# Square Seeds and Round Paths:

# Exploring Patterns within the Art of Classical Labyrinths

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**Abstract**

In this workshop, participants will explore middle and secondary level mathematics inherent in classical labyrinths. These labyrinths are constructed with a square interior and a combination of semi-circles and quarter-circles of different radii comprising the exterior. Some patterns and properties of labyrinths that participants will examine include the number of circuits, lengths of radii, perimeters and areas as functions of number of points used in the construction of the labyrinth.

## A Brief History of Labyrinths

A labyrinth is a maze with a single entrance and a single ending point [1]. Labyrinths of many sizes and designs have been used for the past 4000+ years for a variety of purposes. One famous labyrinth appeared in Greek mythology when Theseus entered the labyrinth and slayed the Minotaur. Labyrinths of various designs are found in many cultures around the world, including Mayan, Roman, Indian, and French cultures. There are four main categories of labyrinths – classical, Roman, Medieval, and contemporary. The labyrinths which are the focus of this workshop are classical labyrinths, named so because they were the first type of labyrinths that archaeologists discovered.

Many people find that walking a labyrinth has physical benefits such as lowering blood pressure and slowing breathing; psychological benefits such as helping people cope with grief, decision making and conflict resolution; and spiritual benefits [2]. Labyrinths are used in many ways: 1) as life-sized puzzles to overcome or conquer (e.g., Theseus' challenge), 2) as sacred environments (e.g., in European Gothic churches such as the Notre Dame of Chartres, France), 3) as medicinal and healing paths (e.g., at the Helen F. Graham Cancer and Research Center in Christiana, Delaware, USA), 4) as artwork (e.g., as displayed in the Delaware Art Museum, USA), and 5) for educational purposes [3]. Life-sized labyrinths are present worldwide [4], and are even celebrated on World Labyrinth Day, the first Saturday in May.

## The Construction of a C-Circuit Classical Labyrinth

There is a distinctive way in which classical labyrinths are constructed. This method, called the "cross and four corners" method, has been passed down from generation to generation. Classical labyrinths' constructions begin with a regular polygon which is considered the "seed." In our examples, a square seed is used. See Figure 1 for a fully constructed, 7-circuit labyrinth.

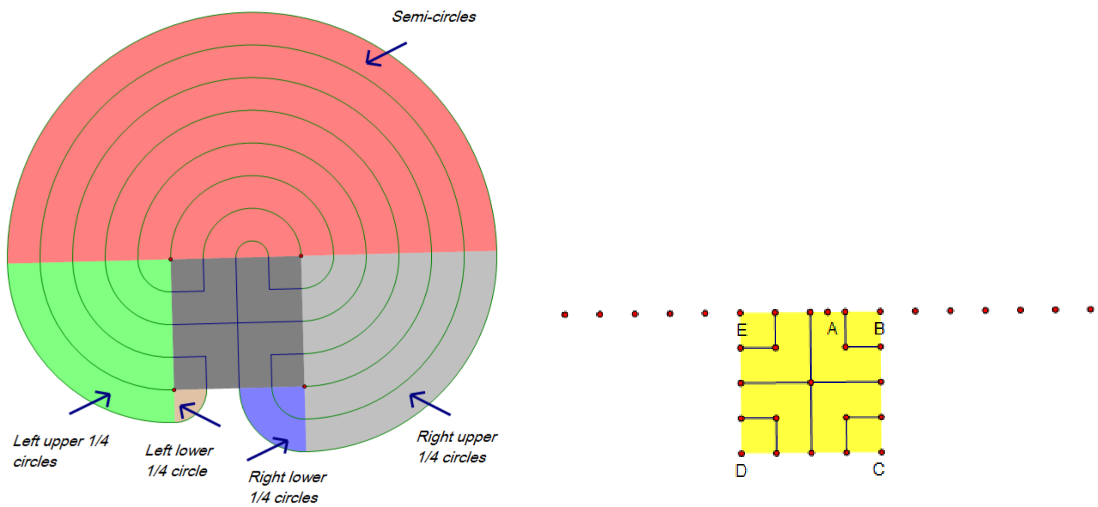


**Figure 1:** *Partitioning of the 7-circuit labyrinth (left) and outline for 7-circuit labyrinth sketch (right).*

Each of the semi-circles on the upper half of the labyrinth is a circuit and there are seven such semi-circles corresponding to seven circuits. The cross is formed by connecting the midpoints of opposite sides of the square, thus forming four equally sized quadrants. For each of the vertices or "corners" of the square to the midpoints of the sides of the square, a number of points N are equally spaced from the midpoint of the square to a vertex of the square. During the workshop, participants will construct a 7-circuit labyrinth on large poster board to gain an appreciation of the geometric features of the labyrinth. Beginning with an outline as depicted in Figure 1 (right), participants will use pins at each of the square's vertices (Points B, C, D, and E), and at the center of the labyrinth (Point A), as well as string as a compass to connect relevant points to form the semi-circles and quarter circles.

Figure 2 illustrates the progression of the construction, and a dynamic video of this progression is available at [2]. Another method of constructing the 7-circuit labyrinth is illustrated in the BRIDGES 2013 paper by Fenyves, Jablan & Radovic [5].

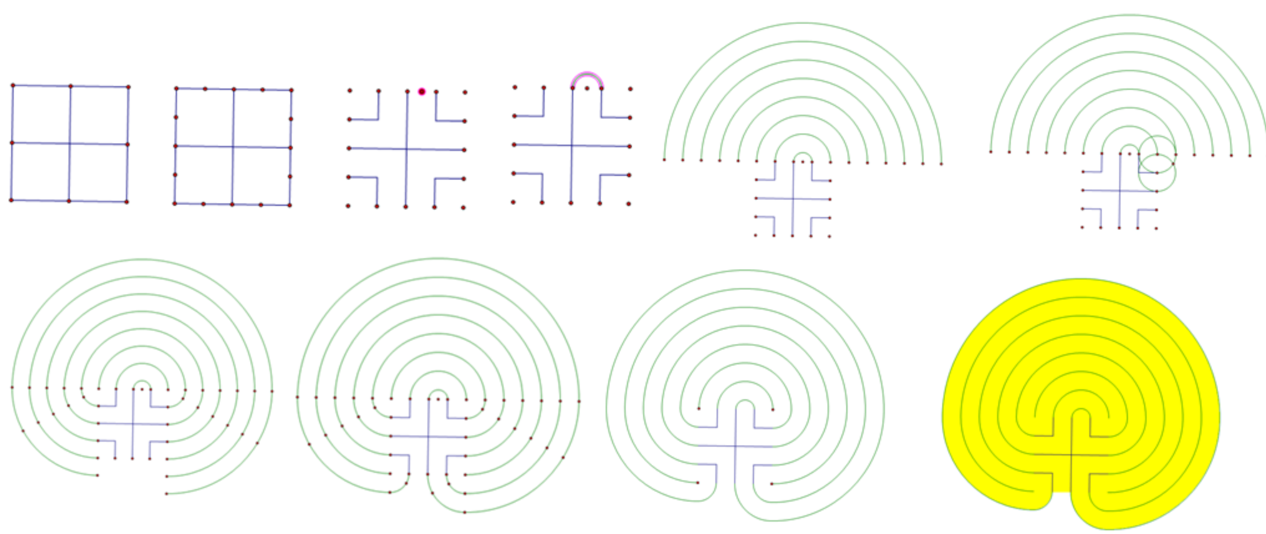

**Figure 2:** *Construction phases of a classical 7-circuit labyrinth with a square seed.*

## Patterns within Labyrinths

A variety of algebraic and geometric relationships exist with the labyrinths, as well as in the solution trajectory for each labyrinth. By examining the pictures of multiple-circuit labyrinths in Figure 3, participants will explore the relationships that are described in this paper, and will have an opportunity to explore other mathematical relationships they may discover.

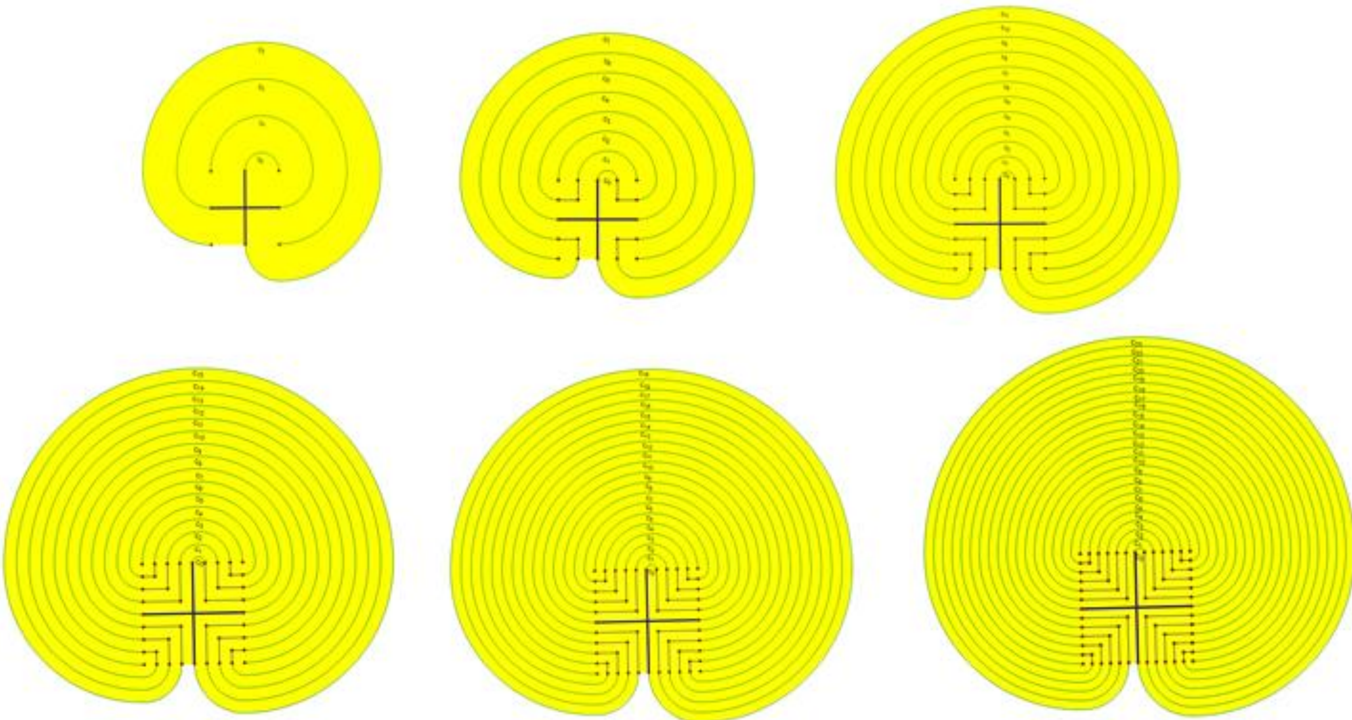

**Figure 3:** *From upper left to lower right: 3, 7, 11, 15, 19, and 23 circuit labyrinths constructed using Geometer's Sketchpad software.*

Linear patterns describe a number of properties that can be derived by examining the pictures of multiple labyrinths. Some of the parameters that yield linear equations with the dependent include number of circuits (K), number of interior intersection dots (D) in the square, and length of the outer semi-circle's radius (R) as a function of the number of dilated points from the midpoint of the square used in the construction of the labyrinth. These relationships will be summarized by participants who will complete Table 1 for each of the depicted circuits in Figure 3 as well as for the general case.

| Number of points from midpoint of square ( N ) | Number of circuits ( C ) | Number of interior intersection dots ( D ) | Length of Outer Semi-Circle Radius ( R ) |
|---|---|---|---|
| | | | |

**Table 1:** *Table of values relating C, D, and R as a function of N.*

Geometric properties regarding perimeter and area of the labyrinths can also be explored. The labyrinths can be partitioned into the square seed, the semi-circles, and quarter-circles. We have labeled the four quarter-circles as left upper, left lower, right lower, and right upper. See Figure 1 (left) for an illustration of the partitioning of a 7-circuit labyrinth. Note that the 3-circuit labyrinth only has three quarter-circles: the left upper, right lower, and right upper.

In determining the perimeter of the labyrinth, we will have participants find each of the arc lengths of the semi-circles and the quarter-circles. We ignore the entrance opening which is part of the square seed. The total arc length (L) on the outside of the labyrinth can be expressed as a function of the number of circuits, C.

In determining the total area of the labyrinth, we used the same partitions as mentioned above and found the areas of each of the semi-circles, quarter-circles, and the square seed. The total area (A) can then be expressed as a function of the number of points from the square's midpoint to the square's vertex, N.

Another aspect of mathematical interest regarding labyrinths lies within their solution paths. A solution path is the consecutive order of the circuit numbers traversed when solving the labyrinth. The solution path for the 3-circuit labyrinth is 3, 2, 1, 0; and the solution path for the 7-circuit labyrinth is 5, 6, 7, 4, 1, 2, 3, 0. Several patterns can be observed when examining the solution paths as they relate to the circuit number (C) of the labyrinth. For example: there is a linear relationship between the first circuit of a solution path and the circuit number of the labyrinth; the second circuit in the solution path is always circuit (C-1); the third to last circuit in the solution path is always circuit 2; and the second to last circuit in the solution path is always circuit (C-1)/2, and the last circuit in the solution path is always circuit 0.

## Conclusion

Labyrinths have been culturally, spiritually, and aesthetically significant. We hope that participants in this workshop will discover many mathematical properties of classical labyrinths constructed with square seeds. In addition to the algebraic and geometric patterns mentioned in this paper, there are also patterns to be explored within the solution path of labyrinths and the "lay out" of a life-sized version of a labyrinth using string.

## References

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