Prelude Op. 11, No. 1 by A. Scriabin: ICVSIM Relations

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Abstract

ICVSIM (Interval Class Vector Similarity) is a function, developed by Eric Isaacson, that allows us to measure and compare the musical interval relations of vectors ICV of two sets of any cardinality. The function measures an average difference between two sets. The octatonic collection and the whole tone collection produce the most distant ICVSIM value of 3.58. Scriabin's Prelude Op. 11, No. 1 contains a total of 89 sets of quintuplets and triplets. Out of these 89 sets, there are a total of 20 unique sets with different ICV values. An ICVSIM analysis indicates that this prelude contains sets of high similarity, as 68.4% of ICVSIM values are equal to or less than 1.00 and the average ICVSIM value, utilized in this prelude, is 0.93.

Introduction

ICVSIM (Interval Class Vector Similarity) is a concept that allows to compare interval vectors of two sets of any cardinality [1]. An interval vector is a string of six digits, representing the amounts of all possible intervals that a set of notes contains. The first number represents the amount of minor seconds and major sevenths, the second number represents the amount of major seconds and minor sevenths, the third number – minor thirds and major sixths, the fourth number – major thirds and minor sixths, the fifth number – perfect fourths and perfect fifths, while the final number represents the amount of tritones. ICVSIM values range from 0.00 to 3.58. The closer the value approaches to 0, the more related the sets are. ICVSIM of 0.00 means that both sets contain identical intervallic identity. The ICVSIM of 3.58 is a relation between the whole tone collection (6–35) with ICV of 060603 and the octatonic collection (8–28) with ICV of 448444 [2]. This is the most unrelated set and highest possible ICVSIM value, since these sets will combine for the highest average intervallic difference of vectors. While an alteration in variable *n* would change the ICVSIM value, it is important to remember that every interval vector in music consists of only six digits and therefore, the variable *n* must always equal to 6. There are two steps taken in order to calculate ICVSIM. First, an IDV (Intervallic Difference of Vectors) of the two sets is measured. If Set A = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and Set B = $\{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$, then the IDV of Sets A and B is equal to the difference of subsets of each vector. Therefore, Set B – Set A = $\{x_7\text{-}x_1, x_8\text{-}x_2, \ldots, x_{12}\text{-}x_6\}$. Second, a standard population deviation formula is applied to the IDV. Let IDV = $\{y_1, y_2, y_3, y_4, y_5, y_6\}$, where n = 6, the number of digits of the interval vector. Solve for S_n.

$$s_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}$$

Analysis

ICVSIM is not a tool that can be used for comparison of musicality of phrases or for theoretical analysis of musical structures in relation to a certain key or tonal centricity, or in other words, an emphasis on particular pitch or pitch class. In fact, ICVSIM is used to compare the interval vectors that are found in atonal music, the music that does not allow one to categorize musical structures based on tonal centricity. In pitch class theory, interval similarity refers to “special inclusion-based relationships” and such relationships are not always commensurate with what actual music offers in the score [3]. ICVSIM relation was used in order to compare the sets that are found in Alexander Scriabin's Prelude Op. 11, No. 1, a piano work, composed in Moscow in November of 1893. Scriabin's compositions can be divided into three musical periods, based on his compositional approach, and this particular work belongs to Scriabin's early period, where the composer is influenced by other Romantics, primarily by Frederick Chopin and Franz Liszt. Even though this prelude is an early and a non-atonal work by Scriabin, ICVSIM reveals essential information about the make up of intervals inside the sets that are created from construction of various harmonies and how these intervals cooperate in relation to each other. Additionally, The ICVSIM relations in this work will allow one to understand the intervallic behavior of tonal structures and the reasons behind certain harmonic progressions. The score of the first prelude is shown in Figure 1 [4].

Figure 1: *Scriabin, Prelude Op. 11, No. 1 in C Major*

Prelude No. 1 is built solely on sets of either five quintuplets (a group of five notes) or three triplets (a group of three notes). The only exception exists in measure 25 of the prelude, the last measure of the work, where Scriabin harmonizes a C major tonality by incorporating a C octave and a C major chord in first inversion. Let us combine such sets of quintuplets or triples into unique elements (E_X), each of which will contain a separate ICV vector, defined by x = {1,2, …, 20}. For instance, E_1 encompasses the first five notes in the right hand. In this example, a set of pitches [CDGA] has an ICV of 021030.

By undertaking such analysis, we are leaving out the final measure of the work, since it is neither a quintuplet or a triplet. There are 48 elements in the right hand and 41 elements in the left hand – a total of 89 elements in this composition. There are, however, only a total of 20 unique ICV vectors because certain elements contain identical ICV vectors, as many of the elements are either reinstated multiple times throughout the piece, or are shown in one or more forms. There are four elements in the first phrase of this prelude, bounded by a slur in measures 1-2, which incorporate two different sets of pitches: [CDGA] and [DEGA]. Both sets contain an ICV of 021030, since both sets of pitches belong to the same class (4–23). Every E_X with its assigned ICV can be seen in Figure 2.

e(M)	ic vector	e(M)	ic vector	e(M)	ic vector
e1(M)	021030	e8(M)	010020	e15(M)	000010
e2(M)	100110	e9(M)	222121	e16(M)	012120
e3(M)	132130	e10(M)	231211	e17(M)	122131
e4(M)	121110	e11(M)	010101	e18(M)	004002
e5(M)	131221	e12(M)	011010	e19(M)	012111
e6(M)	001110	e13(M)	020100	e20(M)	020202
e7(M)	100011	e14(M)	010000		

Figure 2: *Set names along with their respective vectors.*

We are able to find IDV of every two sets and to derive a chart with all possible ICVSIM values in this piece of music. It is shown Figure 3. All values are rounded to the nearest hundredth.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	
E1	0	1.26	0.47	1	0.94	1.12	1.26	0.5	0.94	1.37	1.38	0.76	1.26	1.07	0.9	0.82	0.47	2.08	1.15	1.73	E1
E2	1.26	0	1.21	0.76	0.9	0.58	0.58	0.82	0.69	0.9	0.82	0.82	1	0.75	0.47	0.96	0.9	1.89	0.96	1.26	E2
E3	0.47	1.21	0	0.75	0.82	1.07	1.34	0.69	0.82	1.16	1.34	0.69	1.07	0.96	0.96	0.75	0.58	2.06	1.11	1.7	E3
E4	1	0.76	0.75	0	0.47	0.76	0.96	0.76	0.47	0.47	0.76	0.5	0.5	0.37	0.69	0.82	0.75	1.83	0.82	1.16	E4
E5	0.94	0.9	0.82	0.47	0	0.9	1.07	0.69	0.82	0.58	0.69	0.69	0.37	0.5	0.76	0.94	0.82	2.06	0.94	0.94	E5
E6	1.12	0.58	1.07	0.76	0.9	0	0.82	0.82	0.69	1.07	0.82	0.58	1	0.75	0.47	0.5	0.69	1.5	0.5	1.26	E6
E7	1.26	0.58	1.34	0.96	1.07	0.82	0	0.82	0.69	1.07	0.82	0.82	1.16	0.75	0.47	1.12	0.9	1.71	0.96	1.26	E7
E8	0.5	0.82	0.69	0.76	0.69	0.82	0.82	0	0.69	1.07	1	0.58	1	0.75	0.47	0.76	0.37	1.98	0.96	1.38	E8
E9	0.94	0.69	0.82	0.47	0.82	0.69	0.69	0.69	0	0.82	0.9	0.37	0.9	0.5	0.5	0.75	0.58	1.6	0.75	1.37	E9
E10	1.37	0.9	1.16	0.47	0.58	1.07	1.07	1.07	0.82	0	0.69	0.9	0.37	0.5	0.96	1.25	1.16	2.06	1.11	0.94	E10
E11	1.38	0.82	1.34	0.76	0.69	0.82	0.82	1	0.9	0.69	0	0.82	0.58	0.47	0.75	1.12	1.07	1.71	0.76	0.5	E11
E12	0.76	0.82	0.69	0.5	0.69	0.58	0.82	0.58	0.37	0.9	0.82	0	0.82	0.47	0.47	0.5	0.37	1.5	0.5	1.26	E12
E13	1.26	1	1.07	0.5	0.37	1	1.16	1	0.9	0.37	0.58	0.82	0	0.47	0.94	1.12	1.07	1.98	0.96	0.76	E13
E14	1.07	0.75	0.96	0.37	0.5	0.75	0.75	0.75	0.5	0.5	0.47	0.47	0.47	0	0.58	0.9	0.76	1.68	0.69	0.9	E14
E15	0.9	0.47	0.96	0.69	0.76	0.47	0.47	0.47	0.5	0.96	0.75	0.47	0.94	0.58	0	0.69	0.5	1.68	0.69	1.21	E15
E16	0.82	0.96	0.75	0.82	0.94	0.5	1.12	0.76	0.75	1.25	1.12	0.5	1.12	0.9	0.69	0	0.47	1.53	0.58	1.53	E16
E17	0.47	0.9	0.58	0.75	0.82	0.69	0.9	0.37	0.58	1.16	1.07	0.37	1.07	0.76	0.5	0.47	0	1.7	0.75	1.49	E17
E18	2.08	1.89	2.05	1.83	2.06	1.5	1.71	1.98	1.6	2.06	1.71	1.5	1.98	1.68	1.68	1.53	1.7	0	1.16	2	E18
E19	1.16	0.96	1.11	0.82	0.94	0.5	0.96	0.96	0.75	1.11	0.76	0.5	0.96	0.69	0.69	0.58	0.75	1.16	0	1.16	E19
E20	1.73	1.26	1.7	1.16	0.94	1.26	1.26	1.38	1.37	0.94	0.5	1.26	0.76	0.9	1.21	1.53	1.49	2	1.16	0	E20
	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	

Figure 3: The *ICVSIM chart for Prelude Op. 11, No. 1.*

Out of 190 ICVSIM values, assuming that identical elements are not included, 130 are equal or are less than 1.00, which suggests that 68.4% of the sets in this prelude are highly related. The average ICVSIM value in this work is 0.93. Furthermore, E_{12} is the most intervallically related set in comparison to every other set with an average ICVSIM of 0.71. E_{12} contains three intervals: one major second (two semitones), one minor third (three semitones), and one perfect fourth (five semitones). E_{18} is the most intervallically distant set in comparison to every other set in the piece. This set contains six intervals inside of its vector, which include three minor thirds (three semitones), one major sixths (nine semitones), and two tritones (six semitones). This is a fully diminished seventh chord, symmetrical in many aspects of its intervallic notation, and the existence of only two groups of contrasting intervals out of six possible in the vector is the reason why this particular set is distant.

The symmetry of this work is seen in frequent utilization of quintuplets and triplets in the prelude; that is what allows us to carry out such ICVSIM analysis and compare each element individually to every other element in the work. This is not the case for many musical compositions, since deviations in tempo, note durations, form, and musical structure are more likely to overcome the composer's desire to write music by following a certain pattern. The intervallic relativity is a concept that provides us with a numeric measurement of intervallic breakdown of every set and how these intervals are related. There exists no general range for comparison in ICVSIM relations between tonal and non-tonal music, just as there exists no general range for comparison in ICVSIM relations for classical music of different stylistic periods. However, such analysis can serve as a comparative tool for analyzing both tonal and non tonal music, as well as Western classical music of different genres. The ICVSIM approach is a tool that allows us to define a specific value from the perspective of an interval vector for two sets of notes of any cardinality. While ICVSIM analysis does not provide any explanations for chordal structures or for existence of certain musical progressions in the piece, this technique, nevertheless, allows us to understand the intervallic relations between various structures and elements and may serve as another tool in analytic understanding of music theory.

References

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