Novel Textile Knot Designs through Mathematical Knot Diagrams

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Abstract

A mathematician and a textile practitioner both with an interest in textile knotting and a focus on research in textile design have collaborated to analyze textile practice and realize novel knot designs through mathematical characterization. The design process has made use of century-old drawing techniques used to represent knots and braids. This paper demonstrates how the manipulation of these diagrams was utilized as a design tool to generate and visualize novel patterns.

Introduction

A mathematical knot diagram is a representation of a knot using broken lines to indicate where the knot crosses itself. This century-old mathematical diagrammatic representation technique may be used to characterize a knot and to determine whether two knots are equivalent—a fundamental problem in knot theory [2]. Nimkulrat and Matthews have shown through collaborative work how a drawing technique that has been used to represent mathematical knots and braids in a diagrammatic form may be employed to characterize Nimkulrat's textile craft knots [3]. Although the technique does not aim to portray a physical knot, its visual qualities and underlying analytical processes are useful for describing and revealing the properties of textile craft knots. Table 1 shows a summary of properties.

Table 1: *Summary of differences between textile knot practice and mathematical knot diagrams.*

Property	Textile knot practice	Mathematical knot representation
Ends	May have loose ends.	Continuous curve with no loose ends.
Material	Form of knot is material dependent. The same knot tied with a flexible material will appear very different to that of a stiff material. Material dimensions are pertinent.	Not concerned with materiality. The cross-section of a strand is deemed to be a point.
Tension	Form of knot depends on tension and internal and external spaces are important.	Not concerned with tension. A tight knot is equivalent to a loose knot.
Form	The addition of extra loops changes the appearance of the knot.	If a knot can be simplified to the same representation of another knot, they are considered equivalent.
Color	The appearance of the knot is color dependent.	Not concerned with color.

In [4], further work shows how diagramming led to an idea for the use of a flexible material which gave rise to new textile structures. Given that the knots used in Nimkulrat's practice have loose ends, the representation of braids [1] was utilized to describe an intuitive textile knotting process, making the steps used to tie a knot explicit [4]. However, it was necessary to modify the braid notation to permit strands to temporarily run backwards [4]. The diagrammatic characterizations of textile knots in Figure 1a reveal a property of the craft knots that is not obvious from the knotted work. For both a single knot (Figure 1b) and a group of knots (Figure 1c), the positions of all strands start and finish in the same position; strand *a* starts and ends in position 1, strand *b* remains in position 2, *c* remains in position 3, and *d* starts and ends in position *4*. The coloring of the diagrams makes this property explicit and offers the potential for designing knot patterns in advance of physical knotting. This paper describes how the further use of mathematical knot diagrams has resulted in two novel textile knot patterns.

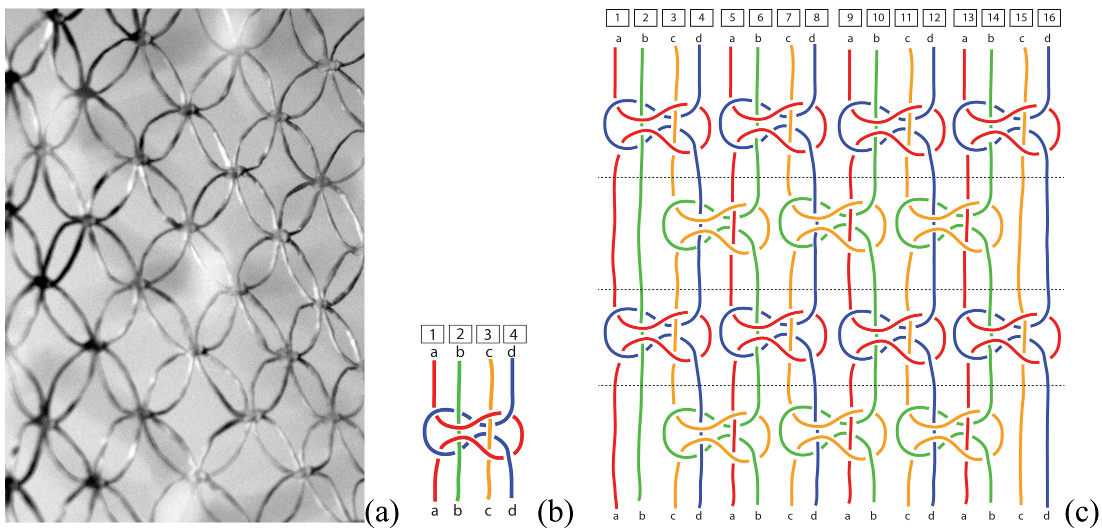


Figure 1: *Multiple textile knots (a), diagram of a single craft knot (b) and diagram of four rows of knots (c), both showing the position of strands.*

An Examination of Knot Diagrams to Further Understand Textile Knots

On examining Figure 1c, it becomes apparent that in addition to start/end positions, there are active and passive strands. The outer strands *a* (red) and *d* (blue) in the first row of knots are active in knotting, while inner strands *b* (green) and *c* (yellow) are passive. On the second row, this is reversed. The third row uses the same strands as the knots on the first row and the fourth the same as the second. From the diagram, it is possible to determine both characteristics—start/end and active/passive. This is undetectable through observation of the physical work alone (Figure 1a). As textile practitioners, both authors recognized design repeats in the diagram. Groups of four strands create knots along a row, and every alternate row employs the same strands for tying. Whilst this represents a two-dimensional mathematical tiling with a clearly defined unit cell (Figure 1b), aspects of tiling were not explored and will form the direction of future work. Both the recognition of the repeat pattern and the active/passive nature of strands became the focus for next steps. Two aspects were investigated: (1) the use of color (to date Nimkulrat had only employed mono-color materials) and (2) changing active and passive strands.

In investigating the use of color to generate an obvious repeating knot pattern, the number of colors was reduced from four to two. Two neutral colors, black and gray, were selected to recolor Figure 1c—*gray* replaced *red* and *blue*, *black* replaced *green* and *yellow*. Recoloring the diagram in several ways revealed that a variety of knot patterns could be attained. Figure 2a visualizes a two-tone textile knot pattern. It is interesting to note that the use of more than one color was an aspect Nimkulrat wanted to explore but had not attempted. Even for a skilled practitioner, determining which color to use, and where, seemed too complicated. The colored diagram has therefore provided an easy-to-use textile design tool.

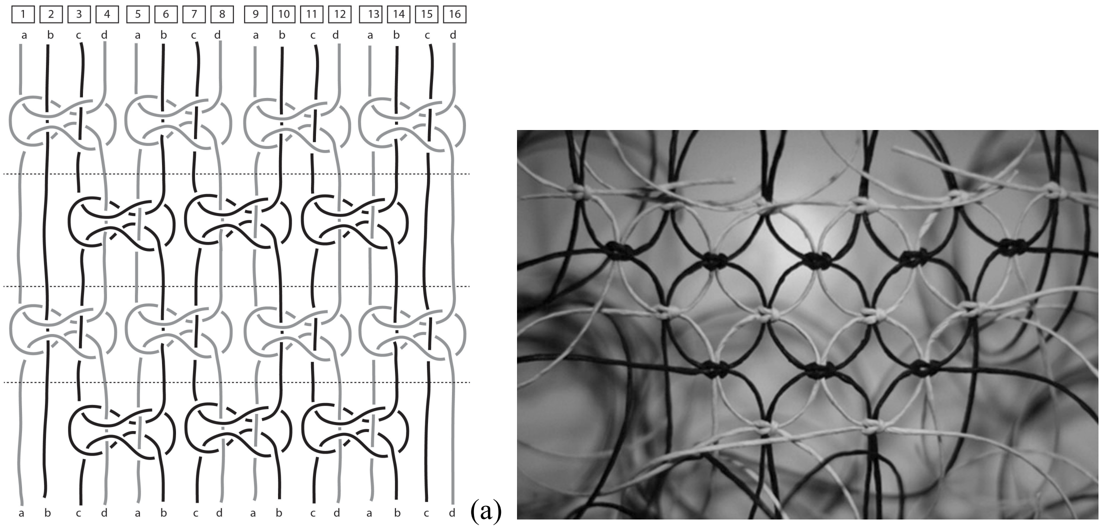


Figure 2: *Diagram of four rows of knots using two colors suggesting a possibility of a two-tone knot pattern (a), two-tone knotted textile based on the diagram in Figure 2a (b).*

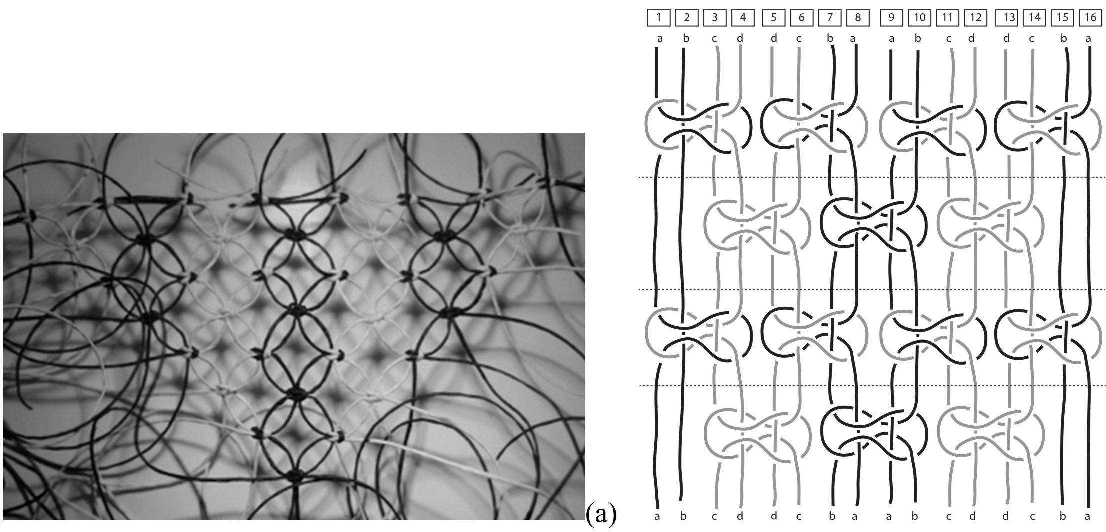
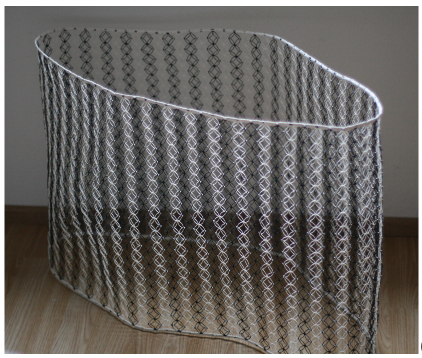


Figure 3: *Two-tone knotted textile in vertical striped knot pattern (a), diagram showing four rows of knots in Figure 3a (b).*

Novel Textile Knot Patterns

Further observation of Figure 2a revealed that (1) a black knot is created when the active strands are black and a gray knot emerges when the active strands are gray and (2) the passive strands that adjoin the knots create single-color rounded rectangles—not a combination of black and gray. These characteristics suggested a knot pattern design possibility—colored shapes could be formed from solid color knots. To confirm this possibility, Nimkulrat knotted Figure 2a utilizing black and white paper string as a material. The physical knotting (Figure 2b) verified that strands alternately played an active role in tying single colored knots—white knots (active white strands) and black knots (active black strands). Furthermore, after four rows, black and white circles became evident. The circle shape is determined by the stiffness of the material. The color of the circle is defined by the active strands of the knots on the top and bottom of circle. The color of the internal scalloped diamond shape is determined by the passive strands of these knots and the active strands of the knots on the left and right of each circle. For example, a white circle has an internal black shape.

The knotting experimentation continued with repositioning black/white strands: Instead of 2black-2white-2black-2white…. an alternative 2black-4white-2black-4white…. was used. Both black and white strings were simultaneously active in tying the knot in one row while in the next row, four strings of the same color were active, forming pure color knots. The iterative knotting process resulted in a vertical striped knot pattern (Figure 3a). This was then analyzed by Matthews to determine whether the pattern could have been foreseen from a colored diagram. Figure 3b confirmed this. Nimkulrat continued to use the striped knot pattern to create larger three-dimensional textile artifacts (Figures 4a and 4b). In making Figure 4b, groups of three strands, instead of four, were used to generate the right angles; analysis of this will also form part of the future work.


(a)

(b)

Figures 4a & 4b: *Three-dimensional knotted textiles utilizing the striped pattern (a) a cylinder, and (b) a chair (62cm x 50cm x 63cm).*

Conclusion

This paper has demonstrated how the diagrammatic method commonly used in mathematical knot and braid theory can be adopted in a craft/textile knot practice as a design tool. Two novel two-tone patterns for textile knotting have emerged—(1) intertwining colored circles and (2) stripes. Through the use of color in the diagram, the positions and roles of strands in a knotted structure become explicit. This in turn is able to provoke an exploration of knotted pattern designs that may not have occurred otherwise. Further work will examine groups of three and five threads and the use of tiling patterns to further generate novel designs.

References

[1] J. J. van Wijk & A. M. Cohen. *Visualization of the Genus of Knots*. in C. Silva, E. Gröller, & H. Rushmeier (eds.) Proceedings IEEE Visualization 2005, IEEE CS Press.

[2] K. Delvin, *The Language of Mathematics: Making the Invisible Visible*, W.H. Freeman, 1998.

[3] N. Nimkulrat & J. Matthews, *Mathematical Textiles: The Use of Knot Theory to Inform the Design of Knotted Textiles*, in Proceedings of the 5th International Congress of International Societies of Design Research (IASDR): Consilience and Innovation in Design, Shibaura Institute of Technology, 2013. <https://dspace.lboro.ac.uk/2134/12840>

[4] N. Nimkulrat & J. Matthews, *From Mathematical Diagrams to Knotted Textiles*, in G. Greenfield, G. W. Hart, & R. Sarhangi (eds.) Proceedings of Bridges Seoul 2014: Mathematics, Music, Art, Architecture, Culture, Tessellations, pp. 219-224, 2014.