A Graph-Theoretic Approach to the Analysis of Contra Dances

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Abstract

Contra dance is an American folk dance comprised of combinations dance moves announced by a caller, similar to those of square dance. Because of their structured nature, we propose a new approach that considers these dances as paths through a directed multigraph. In this paper, we describe the motivation and construction of this extensible approach.

Background

Contra Dance. With the founding of the American colonies in the seventeenth century, European dances also immigrated over to the New World, where some folk dances underwent tremendous change within the melting pot of American society. The contra dance, derived from the English long-ways country dance, is one of these dances. With influences from the French, Irish, Eastern Europeans, and French Canadians, contra dancing evolved into a uniquely American folk dance [2, p. 14–16]. Incubated in New England, contra dance enjoyed a revival in 1970s as it spread throughout the country [2, p. 21]. Contra dancing is now found across the United States and in some Western European countries.

Unlike folk dances that serve to preserve a tradition, contra dance is a living tradition where dancers are still composing new dances. Contra dances use a set of stock dance figures that are combined according to informal syntactical and stylistic norms. This set of figures has grown over the years, borrowing figures from contemporary English Country Dance and Modern Western Square Dance [2, p. 66]. A caller cues each figure in the dance, which can be referred to as either a *call* or a *figure*.

Another difference between contra dancing and other types of country dancing is that a dance is not composed for a particular piece of music. Instead, a contra dance follows a standardized 64-count structure, organized into four 16-count sections, which then divides into eight 8-count phrases. Figures do not cross sectional boundaries, although they can on occasion cross internal phrase boundaries. This guideline allows the dance to be accompanied by any composition that has this same musical structure. The most common types of pieces that accompany a contra dance are old-time fiddle tunes, Irish jigs and reels, and New England contra dance tunes.

In a typical contra dance, dancers line up into two long lines down the hall, called a *set*. Women line up on one side of the set and men are on the other side, with partners facing across the set. In contra dance “men” and “women” refer to the dancers’ role and does not necessarily correspond with dancers’ gender. Starting from the top of the line, pairs of partners join hands in groups of four to form a *minor set*. The other couple in the minor set are *neighbors*. *Proper* dances maintain men and women on opposite sides of the set, but in *improper* dances, the couple closest to the top of the set (caller and band) switch places. These dancers are called the 1s, while the other couple are called the 2s. This nomenclature designates the direction of travel for each partnered pair: after the four dancers perform the 64-count dance, the 1s progress further down the line to find new 2s, while the original 2s progress up the line to find new 1s, as seen in Figure 1. Then the dance repeats. For a dance to work, partners must continually progress in the correct direction with the same partner. When a couple reaches the end of the line, they change their identity (1s become 2s and vice versa) and reenter the dance going in the opposite direction. The caller determines the number of repetitions of the 64-count dance. Video examples of these moves can be seen online at `https://cloud.sagemath.com/projects/f5131d05-a281-4a3b-b456-e738f4182eb1/files/shared/`.

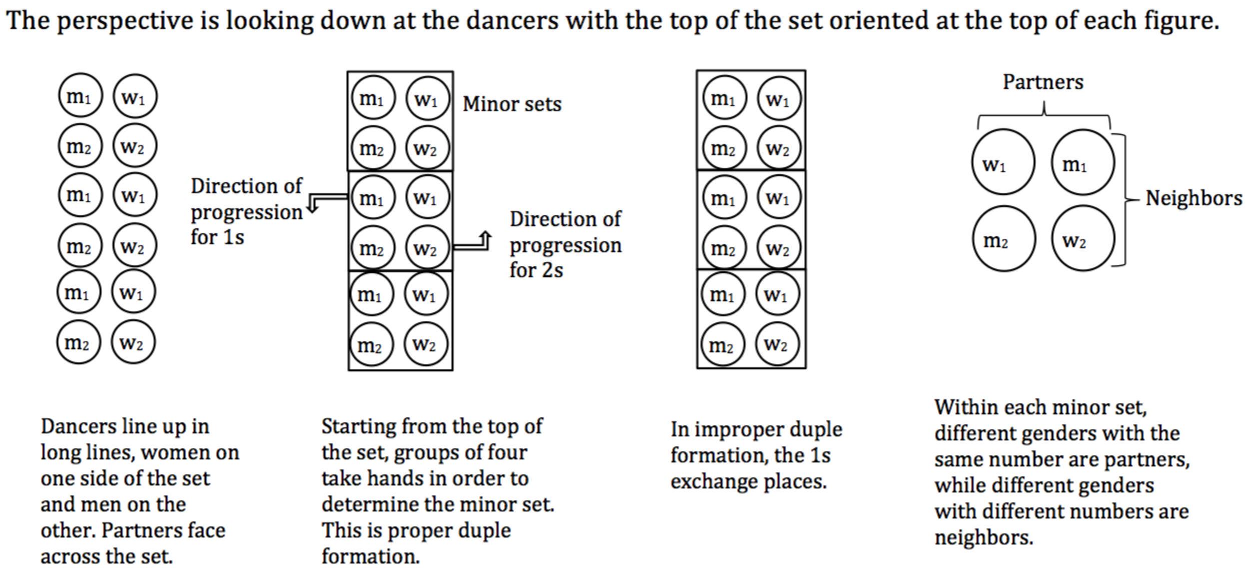


Figure 1 : *Illustration of progression of dancers*

Mathematical Approaches to the Analysis of Dance. A variety of dance has been explored from a mathematical perspective, yet there remains much room for mathematical analysis. For instance, von Renesse and Ecke [7] explore arm configurations and motions in salsa dancing, while Rogers and Buchler [6] discuss square dances using cycle notation and modular arithmetic. Contra dance has been explored mathematically by considering cycles and permutations [1] and using the orbits of couples [4]. Motivating our work, Mui proposed both algebraic and graph theoretic approaches to the analysis of contra dances [5]. Our goal in this project is to create a mathematical representation of contra dance that simultaneously represents all possible contra dances and provides a language for further study.

Methods and Results

A Linear Algebra Approach. We begin by considering the initial configuration of dancers:

$$\begin{bmatrix} w2 & m2 \\ m1 & w1 \end{bmatrix}.$$

Calls are represented by 2x2 matrices that are multiplied by the matrix of positions, leading to the new positions. We acknowledge that calls would not be uniquely determined by these matrices. For example, a circle left four places would return dancers to their original positions, as would a neighbor-allemande right once around. Thus, both of these calls would be represented by the identity matrix, but would be considered different calls and thus would represent different dances. We ignore this issue for the moment and try to determine an example using only left multiplication for the circle left one place. If we were to represent this call by matrix multiplication, we would need to determine a solution to:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w2 & m2 \\ m1 & w1 \end{bmatrix} = \begin{bmatrix} m1 & w2 \\ w1 & m2 \end{bmatrix}$$

which has no solution. If we expand operations to include either left or right multiplication, we still find no solution to represent circling to the left by one position. By expanding beyond simple matrix multiplication, we could consider other linear algebraic solutions. For example, a circle left could be accomplished with the function $cl(x) = x^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

We can see that representing calls by linear algebra operations becomes cumbersome, suggesting that another approach to representing calls may be more promising. This, coupled with the challenges of having multiple calls represented by the same function, leads us to believe that a more productive approach will be the use of graph theory.

A Graph Theoretic Approach.

In Mui's [5] graph theoretic approach, illustrated in Figure 2, the dance Delphiniums and Daisies is represented by a graph called a preferred transition graph. In this type of graph, successive calls are given by directed edges, with preferred options in bold.

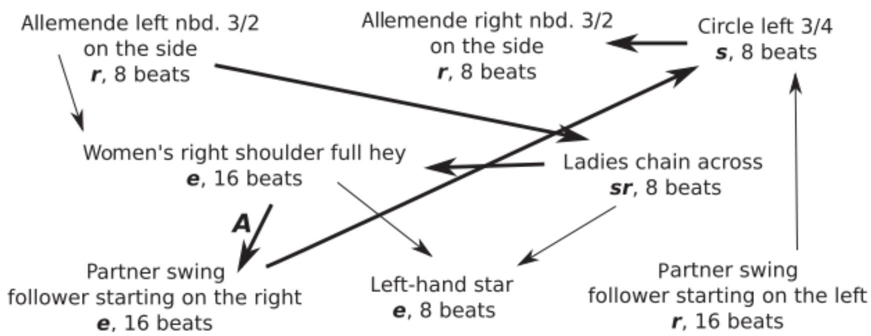


Figure 6. A sample preferred transition graph with the dance Delphiniums and Daisies highlighted in bold.

Figure 2: *Figure reproduced from [5]*

Our approach generalizes this idea by creating a single graph in which any contra dance is represented by a path.

We consider each of the possible eight arrangements of 4 dancers as a vertex in a directed graph, in which edges represent calls which point from an initial setup to a final setup. We define:

$$\text{position } 1 = \begin{bmatrix} w2 & m2 \\ m1 & w1 \end{bmatrix}, \text{position } 2 = \begin{bmatrix} m1 & w2 \\ w1 & m2 \end{bmatrix}, \text{position } 3 = \begin{bmatrix} w1 & m1 \\ m2 & w2 \end{bmatrix}, \text{position } 4 = \begin{bmatrix} m2 & w1 \\ w2 & m1 \end{bmatrix},$$

$$\text{position } 5 = \begin{bmatrix} m1 & w1 \\ w2 & m2 \end{bmatrix}, \text{position } 6 = \begin{bmatrix} w2 & m1 \\ m2 & w1 \end{bmatrix}, \text{position } 7 = \begin{bmatrix} m2 & w2 \\ w1 & m1 \end{bmatrix}, \text{ and position } 8 = \begin{bmatrix} w1 & m2 \\ m1 & w2 \end{bmatrix}.$$

We then consider a dance to consist of a set of eight calls with the restriction that the first call must be called from position 1 and the final call must end in position 5. Position 1 represents improper duple formation, the starting position at the beginning of a dance. While other starting are possible, for this project we are only examining the improper duple formation. In position 5, each couple has progressed: the 1s have moved below the 2s and they are ready to dance with new neighbors from a position 1 configuration.

The edges of the graph represent figures in the dance. For example, an edge labeled "CL3" (circle left 3 places) directs position 1 to position 4, position 2 to position 1, position 3 to position 2, position 5 to position 8, position 6 to position 5, position 7 to position 6, and position 8 to position 7. See Figure 3 for the list of calls that can occur from any starting position included in this study. We break the 64-count dance into

8-count figures, aligning with the overall structure of the dance. While there are figures that are 4, 12, and 16 counts, we combine shorter figures together or divide longer figures into two parts to create an 8-count figure. For example, a full hey would be represented by two consecutive 8-count half-a-hey figures.

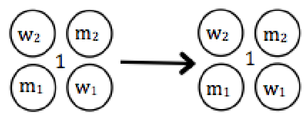
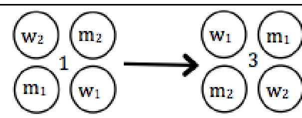
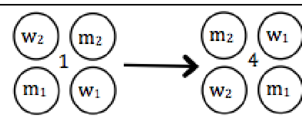
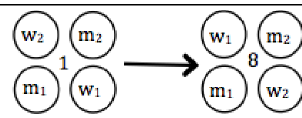
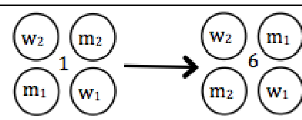
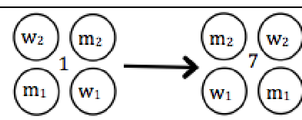
Resulting Transformation	Participants in the figure	Change from Position 1	Figures
Identity	Everyone		Circle Left Four Spaces Long Lines
Identity	Men		Men Do-si-do 1 Men Gypsy 1 Men Allemande Left 1 Men Allemande Right 1
Identity	Women		Women Do-si-do 1 Women Gypsy 1 Women Allemande Left 1 Women Allemande Right 1
Rotate clockwise two places	Everyone		Half a Hey
Rotate clockwise three places	Everyone		Circle Left Three Spaces Balance the Ring and Move one Space to the Right
Same gender exchange places	Women		Women Do-si-do 1.5 Women Allemande Left 1.5 Women Allemande Right 1.5
Same gender exchange places	Men		Men Do-si-do 1.5 Men Allemande Left 1.5 Men Allemande Right 1.5
Couples facing across set switch places	Everyone		Balance the Ring + California Twirl.

Figure 3 : *These figures can be done from any of the eight possible configurations*

In practice, most figures occur along the side of the set in order to accommodate the physical space needed to dance. When a figure takes place in the center of the set, usually only two dancers participate in the figure. For this study, we only include symmetrical figures that follow this space limitation, which accounts for most of the position restrictions placed on figures. Additionally, other moves have restrictions on their starting positions. Two figures that fall into this category are the ladies chain and the right and left through. See Figure 4 for a list of calls included in this project that have only four available starting positions.

We construct a directed multigraph with edges defined as above, and vertices defined by the eight positions, one for each eight-count phrase. The graph must be a multigraph, meaning that a pair of vertices may have more than one edge connecting them, as distinct calls may have the same beginning and ending states. The graph must be directed, meaning that each edge has a designated beginning and end, as the calls move a set of dancers from a beginning position to an ending position. In order to assure a progressed dance, the final vertex must be in position 5, designated "end." The means each couple will be ready to dance with a new minor set at the conclusion of the eight-phrase dance. We include a vertex labeled "start," identical to position 1, which precedes the first figure. A dance is then given by a path beginning at vertex "start" and ending at vertex "end."

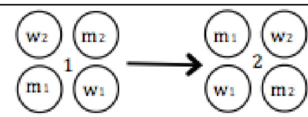
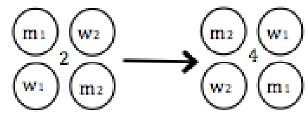
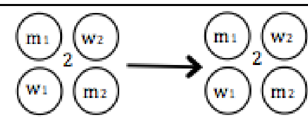
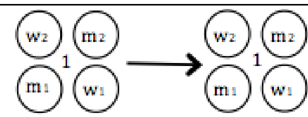
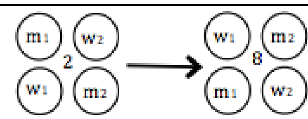
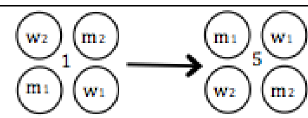
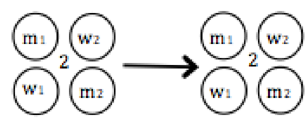
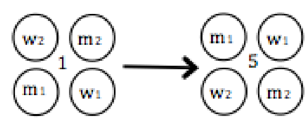
Resulting Transformation	Conditions	Change from Position 1 or 2	Figures
Rotate clockwise one place	Women on the right diagonal		Ladies Chain
Same genders exchange places	Women on the right side of the men along the side of the minor set		Right and Left Through
Identity	Partners on the side of the set		Partners Do-si-do 1 Partners Gypsy 1 Partners Allemande Left 1 Partners Allemande Right 1
Identity	Neighbors on the side of the set		Neighbors Do-si-do 1 Neighbors Gypsy 1 Neighbors Allemande Left 1 Neighbors Allemande Right 1
Couples facing up and down the set switch places with each other	Partners on the side of the set		Partner Do-si-do 1.5 Partner Allemande Left 1.5 Partner Allemande Right 1.5
Couples facing up and down the set switch places with each other	Neighbors on the side of the set		Neighbors Do-si-do 1.5 Neighbors Allemande Left 1.5 Neighbors Allemande Right 1.5
Couples along the side of the set end with the woman on the right side of the man	Partners on the side of the set		Partner Swing
Couples along the side of the set end with the woman on the right side of the man	Neighbors on the side of the set		Neighbor Swing

Figure 4 : *Figures that can be done from only half the possible dancer configurations*

We illustrate this construction using a very simplified version of contra dancing, and provide links to full graphs including all positions, figures, and the full dance length, which would be too large to reproduce here (and is very large and hence difficult to read). In our example, we will only consider a total of four figures (men's do-si-do once, half a hey, right and left through, and balance the ring with a California twirl) and a dance with 24 counts (three figures). We will also restrict ourselves to only four of the eight positions (1, 3, 5, and 7). This graph is printed as Figure 5, where red edges indicate a men's Do-Si-Do once and are printed in dashed lines, teal edges represent a Half-Hey and are printed in a dash-dot, green edges represent Balancing the Ring with a California twirl and are printed in dotted lines, and purple edges represent Rights and Lefts, and are printed in solid lines.

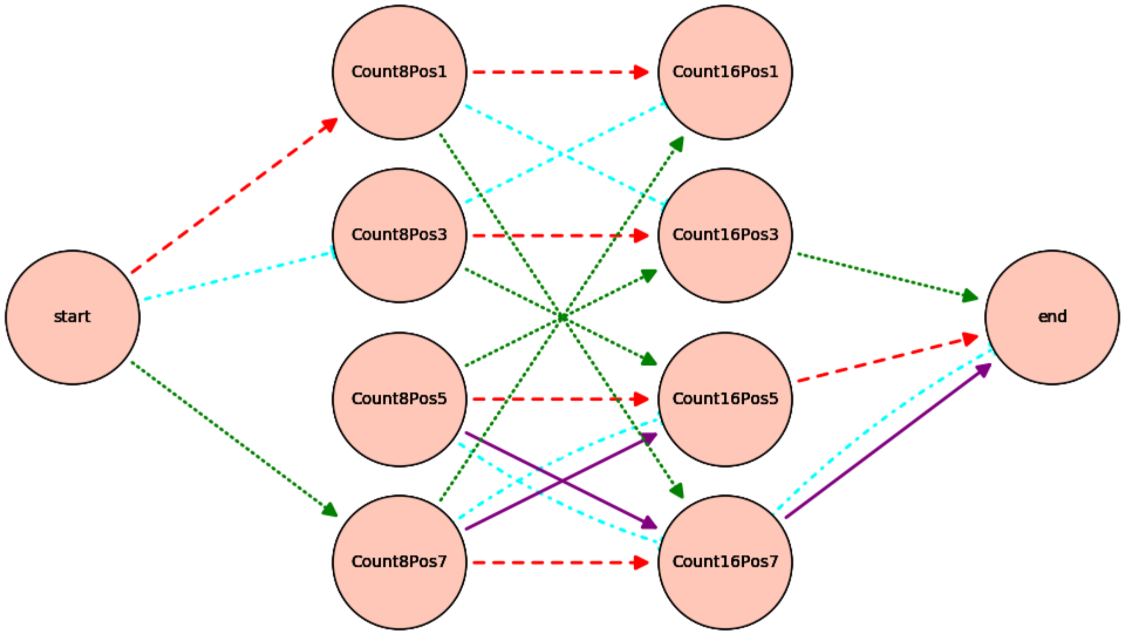


Figure 5 : *Example contra dance graph*

Every possible dance using this restricted setting is represented by a path beginning at the "start" vertex and ending at the "end" vertex. We use the Sage language [3] to count these dances[1], indicating that there are 9 possible dances. The software prints these dances:

```
((start, _8Pos1, 'MDsD1'), (_8Pos1, _16Pos7, 'BRCT'), (_16Pos7, end, 'HH'))
((start, _8Pos1, 'MDsD1'), (_8Pos1, _16Pos7, 'BRCT'), (_16Pos7, end, 'RsLs'))
((start, _8Pos1, 'MDsD1'), (_8Pos1, _16Pos3, 'HH'), (_16Pos3, end, 'BRCT'))
((start, _8Pos3, 'HH'), (_8Pos3, _16Pos5, 'BRCT'), (_16Pos5, end, 'MDsD1'))
((start, _8Pos3, 'HH'), (_8Pos3, _16Pos3, 'MDsD1'), (_16Pos3, end, 'BRCT'))
((start, _8Pos7, 'BRCT'), (_8Pos7, _16Pos5, 'HH'), (_16Pos5, end, 'MDsD1'))
((start, _8Pos7, 'BRCT'), (_8Pos7, _16Pos5, 'RsLs'), (_16Pos5, end, 'MDsD1'))
((start, _8Pos7, 'BRCT'), (_8Pos7, _16Pos7, 'MDsD1'), (_16Pos7, end, 'HH'))
((start, _8Pos7, 'BRCT'), (_8Pos7, _16Pos7, 'MDsD1'), (_16Pos7, end, 'RsLs'))
```

This output provides the ordered list of figures for all dances given these restrictions, as well as each minor set's positions at the end of each figure. For example, the first dance listed would be called as a men's do-si-do once, then a balance the ring with a California twirl, and finally a half-a-hey. This dance would have couples begin in position 1, then staying in position 1 after the do-si-do, to position 7 after the balance with California twirl, then ending in position 5 ("end") after the half-a-hey.

This method can create an easily searchable list of output ready for further analysis. The full code for creating this graph and for calculating the number of dances as well as the full graph of all positions and figures is found at `https://cloud.sagemath.com/projects/f5131d05-a281-4a3b-b456-e738f4182eb1/files/shared/`.

One could alternatively approach the graph theoretic interpretation of contra dances by considering a single vertex for each position and edges joining positions in the same way that we defined. In Figure 6 we have recreated Figure 5 using this methodology while keeping the colors the same, where the half-hey is

[1] We wish to thank the ask.sagemath.org community for assistance with the Sage code.

given by a long dash, Balance the ring with a California twirl is a short dash, men's do-si-do is a dotted line, and Rights and Lefts is a solid line. Notice that we only have four vertices as we are only including 1, 3, 5, and 7. Paths would be required to begin with position 1 and end with position 5, and have a length of 3. While the graph is significantly smaller than in Figure 5, the dances are more difficult to visualize. Thus, we consider using distinct positions at different times (at the end of each phrase) to be a more useful way of analyzing contra dances.

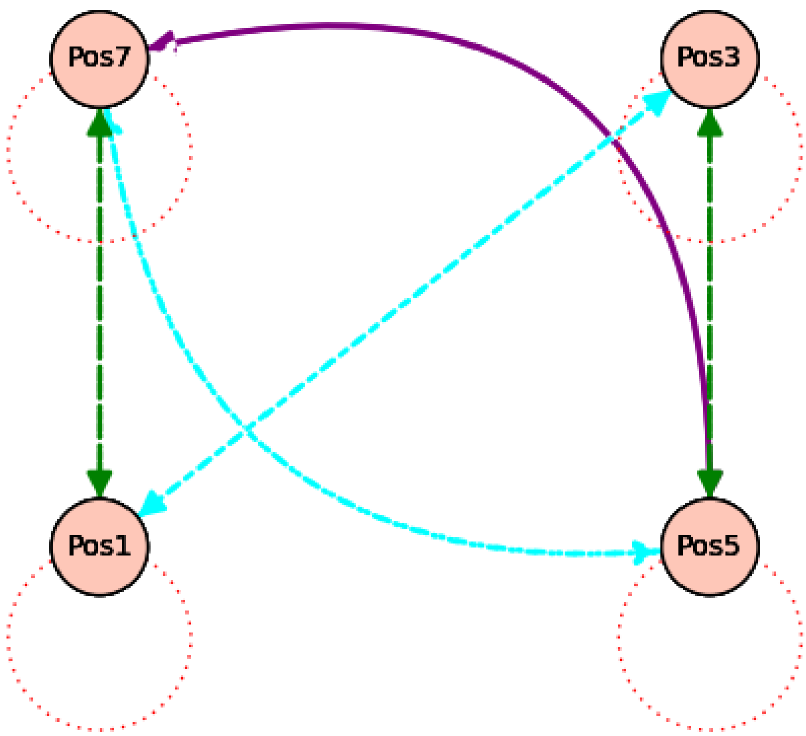


Figure 6 : *Example alternative contra dance graph*

Conclusions

We propose a new method for the illustration and future analysis of contra dances inspired by Mui's approach [5]. Our method for the representation of contra dances as paths within a large directed multigraph may be more versatile than previous graphical approaches, for it allows for easy comparison of different dances, easy construction of new, though possibly very bad, dances, as well as ready visualization of dances.

Future Directions: Next "Steps"

The only rule for dance inclusion in the multigraph was that partners must progress at the end of the eight-phrase dance; however, in practice dance composers adhere to further restrictions. For example, while certain figures can be immediately repeated (like the half a hey and ladies chain), most figures are never immediately repeated in a dance. Further, when figures are repeated, they do not violate a section boundary. In our follow-up project, we plan to include these rules in the model, with the end result of limiting the number of possible dances.

At the outset of this project, we limited our figures to those that kept all the dancers within the minor set. There are many other figures in which dancers leave the minor set. For instance, dancers can move down the hall in short lines of four, interact with the opposite gender/same number dancer behind them (known as the em/ shadow), or travel along the side of the set interacting with next neighbors or previous neighbors. Further, there are other starting positions than just the duple improper position, for instance dancers can remain in the proper position, with all the men on one side of the set and all the women on the other side of the set. Another common starting position is the Becket position, which rotates the duple improper starting position one place clockwise. We plan to expand the model to incorporate these added complexities.

The results from this model inform a larger project to analyze composed contra dances. While this methodology would be able to count how many dances are possible, not every possible dance is reflected in practice. By examining the statistical probability of transitions between figures in composed contra dances, we would be able to wean out dances that do not follow stylistic norms.

Dart [2] documents stylistic change in contra dance compositions through time as a reflection of societal change. Through composer, caller, and dancer interviews, she observes these changes in the types of figures, the types of transitions between figures, and the method of progressing to the next minor set. In our corpus study, we hope to support these anecdotal observations, showing the statistical likelihood that a given figure or transition would occur in any given time period or geographic region.

This project could also be of interest to dance composers as a composition tool. While the graph would illustrate all of the possible pathways, a composer would still need to select those pathways that create a satisfying dance. Perhaps, this tool could also be used to quickly check if a particular composed dance is possible; where at the end of the 64-count sequence the dancers are progressed and with the same partner.

In this paper we create a graphical representation of contra dance that can list all possible contra dances given a set of figures and their resultant transformations. An advantage of this model is its flexibility: it can easily accommodate an expansion, where we can add more constraints, figures, and positions. Further, its accessibility allows anyone to explore possible contra paths through the multigraph.[2]

References

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