Topological Images with Modular Block Print Tiles

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Abstract

Inspired by connected sum constructions of surfaces and knots, I created modular block print stamps to create visual representations of surfaces and knot mosaics. With these stamps, I can create a variety of images. This paper describes the process of making the prints from inspiration to creation, and the influence of Chinese art and crafts.

Inspiration

For years, I thought about how I could combine my interests in mathematics and art, and only recently have I made my first attempts. In college, I carved linoleum and rubber blocks for making prints and greeting cards, as I liked the idea of being able to replicate an image. In graduate school, I studied algebraic structures that were derived from concatenating loops at their intersection points. I drew pictures of genus surfaces and loops on them to visualize problems and make conjectures. I wanted to replicate the image of a genus surface by carving a modular stamp set where I can create surfaces of various genus, as in Figure 1.

Figure 1: *Genus 33 Print*

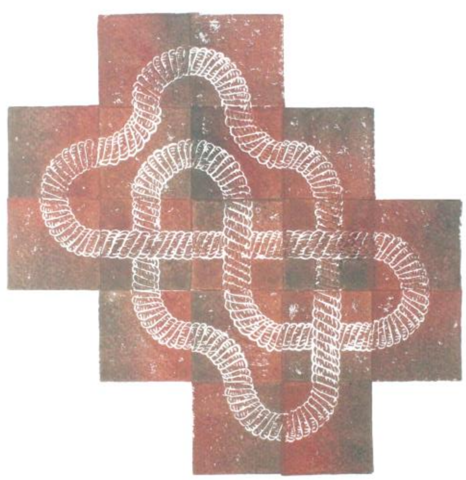

Figure 2: *Gradient Trefoil Print*

While judging the undergraduate poster session at the recent Joint Mathematics Meetings in Atlanta, GA, two students had a poster on hexagonal knot tilings. Since I am not actually a knot theorist, this fascinated me, and I immediately wondered if a game or puzzle was made from this idea (and I found that these already exist). When I learned about Lomanaco and Kauffman's square mosaics [1], and the knot invariant coming from a minimal tiling representation of a knot, I was motivated to make knot block prints (Figure 2).

Chinese arts and crafts are a source of inspiration for my surface and knot block prints. I find it fascinating that Chinese paintings are stamped with name seals directly on the painting. The name seals are not only from the artist, but from the owners of the work of art, so I incorporated seals in my prints. As a child, my mother had a book on Chinese knots, from which I tried my best to construct the knots. Something that struck me about the knot mosaic images is that they look very much like the instructional diagrams for making Chinese knots. This inspired me to not only print the usual math knots, but to also recreate and study knot tiles of Chinese knots.

Designing and Carving of the Blocks

For my prints, I used soft linoleum and soft-rubber printing blocks. I first drafted the designs on newsprint, using mostly drawing implements, a ruler, and an engineering oval template. For the genus surface design, my original inspiration was to make blocks that connect to form an infinite design for wallpaper. The design is based on how mathematicians usually represent holes in a genus surface, with an oval with slight tails to add dimension. I created a partial genus surface out of a square block, with a hole in the center, and four half-holes with 90-degree rotational symmetry, as in Figures 3 and 4. I carved two endcaps to connect to the other side of the holes so I can cap off the ends to make a representation of a closed surface.

For the knot tiles, I took inspiration from Lomonaco and Kauffman's paper [1] and drafted four blocks, as seen in Figure 5. Their paper includes 11 tiles with symmetry, but I only carved the minimum needed because I can rotate the tiles when printing. To make the print more interesting, I drafted a rope design, because I wanted to do a play on words with nautical knots, and I was inspired by images of old nautical knot diagrams. I discovered later that Robert Fathauer's KnoTiles have a similar design with rope images on the puzzles pieces. What was tricky about drafting the rope design was ensuring that the rope image will match with connecting tiles when printing, and I also had to ensure that the shading would be consistent.

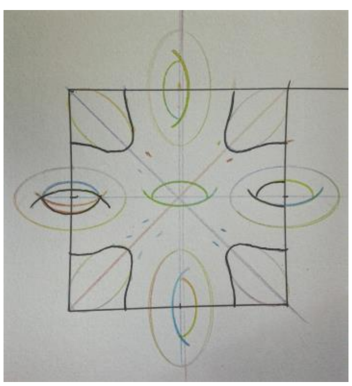

Figure 3: *Draft of Genus Surface Block*

Figure 4: *Partially Carved Block*

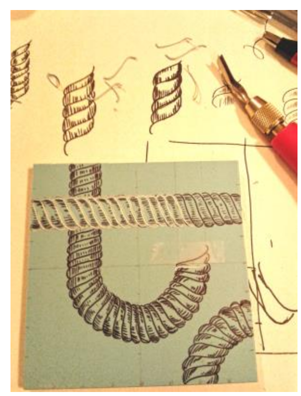

Figure 5: Rope Carving

Printing

Before printing the images, I sketched the designs and made measurements to ensure that the design will fit on the paper I had. For the genus surface prints, I made a few designs of interestingly shaped genus surfaces. I had to make sure that my blocks lined up, so I would print each block one at a time, and then connect the next adjacent block. I like the idea that there are infinite possibilities for what I can print with these few blocks. I did not mind if there was overlap in the ink, because it added to the fact that the print was handmade, and the viewer can clearly see the modular aspect of the print.

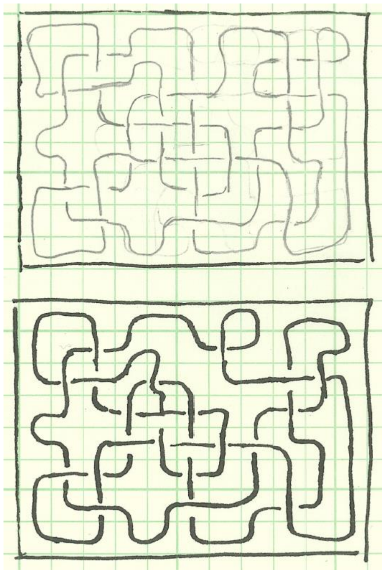

Figure 6: *Knot sketch*

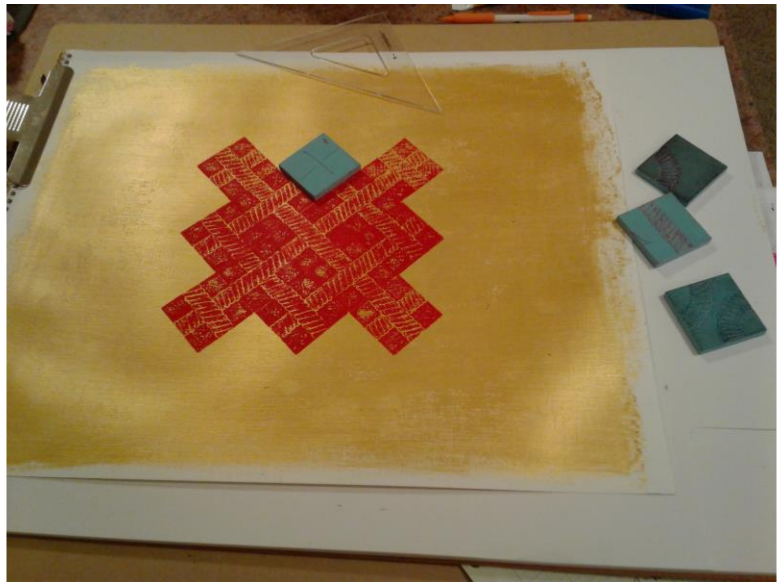

Figure 7: *Printing process*

The process for printing the knot tiles was the same. I drafted designs of knots on the wrong side of engineering graph paper (Figure 6). I started with making different unknot representations, before trying the trefoil. I drafted designs ranging from math knots, knots from nautical knot-tying, and also knots inspired by Chinese knot-tying. The Chinese-style knots were much simpler to draft, because the instructions for tying resemble very much the sort of knot shapes obtained from the curved mosaic tiles I carved. When I actually printed the tiles, I purposely did not make a grid or careful measurements on my paper because I wanted to emphasize the handmade aspect of printing, where the blocks will inevitably be overlapped at the edges of the square (Figure 7). I wanted to create this overlap so it is clear to the viewer that the print is composed of different squares, to relate to the knot mosaic theory.

Symbolism

I incorporated some symbolism in my prints. I carved a "blackboard Z" block for my genus surface prints, to represent the integers to stamp between the empty spaces of the surface print (Figure 12). I carved my main genus block body in a way that resembles an addition symbol, so I interpreted the print not only showing the surface, but also a direct sum of integers. There is one Z for each genus hole in the surface, as what results when computing homology. I also stamped the Z's in red and carved it in a square shape inspired by a Chinese name seal, as I interpreted the Z's as "owning" or representing the surface, the same as someone would stamp their name seal on a work of Chinese art. The collection of name seals shows the provenance of a work of art, and I thought of the Z's as representing the provenance of the surface. I also carved my own name in the style of an old writing-style name seal to sign the print.

For the Chinese knot prints, I printed them mainly in red, as seen in Figures 8 and 9, as these knots are usually tied with red cord. Chinese knots are often tied with a single cord with the ends left loose, or attached to a tassel, but I closed off the ends in the diagram to form a mathematical knot. I also printed a representation of the infinity knot, which is a knot that is used as a popular motif in Buddhist art.

Figure 8: *Endless Knot Print*

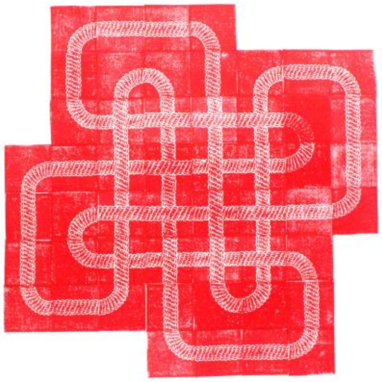

Figure 9: *Chinese Knot Print*

Conclusion

Through the slow process of printing representations of topological surfaces and knots, it brought me to think about the mathematical aspects of connecting pieces, such as connected sums and the mosaic number of a knot. The process of printing knots brought up some questions that I did not initially have after reading [1] or [2], because the knot mosaic invariant is from the minimal dimension of a square mosaic to represent a knot. I had the opportunity to play with finding knot representations with a smaller number of tiles, but I also became interested in filling the space with knot tiles. In one print, I have a simple trefoil in Figure 10, but I wanted to fill up the dimensions of the paper completely with non-blank mosaic tiles. This took several sketches before I found a non-minimal mosaics that would cover the space, a sort of space-filling knot, as seen in Figure 11. I am considering using this art in my future teaching of topology and knot theory, as a teaching tool for play and illustrating topological ideas.

Figure 10: *Trefoil*

Figure 11: *Filling Trefoil*

Figure 12: *"Blackboard Z" detail*

References

[1] S. J. Lomonaco and L. H. Kauffman, "Quantum knots and mosaics," *Quantum Inf Process,* vol. 7, pp. 85-115, 2008.

[2] S. Jablan, L. Radović, R. Sazdanović and A. Zeković, "Knots in Art," *Symmetry,* vol. 4, no. Symmetry and Beauty of Knots, pp. 302-328, 2012.