

Not only Art but also Rocket Science

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Abstract

A few years ago I presented a paper exploring ways that Newton's method for solving equations could be used to create or modify images. I was very pleased with the images so produced [1]. I decided to write a software framework that would allow me to explore various systems of attractors and repellors. This year is the 350th anniversary of the apple falling on Newton's head. I thought it appropriate, therefore, to base my first experiment on rocket science. The earliest rockets were probably developed in China during the 13th century for warfare and celebration. The beginnings of the mathematical treatment of rocket science began in 1666. A young Isaac Newton had returned to Lincolnshire from Cambridge University because of the great plague. He speculated that the force attracting the apple to fall to earth was the same force that kept the moon orbiting the earth. This paper explores ways of applying Newton's laws of motion and gravity to the creation and modification of pictures. The chaotic nature of such pictures is also explored.

Introduction

Whilst rockets were probably developed in China during the 13th century for warfare and celebration, the mathematical treatment rocket science began during the Autumn of 1666, exactly 350 years ago. At that time Cambridge University had closed because of the great plague. Newton, who had returned home, perpetuated the story of the apple falling, which triggered his thinking about why it fell. Over the ensuing years he formulated his theory of gravity and Newton's laws of motion. Newton's laws are still able to describe virtually all mechanical motions with which we are familiar, from the falling of an apple, to the orbits of the planets of the solar system, from our daily lives to rocket science. Although superseded by Einstein's theories differences are only apparent when dealing with extremely fast objects, large gravitational forces, or particles at a quantum level. In this paper I explore ways in which those same laws can be used not only to generate pictures, but also to modify existing pictures. I investigate the issues of unpredictability along borders and show that the pictures display properties often associated with chaos theory. Some slides that relate to this paper can be found at [5].

Newton's Laws

Newton's laws of motion are three laws of physics that, together with the laws of gravity, are the basics of classical mechanics [2] [3]. They describe the relationship between a body with a mass, the forces acting upon it and the subsequent motion. Although they have been superceded by Einstein's relativity they are extremely accurate within our normal experience.

1. The first law of motion says that an object in motion will not change its velocity unless acted upon by an external force.

2. The second law of motion describes the relationship between a body, the forces acting upon it and the resulting acceleration.

$$F = ma \quad (1)$$

where F is the net force acting upon the body of mass m and a is the resulting acceleration.

3. The third law of motion states that if a body A exerts a force on another body B then B exerts an equal and opposite force on A.
4. Finally, Newton's law of universal gravitation states that every object attracts every other object with a force that is proportional to the product of the two masses and inversely proportional to the square of the distance between the two objects.

$$F = \frac{GMm}{d^2} \quad (2)$$

where M and m are the two masses, d is the distance between the two masses and G is a constant which depends on the units used. G is known as the gravitational constant. See figure 1.

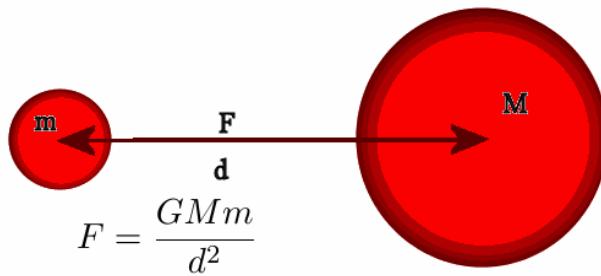


Figure 1: The force of attraction between any two masses is proportional to their product and inversely proportional to the square of their separation.

The Basic Algorithms for creating a Picture

The software developed for creating pictures uses Newton's laws of motion and Newton's laws of gravitation. The gravitational constant G has been adjusted to keep the graphics within the confines of a screen. A large number of masses have been catered for, some of which are fixed in space and some allowed to move. Additionally, any pixel can be the starting position of a space craft that can move around this artificial universe. The forces acting upon the craft were recalculated after small time intervals and a new position and velocity determined. Each dot on the trajectories denotes a recalculation, [5], see also figure 2. The maximum size for any picture is 100 megapixels. There are several ways to create artwork from these laws.

1. The first method is to record the final destination of each pixel as it moves around the screen. The colour of the pixel can be set, depending on its final destination and maybe adjusted depending on the time, or number of steps taken. The output can take the form of a static graphic or an animation. In figure 2 a large mass is surrounded by a number of smaller masses. Assuming a spacecraft is situated at a pixel position then its motion can be plotted. Eventually, the craft will hit the central mass, or one of the smaller masses or escape completely. See figure 2.
2. The second method starts with an image. The pixels are then moved around individually. A design after a fixed number of steps is recorded. Again the output can take the form of a static graphic or an animation. See figure 5.

3. The third method allocates a colour to a pixel depending on the gravitational force acting on it. See figure 6.
4. The starting position is usually taken to be the centre of a pixel. I have also considered the situation where there are four starting points, one at each of the four corners of the pixel. Each pixel is coloured black or white. White if all four corners reach the same destination and black if one or more end at different destination. I call the resulting picture a border of uncertainty. This border gets longer the more accurately it is measured ; a feature of fractals. See figure 7.

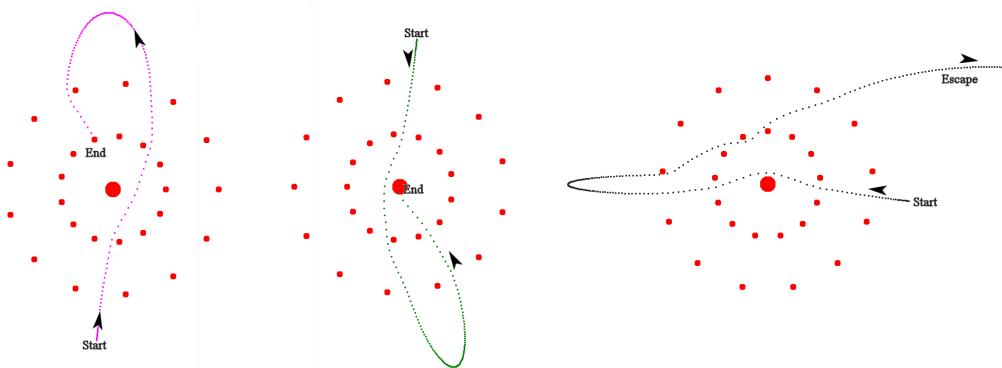


Figure 2 : The movement of rockets around the universe.

Pictures of Destiny

Figure 3 shows an arrangement of masses set around a large central mass. Each pixel moves under the influence of Newton's Laws. Its colour depends on its final destination and the number of steps it takes to reach its destiny. In this case the pixel either hits one of the smaller masses - coloured with blue/green, hits the centre - coloured with red/orange or leaves the screen altogether - coloured grey/ black. Changing or altering the number of colours can change the aesthetic appearance. Figure 3 shows three results where the destinations are separated and one where the three images are together. Figure 4 shows the results of having masses arranged in the shapes of the constellations Orion and Ursa Major. I should point out that the arrangements of the points for Orion and Ursa Major bear no relation to the actual positions in our universe, but are as they would be seen in the night sky.

May the force be with you

Newton's laws were used to calculate the vector sum of the forces acting upon each pixel. The colour of the pixel was then set depending upon the magnitude of this force. Some results are shown in figure 6, (a) is based around a pentagonal layout, (b) a 19-gon and (c) a heptagon.

Moving the pixels of an image

If we take an image, in this case a picture of Newton, and then track the movement of each pixel, separately, a series of images can be produced which can form an animation. Figure 5 shows the 10th, 50th, 90th and 130th frames of such an animation.

Border of uncertainty

If a picture is created by charting the final destination of a pixel then there will be pixels for which the destination depends upon which part of the pixel is the starting point. An interesting development is to separate the pixels into those for which there is little doubt about its final destination and those where there is doubt. A simple way is to consider each corner of the pixel and work out its final destiny. If each corner ends up at the same place then there is little doubt. If at least one corner is different then there is some uncertainty. Figure 7 shows the results of such an analysis. An English meteorologist, Lewis Fry Richardson did some experiments for determining the length of Britain's west coast [4]. An experiment that can be done with a good map and a pair of compasses. Set the compass to some large distance, say 128 km, them mark off points on the coast that are 128km apart. A rough estimate for the length of the coastline can be made. Repeat with the compass set to 64km, then 32km, 16km and so on. He plotted the results on a logarithmic scale. The measuring unit along the x axis and the estimated length plotted vertically, also on a logarithmic scale. The results turned out to be roughly on a straight line. Mandelbrot adds 1 to this slope getting a number D which he calls the fractal dimension. The behaviour of the length of the border of uncertainty displays similar characteristics to Richardson's experiments. I have measured the length of the border in pixels. The number of uncertain pixels was estimated. Multiplying this by the size of the pixel gives the border length. A graph of pixel size and border length provided a linear graph when both were plotted on a logarithmic scale. See figure 8.

References

- [1] <http://archive.bridgesmathart.org/2010/bridges2010-183.html>
- [2] https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion
- [3] School Maths Project, *Advanced Mathematics Book 1*. CUP 1973 1986 -ISBN 0-521-20241-8
- [4] Hans Lauwerier, *Fractals images of Chaos*. Penguin Books 1991 -ISBN 0-14-011411-0
- [5] <http://www.grandadstan.com/sjpdf/Finlandsldes2016.pdf>

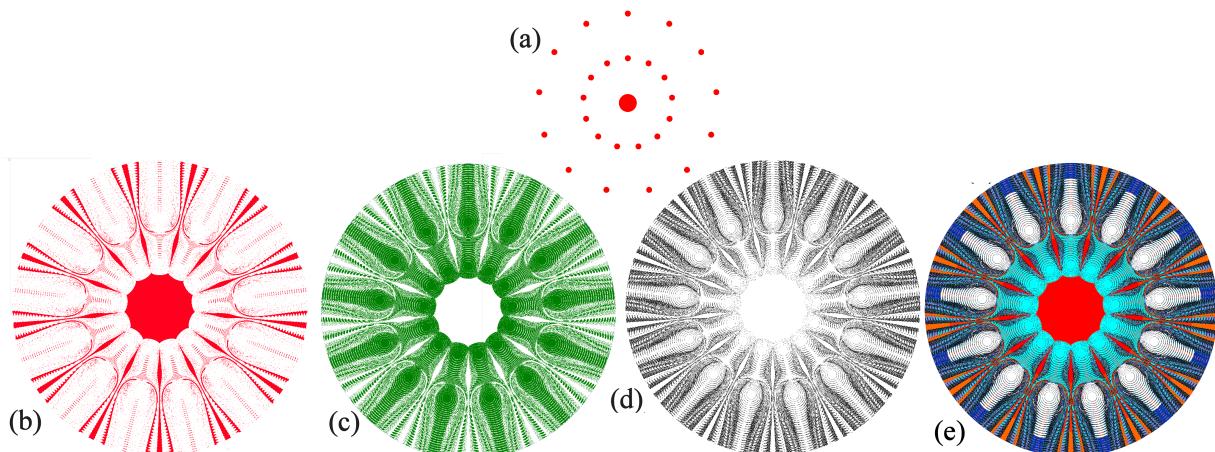


Figure 3: The colour of the pixel depends upon the final destination. (a) shows the arrangement of masses, (b) shows those that hit the central mass. (c) those that hit one of the smaller masses, (d) those that escape completely and (e) all the results in one image.

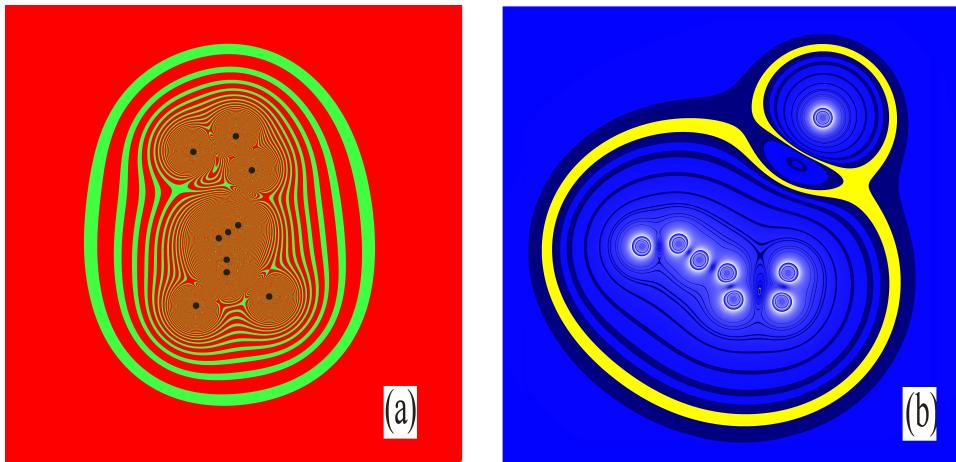


Figure 4 : The masses are arranged as the constellation (a) Orion, and (b) Ursa Major.

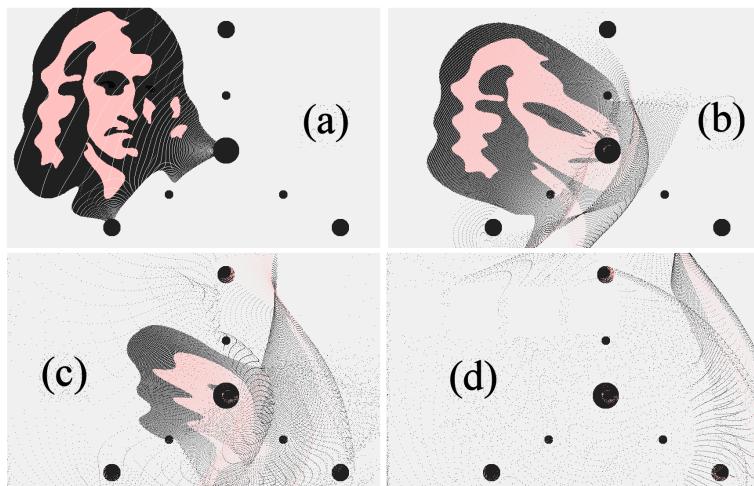


Figure 5 : The (a)10th, (b)50th, (c)90th and (d)130th frames from an animation tracking the movement of individual pixels from a picture of Isaac Newton.

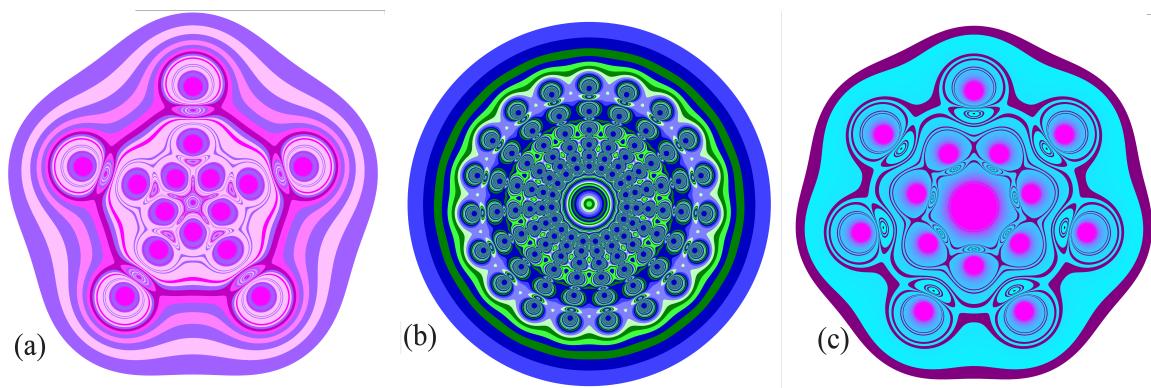


Figure 6 : The forces around various arrangements of masses. The colours of the pixels reflect the magnitude of the force acting upon it. (a) is based around a pentagonal layout, (b) a 19-gon and (c) a heptagon.

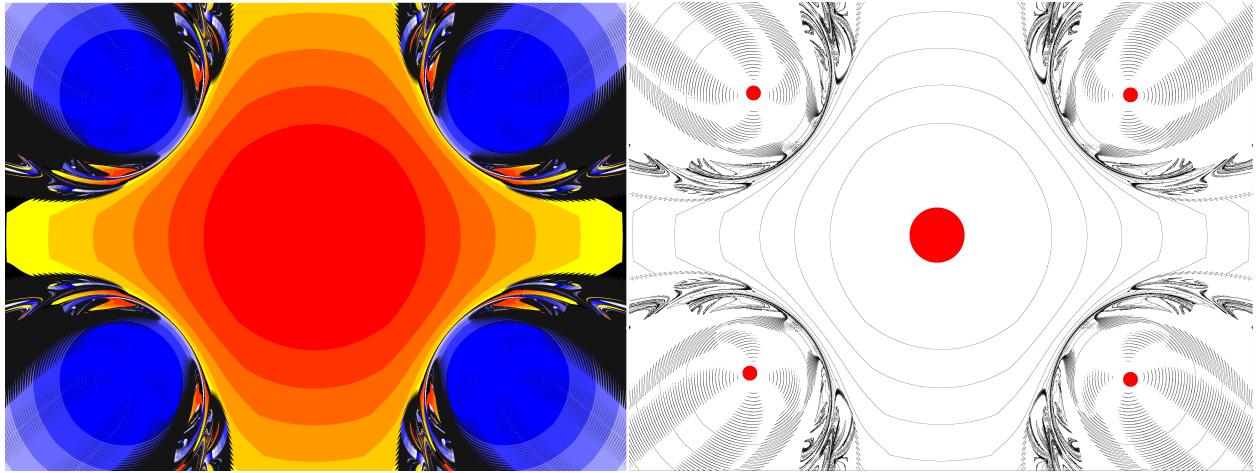


Figure 7 : Picture (a) shows the original picture where the colours represent the final destination. Picture (b) has white pixels where there is little doubt about the final destination, whereas the black pixels produce a different outcome depending upon the chosen corner. I call the black pixels a border of uncertainty. The red circles indicate the positions of the masses.

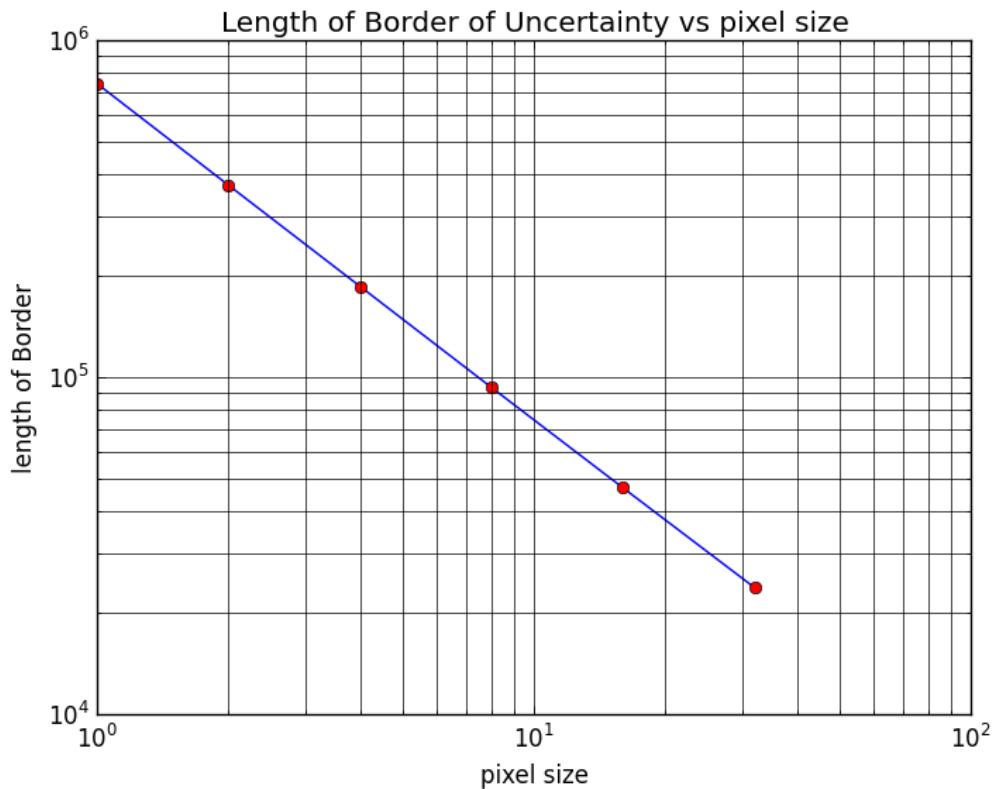


Figure 8 : When the length of the border of uncertainty is plotted against the pixel size, both on a logarithmic scale, then a constant can be calculated. This constant is a measure of the degree of meandering known as a fractal dimension (D). $D = 1 + \text{slope of line} = 1.78 \text{ approx.}$