.a., soccer ball or football), Figure 5. The first part of the activity is done on paper to introduce students to the mathematics of perspective and give them a better understanding of the geometric structure. The end result is a shimmering conversation piece we call a **Disk-O-Ball**, which emanates the beauty of math.

Anything made out of playing cards immediately piques people's interest. The **12-Card Star** workshop uses oversized playing cards as puzzle pieces to be assembled, Figure 6. The cards are slotted and folded to come together as a visually engaging paper sculpture which challenges spatial reasoning abilities. The connections of the twelve cards are isomorphic to the edges in a cube, so the workshop is a natural opportunity to introduce the concept of isomorphism to students of any age.

As artists, we are always trying to find interesting materials to work with. Pencils are iconic objects in math class that can be turned into a surprisingly whimsical sculpture, Figure 7. This assembly uses **72 pencils** and eight rubber bands to create a fun geometric and dexterity challenge. The construction stretches participant's spatial reasoning skills from the three familiar XYZ directions to the four directions corresponding to the long diagonals of a cube.

Figure 8: *SOMA puzzle from cardboard boxes.*

Puzzles are a fantastic way to get students excited about math, logic, and spatial reasoning. In this workshop, participants assemble wood cubes to discover and create Piet Hein's classic **SOMA puzzle**, Figure 8. Playing with their own personal set allows participants to explore many assembly challenges before constructing a large-scale version from cardboard boxes. This oversized, physically imposing puzzle invites collaboration and mathematical conversations as the group manipulates it together.

Figure 9: *Giant cardboard construction. (6 foot diameter)*

Cardboard is an inexpensive medium for making large-scale, visually engaging geometric sculptures. In this activity, participants are led to assemble flat components, using glue and clamps, into an intricate mathematical **cardboard sculpture**, Fig. 9. Along the way they are discovering and extending patterns while developing visual thinking skills. The final result is an impressive centerpiece that highlights the creative side of math and can be publicly displayed to showcase mathematics as an art form.

Figure 10: *Wood dome. (5 foot diameter)*

Figure 11: *Cardboard dome. (12 foot diameter)*

Domes are familiar structures that students love to build because they become physical spaces that illustrate how math applies to architecture. We offer a series of progressively more substantial dome activities, ranging from a pedagogical paper model to functional cardboard and wood models. See Figures 10–11. The assembly becomes an exercise in collaboration that results in a human-scale structure that clearly manifests its mathematical foundations.

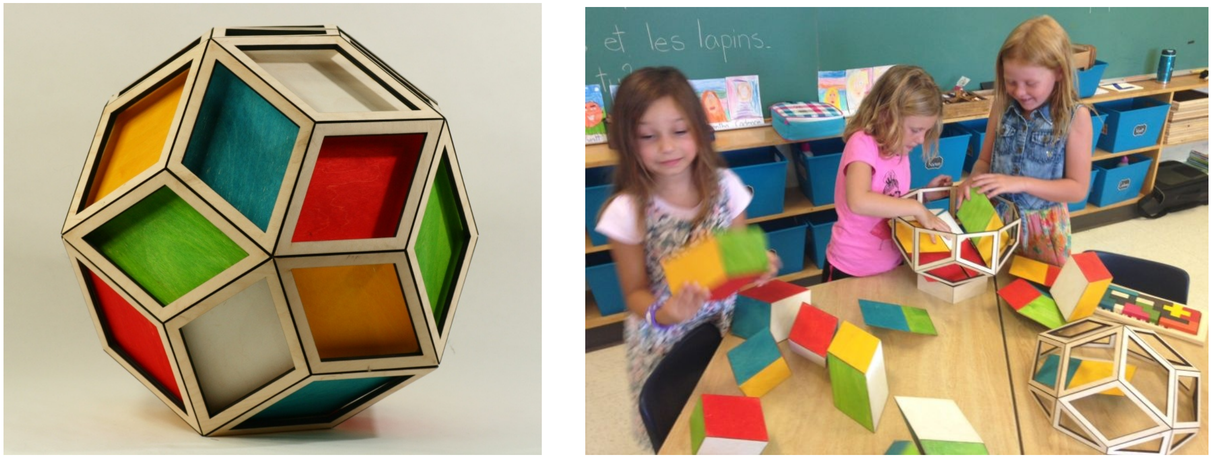

Figure 12: *Rhombic triacontahedron puzzle, wood. (16 inches)*

The **rhombic triacontahedron puzzle** is one of our favorites because it blends geometry, logic, color matching, and combinatorics, Figure 12. We led paper and cardboard versions of this workshop at Bridges 2016 in Jyväskylä [4]. It is a zonohedral dissection puzzle which challenges students because the rhombohedral parts look like slanted cubes, but don't have 90-degree angles, so come together in unexpected ways.

Figure 13: *Wood sculpture, "Winter." (25 inches)*

Sculptures made from laser-cut plywood can be mathematically rich and visually appealing. We have developed techniques using cable ties for assembling the parts, which allow for exciting construction events with low material costs, Figure 13. A set of four progressively more challenging **wood sculptures** are themed to the seasons, so they can be built at intervals across the school year. Follow-on activities lead students to modify the designs to make their own customized sculpture. Aspects of puzzling, logic, spatial reasoning, and pattern discovery are developed. The resulting sculptures can be studied mathematically or enjoyed purely as works of art.

Three **polylink puzzles** derive from regular polyhedra, but the faces join by linking instead of meeting edge-to-edge, Figure 14. When made with colored craft sticks and glued together, these structures become beautiful display pieces. But their apparent simplicity obscures the difficulty in understanding and assembling them, so they will present even advanced students with worthy challenges and opportunities for growth.

We are still refining several other workshops that are not fully documented yet and we have plans for many more that we will write up as time permits. So check the MakingMathVisible.com website periodically for updates.

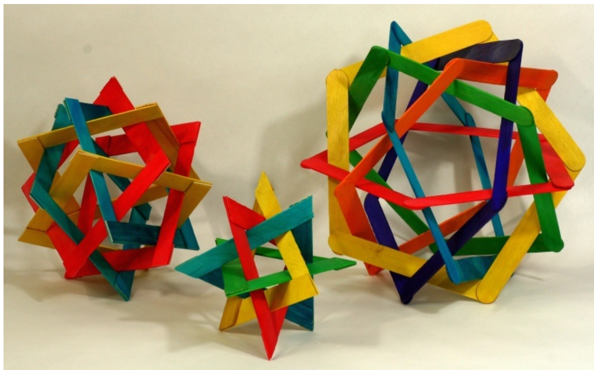

Figure 14: *Popsicle-stick polylinks. (6–11 inches)*

Figure 15: *A classroom making math visible.*

Conclusions

We have observed that creating and displaying beautiful mathematical objects inspires viewers to enjoy and appreciate various topics in math. It is our hope that by placing visually engaging constructions prominently in our everyday surroundings, mathematical thinking and discussion will become normalized. Perhaps a negative image of math in popular culture can be gradually remedied by involving the public in exciting math/art activities that change people's attitudes and their perceptions of what math is. To this end, we have designed objects and construction workshops that we have carefully documented so they can be widely replicated. Although our workshop write-ups are in the form of lesson plans that teachers can use to enrich their classrooms, e.g., Figures 1 and 15, we hope to see the content used by a much broader public. We have received positive feedback from teachers and students that encourages us to continue these contributions to math education and general culture. We hope this ongoing project will help improve society by leading to happier, better educated, and more rational citizens. We urge everyone to participate in Making Math Visible.

References

[1] Brent Davis et al., *Spatial Reasoning in the Early Years,* Routledge, 2015.

[2] G. Hart et al., "Forming a Museum of Mathematics," in *Science Exhibitions: Communication and Evaluation*, ed. Anastasia Filippoupoliti, MuseumsEtc publ., 2010.

[3] G. Hart and E. Heathfield, http://MakingMathVisible.com (as of Feb. 1, 2017).

[4] G. Hart and E. Heathfield. Rhombic Triacontahedron Puzzle. In Eve Torrence, Bruce Torrence, Carlo Séquin, Douglas McKenna, Kristóf Fenyvesi and Reza Sarhangi, editors, *Proceedings of Bridges 2016*, pages 609–614, Phoenix, Arizona, 2016, Tessellations Publishing, online at http://archive.bridgesmathart.org/2016/bridges2016-609.html

[5] Daniel Jarvis and Irene Naested, *Exploring the Math and Art Connection,* Brush Education, 2012.

[6] Paul Lockhart, *A Mathematician's Lament,* Bellevue Literary Press , 2009.

[7] Zbigniew Michalewicz and Matthew Michalewicz, *Puzzle-Based Learning,* Hybrid Publ., 2008.

[8] Wei Zhang, *Exploring Math through Puzzles,* PQ Innovations, 2011.