Using African Designs in Virtual Manipulatives for Geometrical Concept Development

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Abstract

Promoting interest and engagement in mathematics among learners in early high school grades is challenging in the South African educational scenario which is typified by traditional pedagogies, desolate classrooms, a lack of interesting resources and something of a rote learning culture. To enhance the attractiveness and the cultural congruence of learning materials for early high school grades we have used GeoGebra to design a set of virtual manipulatives, which incorporate Southern African indigenous art designs, for promoting concept formation in geometry. We explain the underlying pedagogy of the development and show an example. Plans for incorporating the material into existing programs and for researching the learning outcomes are also presented.

Introduction

The challenges faced by teachers and learners of mathematics in South African schools are well documented [1]. They include lack of adequate teacher training and professional development, lack of teaching and learning resources, desolate mathematics classrooms and reliance on outmoded pedagogies. In-school challenges are compounded in many communities by a socio-economic disadvantage which presents as low levels of parental literacy and numeracy, lack of parental and community support for children and a lack of role models who have succeeded as a result of education.

Learner achievement in mathematics is consequently poor with South Africa ranking low on international benchmarking measures such as TIMMS [2]. Annual National Assessments at various grades also yield low averages, and weaknesses are compounded from one grade to the next [3].

For those learners who persevere with mathematics, some areas of the CAPS curriculum present major conceptual difficulties. An area of pronounced weakness is geometry. Many senior high school learners have difficulty in constructing logically correct answers for geometrical questions as they lack the basic conceptual tools required [4]. The roots of these difficulties can be traced to problems with earlier development of concepts of shape and space as well as limited opportunities to develop logical arguments.

A Techno-Blended Approach to Mathematics Development in Schools

To engender enthusiasm and interest we have included recreational and conceptual manipulatives, developed in GeoGebra, in a curriculum-aligned, off-line, digital teaching and learning package. The TouchTutor™ package, developed over the past five years in South Africa, serves as the basis of a comprehensive techno-blended model [5] for the teaching and learning of mathematics.

The package includes a wide range of learning resources including video, PDFs, reference facilities and language support for mathematics, a self-testing system and a range of GeoGebra applets that are designed for learner investigations.

The rationale for building and implementing such an integrated learning system rests on research reviews showing the potential enabling and liberating role of digital learning [6] and claims of positive learning outcomes associated with mobile learning [8, 9]. The GeoGebra applets that form part of the package were developed to be recreational but also educational. Through the project, we investigated the value of the applets and it was found that learners believed that using the applets improved their understanding of applicable mathematics concepts.

Southern African Indigenous Art Designs as Stimulus for Learning about Geometry

Many authors [10, 11] have documented the richness of incorporation of mathematics in African designs in the form of tessellations, sona, string figures, fractals and number patterns in games. Gray and Sarhangi [11] illustrate how beautiful cultural designs can easily be incorporated into the secondary school mathematics classroom. The inclusion of cultural designs narrows the gap between the learners' world inside and outside the school environment [9, 10].

Gerdes [9] argues that it is increasingly necessary to "multi-culturalize" the mathematics curriculum as mathematics in Africa is often seen as useless and foreign. Including aspects of ethno-mathematics will improve learners' confidence in the value of mathematics. Ethno-mathematics involves a valuing of indigenous mathematical knowledge, practices and applications in arts and crafts and the incorporation of these into the teaching and learning of mathematics.

The inspiration for the design in the applet described comes from the beautiful designs using basic geometrical shapes that the Ndebele woman use to decorate their homes. The Ndebele, a tribe in Southern Africa, has a culture rich in decorative geometrical patterns (Figures 1 – 3) applied to architecture, fabric and beadwork [12]. Other related South African cultures have their own decorative patterns.

Figure 1: *Painted walls*

Figure 2: *Beadwork bracelet*

Figure 3: *Fabric design*

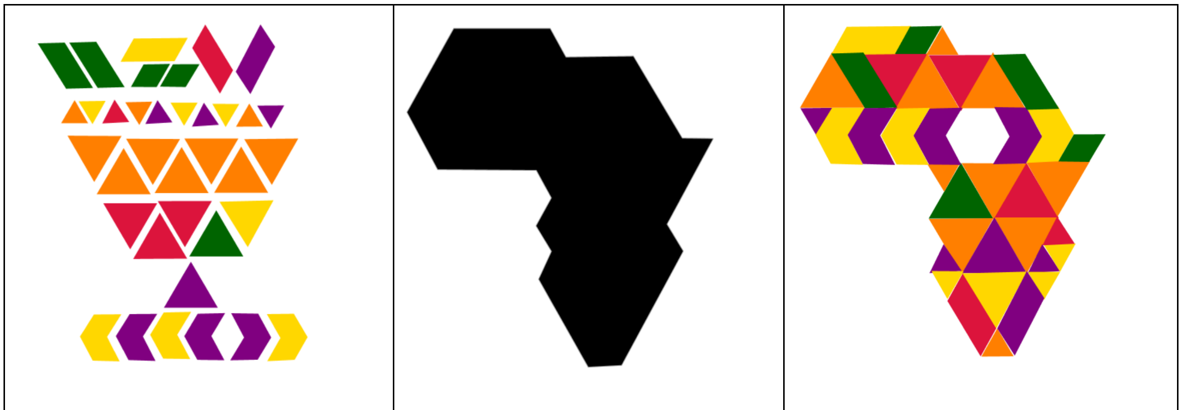
An African Map Tessellation with Ndebele Patterns Created with GeoGebra

To enhance the attractiveness and the cultural congruence of learning materials for early high school grades we have used GeoGebra to design a set of virtual manipulatives for promoting concept formation in geometry. One such applet is illustrated in Figure 4 on the following page.

The puzzle pieces are created as rigid polygons in GeoGebra. We used polygons similar in shape and colour to those used in Ndebele designs and included a set that can be combined to form a stylized map of Africa. Each shape may be translated or rotated by the user of the applet to create their design (Figure 5).

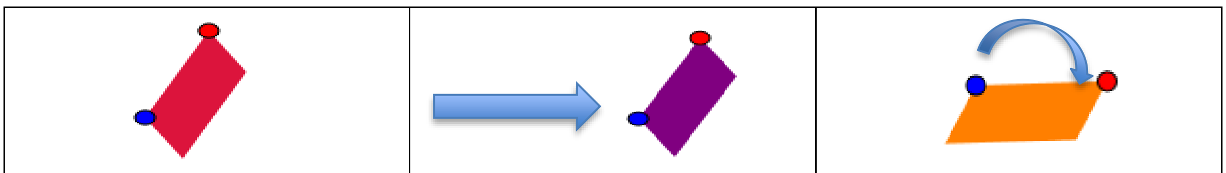
Underlying Pedagogy

The learners involved in the project have typically experienced a learning environment poor in stimulus and challenge, particularly in space and shape. We believe attractive virtual manipulatives, which are culturally familiar in design, could be a catalyst to developing greater confidence and competence in geometry. Clements [13] contends that manipulatives can be used very successfully in the instruction of mathematics.



Puzzle pieces	Puzzle outline	Completed puzzle

Figure 4: *Elements of an Africa Map puzzle inspired by Ndebele designs.*



Puzzle piece	Translation	Rotation

Figure 5: *Manipulating rigid puzzle pieces in GeoGebra.*

Applets implementing simple tessellation type puzzles provide an opportunity to identify basic polygons (such as the triangles, rhombi, parallelograms, and hexagons in the puzzle described here) and to manipulate them by applying basic transformations such as translation and rotation. The degrees are shown during the rotation of the shapes, visually linking the degrees and the orientation of the rotated shape. There are many applications that can be demonstrated for e.g., the effects of 180° rotation and another 180° rotation returning the shape to its original position. The vertical and horizontal movement of the shape can be described to reinforce the effect of translation. Fitting pieces to the map teaches visual planning.

There are good opportunities for class discussion about ways of combining shapes to create bigger shapes, both in the form of larger regular polyhedral, quadrilaterals or more complex compound shapes. The design could also easily be reproduced on card allowing the learners to physically manipulate the objects, similarly to tangrams. In the digital form, applets can also assist learners to become familiar with the use of technology in a non-threatening manner.

Pilot testing of the example applet indicated that it was easy to use, enjoyable and motivating. It challenged learners' design ability and required iterative techniques. The range of new composite figures which could were formed within the puzzle was interesting, as was the many different solutions which varied in the extent to which symmetry and colour patterns were used. The activity provided fruitful opportunities for discussion on classification of polygons and their properties, transformations, and tessellations. The cultural context of the design was clearly identified as African.

Further Development and Research

The current indigenous art applets use simple shape identification, combinations and transformations for the junior grades. We already have various other applets that help learners in the senior grades to visualize the more complex theorems but none of them incorporates indigenous art. Therefore, we intend to extend the number of applets by identifying more indigenous patterns that lend themselves to dynamic investigation by senior learners.

The material will be made available via the TouchTutor™ system to groups in under-resourced rural schools in the Eastern Cape province of South Africa [9].

Pilot testing has provided a framework for studying learner interactions with the material to gauge effects on engagement with mathematics, motivation and, more specifically, the development of basic concepts in Space and Shape which form part of the current new curriculum.

Conclusion

Mathematics teaching and learning can be enhanced with a creative use of materials so that local cultural content resonates with and motivates learners. Future development, implementation and ideas will be guided by learner responses to these materials.

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