Visual Arts and Mathematics Education: Looking for Integrative Phenomena

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Abstract

The starting point of our article is to identify integrative phenomena, that are suitable for both visual arts and mathematics and, moreover, for teaching that preserves the nature of each of these subjects and their specific methods of acquiring and processing information. The article consists of two separate, yet interconnected studies: a teaching experiment, implemented in a Finnish 7th grade lower secondary school, and a survey aimed at preservice mathematics teachers with the question: `What are mathematical phenomena?´. The teaching experiment revealed that integrative phenomena suitable for use in school must be sufficiently open, to enable pupils and teachers to have a genuine joint investigative learning experience. Following this finding, we looked for more general principles and, from the survey, we found that the phenomena could either be related to concepts that are shared by each of the integrating disciplines, such as ratio and representation, or they could be more dynamic, typical processes of knowledge- acquisition and construction, such as sorting and classification.

Introduction

The Finnish national core curriculum for basic education, determined by the Finnish National Board of Education, has been recently reformed and the new curriculum will be introduced in August 2016. It includes the objectives and core contents of different subjects, as well as the principles of pupil assessment, special-needs education, pupil welfare and educational guidance. The principles of a good learning environment, working approaches as well as the concept of learning are also addressed in the core curriculum. In the reform, emphasis has been placed on collaborative classroom practices that include interdisciplinary, phenomenon- and project-based studies where several teachers may work together with the pupils. The main focus is on increasing curiosity and motivation as well as combining knowledge and skills from different subjects and therefore enhancing understanding and multi-literacy [1].

In this article, phenomenon-based teaching and learning as implemented in visual arts and mathematics is discussed, together with the goal of preserving the nature of these disciplines and their typical working methods. Two interconnected studies were conducted, which we present here in parallel: a teaching experiment implemented in Finnish 7th grade (age 13) lower secondary school, and a survey of Finnish preservice mathematics teachers' views of mathematical phenomena. In these studies, we examine how to describe phenomena that are of interest to pupils and at the same time clarify the nature and traditions of the disciplines involved. Furthermore, we identify phenomena, that enable teachers to design and construct discipline-based integration and interdisciplinary learning continuums and to create conditions for building broad, open and collaborative classroom practices.

Integrated and Holistic Teaching

A central aim of the Finnish National Core Curriculum for compulsory basic education 2014 is the interdisciplinary study of phenomena. Schools are required to design and provide at least one such study-period per school year for all pupils. This is a part of larger whole focusing on developing organizational culture and learning communities and generating innovative learning environments and working approaches [1].

Halinen and Jääskeläinen [2] suggest that teaching could be integrated in three ways. Firstly, the focus could be on pupils, with learning continuums grounded on their experiences, questions, and interests; secondly, the focus could be on learning communities, with the collaborative aspects represented; and thirdly, the focus could be on building interdisciplinary studies [2]. In secondary schools, discipline-based integration offers a way to organize integrative learning continuums. Tani, Juuti and Kairavuori [3] raise the question of what might be an appropriate level of subject-specific knowledge in order to teach subjects in an integrated curriculum [3].

When visual arts and mathematics are integrated, teaching is considered holistic. Räsänen [4] has suggested that her model of teaching integrative visual arts could be a frame for outlining all art and craft subjects. In this model, knowing is based on observations, emotions, forms representing each field, and cultural symbols. In the teaching and learning process, knowledge gained from observations and emotions is connected to conceptual and cultural knowledge. These are also the key elements of investigational, experiential mathematics teaching and learning [4].

Investigative Learning in Visual Arts and Mathematics

Learning in visual arts requires, according to Räsänen [4], an ability to identify specific characteristics of a phenomenon and moreover, discover how these features could be illustrated or visualized and what kinds of metaphorical meanings they hold. Activities can either produce something or interpret others' artistic work. Quite often the knowledge behind a work of art is tacit and understanding or interpreting the work requires a fine-grained ability to distinguish relations and cultural symbols. Accordingly, we can talk about investigative learning in visual arts [4].

In mathematics, investigative learning is often seen as a synonym for problem-solving and inquiry-based mathematics. In this paper, by investigative learning we mean that pupils have opportunities to work with non-routine mathematical problems or unfamiliar situations, individually or collaboratively, and thereby develop their own meaningful solutions [5]. When working on problem-solving tasks, pupils learn what different means in terms of solutions, and how to produce different solutions. In this case we can talk about agency, i.e pupils' own mathematics, which is, nevertheless located within the cultural framework shaped by the mathematics community [6].

Searching for Phenomena to Integrate Visual Arts and Mathematics

In this paper, we search for phenomena that integrate visual arts and mathematics. The results of the two interconnected studies shed new light on practical curriculum integration and the challenges this presents regarding teacher preparation.

First we introduce a case study conducted in a Finnish lower secondary school in autumn 2014. One 7th grade class (20 pupils) participated in a teaching experiment based on two themes integrating visual arts and mathematics [7]. The teacher implemented an action research on the basis of the national curriculum for 7th grade pupils. She planned and carried out the teaching, videotaped the learning situations, and kept a diary of her and the pupils' work. She also collected pupils' written and visual artefacts for data. Two of these projects are presented here to clarify the differences between these projects.

Using these teaching experiments as reference, we then began to plan how these integrative aspects could be integrated into teacher education. In autumn 2015, we conducted a survey at the University of Helsinki at the beginning of a mathematics student teachers´ meeting regarding curriculum integration. The students were invited to think about the topic individually and answer an open question: What are mathematical phenomena? The students were told that they do not have to return the form if they do not wish to participate in the study. In total, 51 forms were returned.

Analysis

The data collected from the action research in school consisted of 7th grade pupils' products from mathematics lessons and visual arts lessons [7]. Two themes were introduced and implemented: 1) personal meanings when producing and interpreting integers, and 2) sorting and classifying as methods in visual arts.

Integers is one of the key topics in the curriculum concerning the 7th grade mathematics teaching and learning and is normally introduced at the beginning of the autumn term. In the class under study the pupils worked in the mathematics lesson in small groups of 3 or 4. Their task was to choose at least five numbers or symbols from the following list ($1, -5, 4, 7, 32, 8, 11, \infty, 13, -10, -1, 0, 49, 64$), and to logically and precisely justify their selection. The pupils' answers were then interpreted together with the pupils using Brown's [8] categorization (p. 151): 1) apperceptual scheme – the object itself, 2) appresentational scheme – the object seen as a sign, 3) referential scheme – the thing signified, and 4) interpretational scheme – the connection between sign and signified [8].

The pupils work then continued in the visual arts lesson. This time the task was to choose an integer and then to visualize it on 20 x 20 cm paper either by drawing or painting. If there was enough time an extra task was to visualize the opposite number. The thinking process in this task thus differed from the first task. The pupils' pieces of work were then categorized together with the pupils using the same categorization they had learned in their mathematics lesson. Next, the drawings were organized according to their numbers on a number line. Later, the teacher observed that the pupils' drawings represented various cultural systems and that the nature of their understanding varied. Consequently, she discussed with the pupils, positioned their pieces of work along two axes, and hung them on the school wall.

Different methods of making art is one of the contents listed in the visual arts curriculum for the 7th grade. In connection with this, the second theme of the teaching experiment – sorting and classifying as methods in visual arts – was introduced in the visual arts lesson by exploring contemporary artist Antti Laitinen and his work Forest Square (www.anttilaitinen.com). In his work, Laitinen removed a 10 x 10 metre patch of forest and sorted it into its consistuent materials, such as earth, moss, wood, spruce needels, etc. He then rebuilt the piece of forest by arranging its different materials by colour, texture and form. The pupils were given a similar task within their small groups to identify the different recognizable textures from either a plant with roots, or a fruit. They were to then reorganize the object again by sorting and classifying its textural components and forming a visually interesting work. The whole process was photographed, and the weight and volume of each component were measured.

The work continued in the mathematics lesson. Based on the measurements made, each group calculated the ratios: how much each textural component weighed compared to total mass of the plant or fruit. Findings were reported as fractions, decimal numbers and as a percentage. Finally, the small groups presented their work to the class and, together with the teacher, sorting and classifying in mathematics was discussed.

In the analysis of the survey of Helsinki University mathematics student teachers, the survey responses were transcribed into electronic format, and analysed qualitatively using the grounded theory approach. In the first phase of the analysis we looked for descriptive words (marked below in different text styles), numbering as many as 52 in total.

> #23
> I can't give any example of a particular mathematical phenomenon, but it would be natural to combine it with **logical reasoning**: architecture, golden section in *nature* and art.

We used these descriptive words as a tool for categorizing the data (see Table 1). The following table lists a total of 202 excerpts from the data.

Table 1: *An example of the analysis of preservice mathematics teachers' responses*.

Frequency	Descriptive word	Excerpts from the data
5	modelling	mathematically modelled, modelling disease, mathematically modelling, can be modelled, can be modelled
7	art	from art, from visual art, art, visual art, in visual art, from visual art, in visual art
4	observation	observations, the differences and similarities of observations, observation, visually observable

Finally three categories were established: 1) Internal phenomena of mathematics, 2) Mathematical phenomena in nature, and 3) Mathematical phenomena in cultural settings.

Findings

The first theme of the teaching experiment integrating visual arts with mathematics was `creating personal meanings and interpreting integers´. This was first explored in the mathematics cass by asking pupils to form sets of numbers. During the group activity the pupils placed numbers into categories as follows (Table 2). The `The object itself´ - category remained empty, while the majority of sets were placed in the category `the connection between sign and signified´. The pupils´grounds og selecting this category were mathematical. Surprisingly, none of the groups presented a fully nonmathematical set, such as hockey players' numbers.

In the visual arts lesson the drawings were examined together with the pupils using the familiar categorization from the mathematics lesson. The examples drawings in Figure 1 show how the pupils approached their own number in many different ways. The first drawing clearly illustrateds as well as its numerical value. The second drawing represents the shape of the number in the form of swans' necks, as well as its numerical value in several instances. Many pupils chose a number which representing a system, such as money, shoe size, awards or temperature scale. In one drawing the chosen number 3 is displayed along with its numerical value its ordinal number, and metaphorical meanings of the number taken from well-known fairy tales.

Table 2: *Frequencies and examples of sets of numbers presented by pupils.*

Category	*f*	Examples of sets of numbers presented by pupils
Apperceptual scheme – the object itself	0	
Appresentational scheme – the object seen as a sign	7	\`single-digit numbers´ \`all figures have circular forms´
Referential scheme – the thing signified	3	\`consecutive numbers´ \`numbers are between zero and infinity´
Interpretational scheme – the connection between sign and signified	14	\`numbers are every fifth number´ \`previous number is divided by two´ \`all the numbers are positive´ \`none of the numbers can be divided by two´

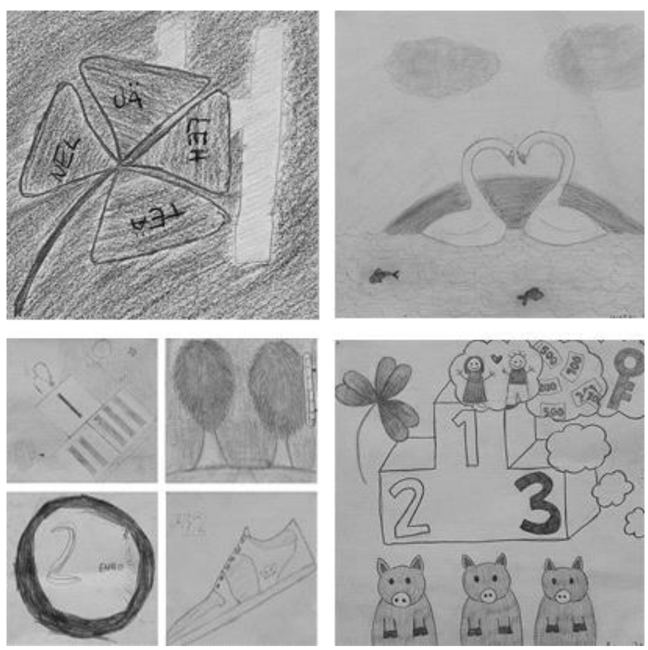

Figure 1: *Examples of pupils' drawings derived from the first theme of the teaching experiment: \`creating personal meanings and ingerpreting integers´.*

During the lesson, the pupils' work was organized on a number line and concepts such as opposite number and infinity were discussed. Later, when the teacher reflected on the results of the pupils' work, she noticed that the drawings clearly differed from each other, ranging from literal examples to abstract interpretations and from personal to common understanding of the information presented. Some drawings were easy to interpret, others more personal to the artist. Some drawings clearly presented a commonly known systems, others presented personal interpretations of them. The teacher arranged pupils´ work on the school wall along two axes: *Personal knowledge – common knowledge*, and *Own conseption - known system* (Figure 2).

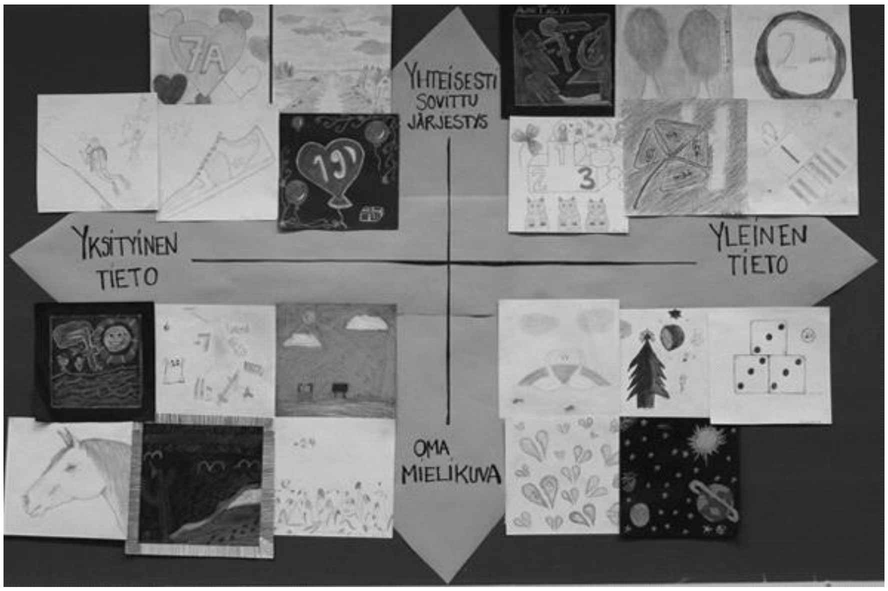

Figure 2: *Pupils' drawings organized on two axes. X-axis: personal knowledge – common knowledge , y-axis: own conception – known system.*

Open themes are typical in teaching and learning the visual arts. Nonetheless, the first theme of this teaching experiment, showed that increasing openness in mathematics teaching and learning brought a new kind of investigative and interactive atmosphere to the classroom. The pupils' discussions on infinity, for instance, were particulary rewarding. In visual arts lesson, the task engaged the pupils in talking about the semiotic features of art and the ways in which images and works of art refer to the prevailing culture. In both tasks it was possible to distinguish the same path of understanding: making observations, working independently, seeing in a new ways and understanding.

The second theme of the teaching experiment was `sorting and classifying as methods in visual arts´. The theme combined group work, methods of contemporary art, photography, experimentation, playful learning, composition skills, and use of image processing software. This work resulted in two concepts, equivalence and representation (equivalence relation). Representation is central to learning in visual arts, but also important in mathematics. Moreover, pupils gained experience of intuition directed work. Mathematics and art were again successfully combined. Figure 3 shows the various stages of the work.

In the mathematics lesson, the key concept was ratio. Some pupils were surprised to discover `how little the flower is compared to how much soil it needs´. While percentages, fractions and decimal numbers were already familiar to the students, several interesting discussions arose on measurement inaccuracy: `my precentafes don´t add up to 100%´. In this task, mathematical thinking was most clearly evident when pupils were sorting and classifying, even though the pupils did not themselves identify these as mathematics and more likely considerd measurements and calculating as real mathematics.

Figure 3: *Examples of pupils' works derived from art considering the second theme:'sorting and classifying as methods in visual arts'.*

The preservice mathematics teachers' suggestions for mathematical phenomena were summarized in categories, that proved to be almost equal in size. The first category `Internal phenomena of mathematics´ (28%) comprised mathematical entities like regularities, linearity, periodicity, convergence, infinity, patterns, logical reasoning, symmetry, problem solving etc. In the second category `Mathematical phenomena in nature´ (34%) student teachers mentioned mathematical events, observations, natural, chemical and physical phenomena, geometric phenomena, and examples such as the golden section, fractals, constellations, the annual- rings in trees, and snowflakes. In the third category `Mathematical phenomena in cultural settings´ (38%) mathematical phenomena like currency, finance, taxation, wages, music, art, probabilities, games, models, architecture, construction and building, cooking, travelling, etc. were mentioned.

Both the action research and the mathematics student teachers survey revealed that when integrating visual arts and mathematics abstract concepts and the links between them are key components of successful integration. Considering mathematics integrative learning continuums might include phenomena that are based on mathematical features like structures, regularities, dependencies, or relations. On the other hand, the dynamic phenomena and processes used when collecting and working with mathematical information are interesting. These include, e.g., logical reasoning, problem solving and proofs. Another interesting approach to construct integrative learning continuums an still simultaneously maintain the inherent working methods in mathematics is to explore phenomena occurring in organic nature or in cultural settings. These can most often be modelled and described using mathematical instruments. It should be noted that we can also generate mathematical phenomena, such as music, architecture and art, as well as manage social and economic phenomena with mathematical tools.

Discussion

When we are building interdisciplinary, integrative learning continuums in basic education (7^{th}-9^{th} grade), it is important to concider how this could be done so that the nature of each subject and the typical ways to build knowledge are taken equally into account. Our research indicateds, we discovered that the considered phenomenon must be sufficiently open; only then can collaborative learning bring together the whole class, pupils and the teacher, and motivate them all to learn. Such phenomena may be related to concepts that many disciplines have in common, such as representation or relationship. They may also be dynamic and related to the acquisition of more disciplinary knowledge or knowledge building processes, such as sorting and classification, or they may concentrate on the different natures of the disciplines beeing integrated. Our experiences have shown that it is essential to have conceptual thinking as a starting point when planning integrative instruction.

For most school subjects the integrative art education model suggested by Räsänen [4] could work as a starting point. The observations, emotions, forms representing each field, as well as cultural symbols can be a unifying factor in the learning continuums the teachers are constructing. [4]. In this, we do not forget the pupils' identity work, but we pause to reflect on what is there, in the pupils' lives or everyday events, that can be identified as mathematical order, or disorder, which the pupil could observe, describe and model with mathematical tools. Such documentation and collection of data is also involved in many of works of contemporary art and their creative processes. In this regard, our studies cast new light on questions such as: What kinds of phenomena are interesting and appropriate to curriculum content? How can we implement curriculum integration in practice? How can discipline-based integration be carried out so that each subject maintains its uniqueness? And, how can we support future teachers, both theoretically and practically, in these challenges?

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