Random Walks on Vertices of Archimedean Tilings

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Abstract

Random walks have been studied by mathematicians and statisticians for over one hundred years, and have recently been used as the basis for some two- and three-dimensional artwork. In this paper, two-dimensional images are created based on random walks on the vertices of Archimedean tilings of the plane.

Introduction

I recall the day at lunch when the student sitting across from me asked about a problem involving random walks. I took out my computer and wrote a short program which produced a random walk on a square lattice. Some of the walks were rather uninteresting, looking like long, crooked lines. What surprised me was that some walks had a remarkable degree of self-intersection, and created interesting visual results. Thoughts extended to random walks on the vertices of Archimedean tilings – informally, tilings consisting of regular polygons, with the same set of polygons meeting at each vertex. A convention for describing such tilings is to assign a label consisting of the numbers of sides on the polygons meeting at a vertex, so the Archimedean tiling of the plane where one square and two octagons meet at each vertex is labelled either 4.8.8 or 4.8^2.

Finding the Paths

The main device used here in exploiting the self-intersecting nature of random walks is to layer translucent objects (in this case, circles or disks) on the nodes as the walk is traversed. In this way, a texture is created by virtue of the fact that some nodes are visited more frequently than others, so that more objects are layered upon one another in the process. The algorithm to generate the walks is the usual one. Once a tiling of the plane is chosen, an initial vertex is selected. A random neighbor vertex is then selected (here, with equal probability); as this process is iterated, a random walk is generated. In Figure 1, two random walks on the vertices of the tiling 4.6.12 are shown. Translucent circles are placed on the nodes as the walk is traversed, so that nodes visited more frequently appear darker.

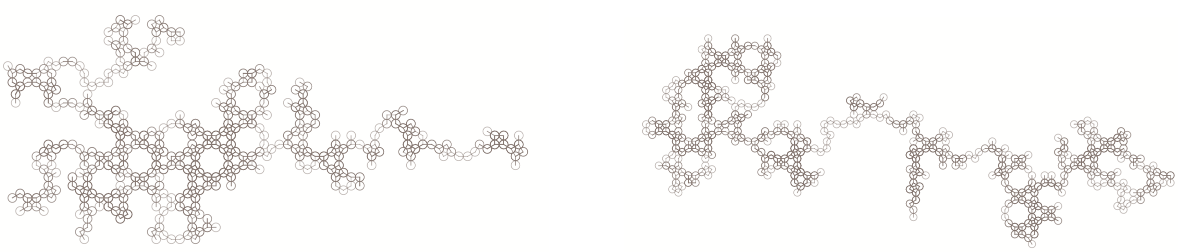

Figure 1 : *Random walks on 4.6.12 of length 1500.*

What determines whether or not a random walk is "interesting?" Of course there is no unambiguous answer to this question. To create the image in Figure 2, over 100 random walks were inspected; one had an initial path which resembled the form of a bear, and this walk was truncated to 1236 nodes. Layers of circles and disks were then added, as will be described in more detail in the next section.

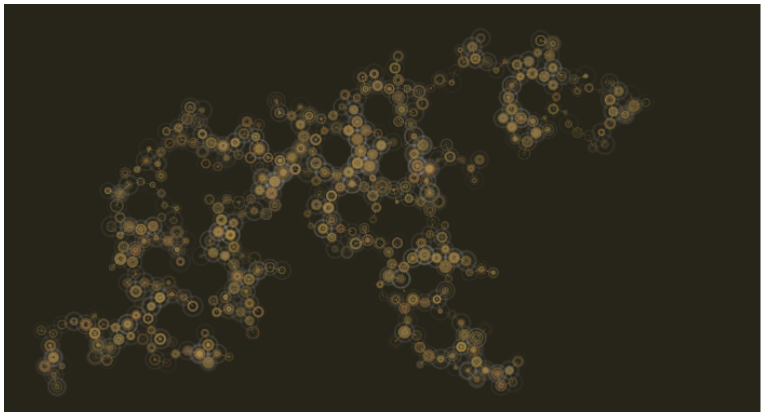

Figure 2 : *Bear.*

Layering Circles and Disks

Figure 3 below is based on the Archimedean tiling 4.4.4.4, which is the usual plane tiling by squares. Once a sufficiently interesting walk was selected, the layering process began. In this case, the walk was 1000 steps.

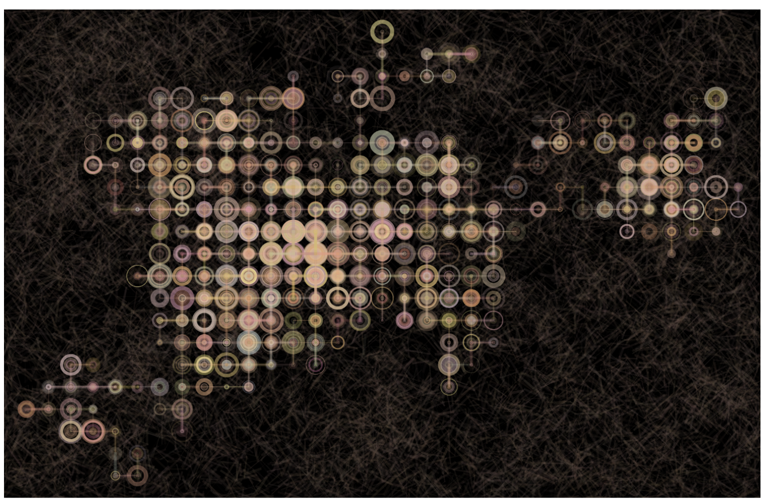

Figure 3 : *Random Walk 4.4.4.4.*

As shown in Figure 4, four layers were superposed to create the walk. Three of the layers were composed of circles, while the fourth focused on the steps of the walk itself. Texture was created by assigning each circle a random radius, thickness, color, and transparency, as well as similarly creating the steps in the walk. There were a wide array of artistic decisions to be made in the process, since too much randomness results in elements of the image that do not seem to relate to each other, while too little randomness creates a flat image with little texture. Each circle of each layer was generated with six distinct random parameters.

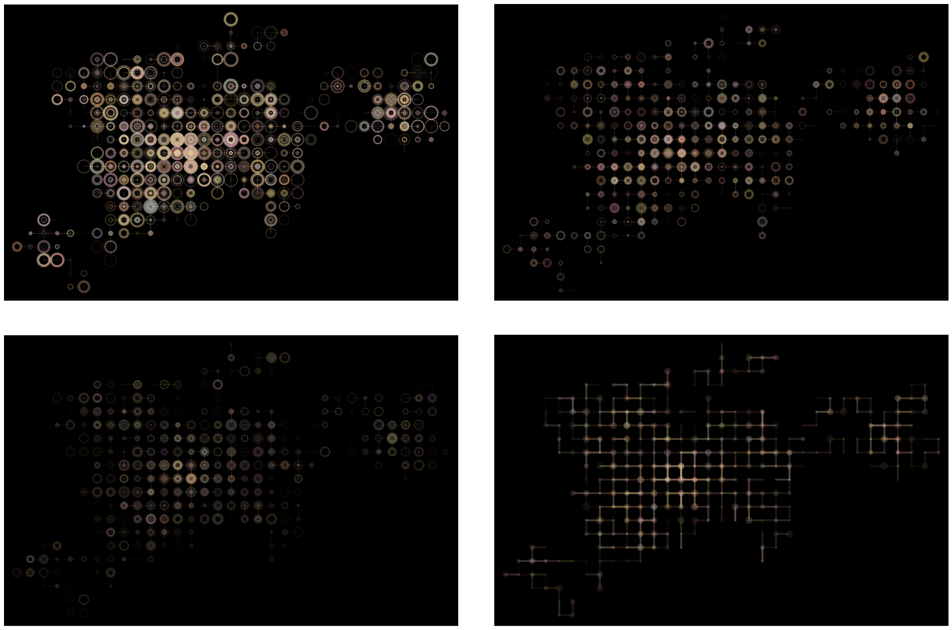

Figure 4 : *The four layers for Random Walk 4.4.4.4.*

Finally, to give the background some texture, these four layers were superposed on a random walk of 40,000 steps. The coordinates of each step were randomly generated numbers in $[0, 1]$, where steps wrapped around when they went off the edge of the image.

Figure 5 is based on the Archimedean tiling 4.8.8, where one square and two octagons meet at each vertex of the tiling. Once the layers of circles on nodes of the tiling were superposed, the image seemed

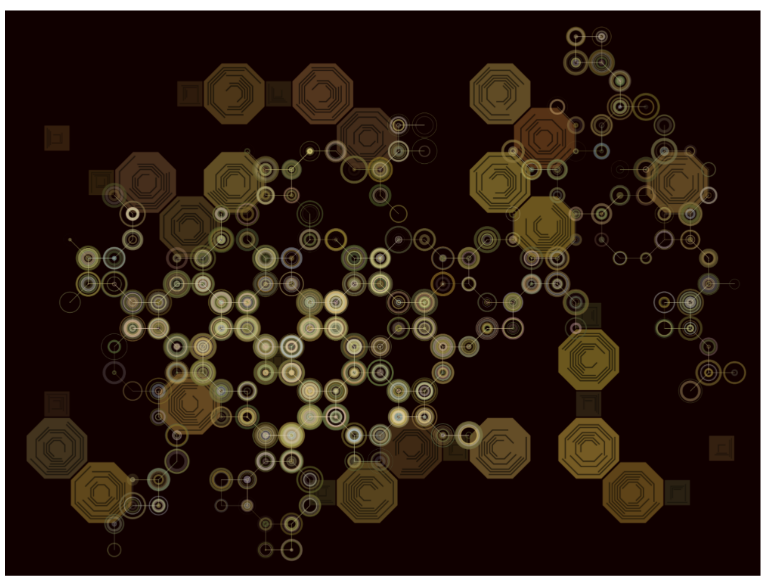

Figure 5 : *Random Walk 4.8.8.*

unbalanced, being much more intense on the lower left. To create balance, additional octagons and squares were individually placed toward the top and right of the image. The colors of these elements were individually decided, but the interior structures involving polygonal segments were randomly generated.

Figure 6 shows a close-up of a section of *Bear.* While not apparent in Figure 2, this close-up shows the interaction of the circles and disks generated with random elements.

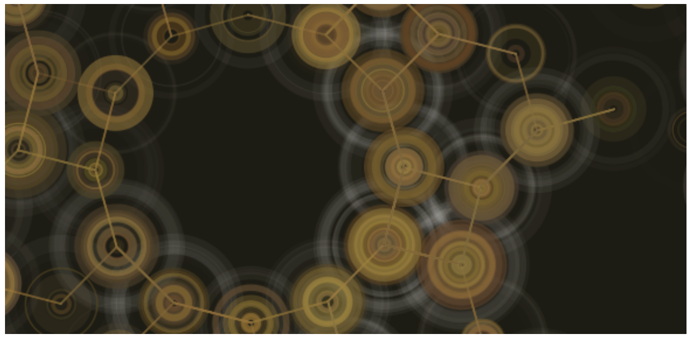

Figure 6 : *Bear, close-up.*

In a similar way, Figure 7 shows detail of a section of *Random Walk 4.8.8.* Here, elements are generated with more randomness than those of *Bear,* resulting in a higher intensity.

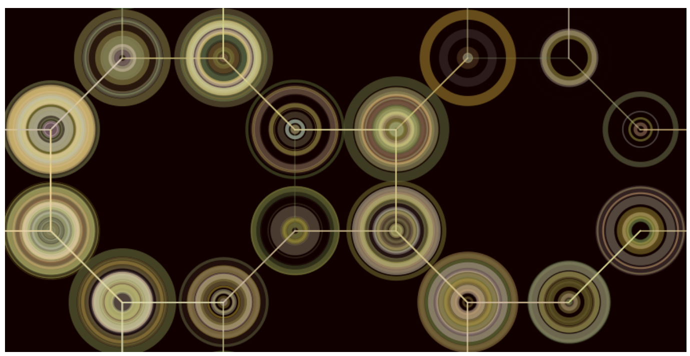

Figure 7 : *Random Walk 4.8.8, close-up.*

Concluding Remark

The use of randomness in computer-generated art is not new (for examples, see Coyne's artwork generated by context-free grammars [1], the article by Schönlieb and Schubert [2], or the work of Tarbell [3]). Perhaps overlaying random graphical elements on regular and semi-regular tilings creates a novel way of artistically capturing the dynamic interplay between structure and chaos.

References

[1] Coyne, Chris, *Context Free Art,* 2015. `http://www.contextfreeart.org` (as of Feb. 12, 2015).

[2] Schönlieb, Carola-Bibiane and Franz Schubert, *Random simulations for generative art construction—some examples,* Journal of Mathematics and the Arts, Vol. 7, Iss. 1, 2013.

[3] Tarbell, Jared, *Gallery of Computation,* 2014. `http://www.complexification.net/gallery/` (as of Feb. 12, 2015).