Integrating Origami Art with Mathematics in a College General Studies Course

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Abstract

This paper presents research conducted with a unique college level course called "The Art and Math of Origami". As the title suggests, this course teaches about the art of paper folding while integrating mathematical concepts. Data collected over a seven year period is presented illustrating the impact Origami can have on spatial ability, a mathematical skill linked to geometry and with ties to careers such as architecture and engineering. The most current year's research also shows potential benefits related to students behaviors including creativity and attitude. Results, as a whole, are promising illustrating that Origami itself has an inherent connection to mathematics and can benefit those that practice it.

1 Introduction

Linn and Peterson [9] define spatial visualization as a person's capacity to perform a series of mental manipulations of an object in both two- and three-dimensional forms. Referred to more generally as spatial ability, this skill is used in a variety of ways from every day to career-related tasks. For instance, have you ever had to build a piece of furniture, play a video game, or organize items in a confined space? Examples of careers using spatial skills range widely from engineering and computer programming to architecture and mechanics. In all cases, a person's ability to perceive, recall, create, and arrange spatial images allow them to complete tasks. This is at the core of spatial ability.

Spatial skills are typically categorized as mathematical in nature with a link to geometry, the study of shape. A branch of geometry known as transformational geometry applies well to spatial ability since it defined as the study of objects by looking at two- and three-dimensional figures through transformations such as rotation, reflection, and translation [10]. This skill has proven to be malleable, at all ages, from our youth through adulthood via training and practice [12].

An art that is very spatial in nature and the focus on the work presented here is the ancient art of paper folding, Origami. At first mention, a novice of paper folding likely thinks of learning Origami as folding a crane or a paper airplane. This is not uncommon with Origami often thought of as a craft or a past-time with little connection to mathematics or spatial ability. Origami studied in more depth is actually an intricate art involving the physical and mental manipulation of shape from a two-dimensional sheet of paper resulting in a three-dimensional object. One simply needs to do a search of the words Origami and mathematics on the internet to find evidence of it. Books, websites, and journal articles all speak to the geometric nature of Origami models and the rich mathematics within the folding process itself [1].

It is the richness of the art of Origami that has led to the creation of and subsequent instruction of a four credit college-level course called "*The Art and Math of Origami*". As part of the 15 week course, students are exposed to topics including culture, history, Origami as a form of artistic expression, and mathematics while practicing Origami regularly. The research to be discussed here focuses on the impact of the regular study of Origami with an in depth analysis of the mathematical benefits related to spatial skills.

2 Design

2.1 Purpose

As I noted, this study builds upon investigations on the impact of Origami on mathematical achievement. Study began with middle-school age children and the impact Origami would have on achievement if blended into a traditional geometry curriculum [2,3]. An experimental study design was used comparing two groups of children, one that received regular instruction on Origami blended into their geometry curriculum and one that received only traditional instruction on the same topics over a one-month period. A series of mathematical spatial tests were used to compare Origami's impact with a comparison of two different types of instruction, traditional versus Origami-infused mathematics instruction. Results from this investigation showed that students, regardless of instructional style, showed gains in learning but nothing yielding statistical significance. This offered some support to the claim that Origami can be used as a tool for improving spatial skills but results were limited due to the short length of the study (one month) and factors such as outside spatial abilities not accounted for within the research [1,2].

My investigation of Origami's impact on mathematical ability continued shifting focus from middle-school age to college-age students [4]. On the premise of the previous study, I conducted research within the semester-long college level course, *The Art and Math of Origami*. The work presented here offers a holistic look at the influence of the art of Origami on the full population of students that have taken this college course to date.

2.2 Sample

The sample of this study consists of all students electing to take the course *The Art and Math of Origami* over the seven year period. The course is offered every spring with the capacity of the course capped at 26 students for any one semester. Class size for each term included within this sample ranged from 22 to 26 students. This course falls within the college's general studies curriculum and is identified as a natural sciences and mathematics elective course. Students of any major are open to take the course and are typically junior/senior level. Among those within the sample, 105 were female and 49 male. In terms of ethnicity, the majority of the sample was Caucasian (118 or 77% of the sample) with 36 students of other ethnicities (including African American, Asian, Hispanic or other).

An additional tool used to identify the sample relates to students' exposure to spatial experiences. With spatial abilities often used in everyday life as well as in specific fields of study, students could bring pre-existing abilities to the course. These existing abilities could potentially influence the impact Origami might have on their skills [1]. To account for this possibility, a survey was used at the start of the course to measure pre-existing spatial skills. Students were asked to rate their abilities on six common spatial tasks (such as playing video games regularly or working with their hands) on a 1-5 Likert scale. Ratings from this scale were then used to identify students as "low spatial" or "high spatial" experience levels. In addition, students were asked to identify their major area of study. These were then categorized into STEM areas such as biology, mathematics, and computer science or non-STEM such as criminal justice, history, art, etc. Data related to these categories is discussed later in Section 3.2 and 3.3.

2.3 Instruments

There were several instruments utilized for this study: a survey to measure pre-existing spatial skills (noted in 2.2), three tests measuring spatial abilities, and the recent addition of a post-test with open-ended questions to gather additional evidence of Origami's impact on students (with last set of students taking course in 2014). Each of the three spatial tests was taken from an intelligence test called the Kit of Factor-Reference Cognitive tests [5] found to best represent the main skills associated with spatial ability [1]. Each test evaluates a distinct skill associated with spatial ability. (See Table 1.) Tests are taken in two parts to offer the test taker a chance to acclimate to the test question types. Each section lasts between three to six minutes depending on the complexity of the problems answered. The "paper folding" test is the most closely aligned to the art of Origami asking an individual to visualize a series of folding steps with a sheet of square paper, followed by punching an imaginary hole through all the layers of the paper, then identifying which of five pictured unfolded figures represents the true result of those actions. As the description suggests, this tests involves both 2-D and 3-D mental visualization. The score is based out of 20 with 10 questions on each of two sections taken with a point awarded for each correct response made. The "card rotation" test is literally as the name suggests. It involves taking a 2-D object and comparing it to ten other images, determining if the image was rotated or manipulated by more than just a rotation. This tests purely 2-D spatial skills. For this test, each of 20 questions has 8 responses making a total of 80 points possible (1 point per question answered correctly). The final test is called the "surface development" test. This test involves comparing a 3-D mathematical solid object with its corresponding 2-D mathematical net and being able to associate the 2-D figure to the 3-D version. This test involves both 2-D and 3-D visualization as well as the ability to move between forms. For this test, each of 12 questions has 5 possible responses so scoring is based on a maximum of 60 points. All tests were field tested with reliability found to be within an acceptable range [6].

The set of spatial tests were used as the pre- and posttest for this study. The tests were given at the start of the college course, prior to treatment, and again at the end of term, after treatment was completed. The tests took place approximately three months apart so a test-retest effect was unlikely. In addition, procedures and timing of tests were based on field testing methods established [5,6]. (For additional details, see the full report of the 2006 study [1].)

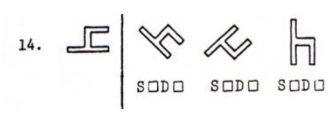
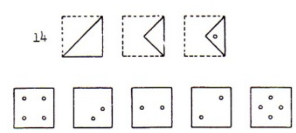
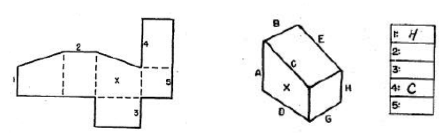
	Minutes per section	Total responses	Sample*
Card Rotation Test	3	8 visual prompts with 10 responses per prompt	 Only 3 responses shown
Paper Folding Test	3	20 questions	
Surface Development Test	6	12 questions w/5 parts per question	

Table 1. *Spatial Ability Test Question Samples*
*Reprinted by permission of Educational Testing Service, the copyright owner.

2.4 Treatment

Treatment of this study consisted of students learning the art of Origami over a 15 week semester with four hours of classroom instruction per week. Origami is taught by me at every session beginning with basic level models and progressing to lower intermediate to intermediate models. Students are taught the visual notations of Origami, how to read and interpret printed Origami directions, and the terminology associated with the art. Students are exposed to various forms of Origami including traditional, modular, and unique material (using non-traditional materials). Two Origami books serve as main textbooks and are referenced throughout the course including Kenneway's Complete Origami [8] and Montroll's Teach Yourself Origami [11].

Beyond the intricacies of Origami, students are exposed to mathematical topics throughout the semester. Math is taught and explored through the fold patterns of completed Origami models when unfolded as well as during the process of folding itself. Topics often fall in the area of geometry such topics as symmetry, transformations, plane figure relationships, properties of polygons, etc. Books offering a similar methodology that are referenced in the course include Unfolding Mathematics with Unit Origami [7] and Unfolding Mathematics with Origami Boxes [13]. Figure 1 below is an example of one of the modular models explored which consists of a single "unit" (far left) that is combined to make various 3-D polyhedral shapes. (All models shown to right of unit are made with the same individual unit. The number of pieces and tucking pattern alters the final model formed.) Mathematics explored with this model includes the mathematical nature of the folding pattern resulting in the parallelogram shape and the polyhedra resulting from multiple units. Figure 2 illustrates the fold pattern of the unit when it is fully unfolded. This unit, in particular, is useful for discussing rotational symmetry as well as triangle relationships.

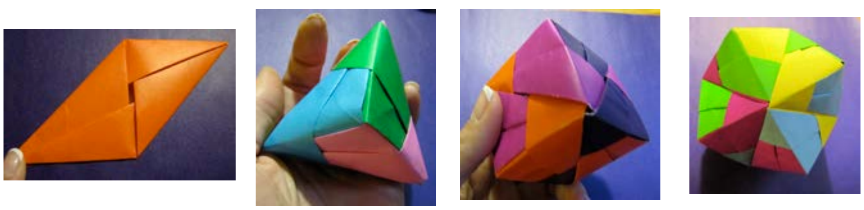


Figure 1. *Starbuilding unit & resulting modular Origami*

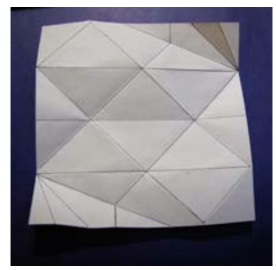


Figure 2. *Fold pattern for a starbuilding unit*

Student's actual academic work completed as part of course requirements also includes Origami. A series of written essays are completed, each with an accompanying Origami model that is linked to the writing. Traditional quizzes are given that test students' ability to identify and discuss mathematics highlighted through Origami models and fold patterns. In addition, students must create a collection of 5 Origami models without direction from the instructor that serve as their final project. Each model is accompanied by a written paper (and in one case, an instructional video created by the student). A sample collection shown in Figure 3 illustrates the variety of the Origami practiced throughout the course.

Figure 3. *A student's Origami collection*

As a whole, Origami is an integral part of the course that serves as the treatment for this study. In the beginning of the term, the instructor serves as the main teacher of Origami. However, over the course of the semester, the students become self-guided with the instructor serving as a facilitator in the process. Students are often teamed to allow student-student learning and slowly increase students' ability to work independently. In terms of mathematics, it is estimated that about 20% of the time is spent on purely mathematical tasks. The main focus on the course is on the art of Origami, rather than the teaching of direct mathematical concepts.

3 Results

3.1 Change in Spatial Skills

To begin the process of analysis, a paired-sample t-test was run using the pre- and posttest results from the three spatial tests (discussed in Section 2.3). Data reported in Table 1 represents all students in the sample that took both the pre- and posttest. Data is broken down by each spring the course was offered. The final row noted "all" represents all semesters combined. Mean, standard deviation and gain on each test is noted. As shown, the vast majority of tests produce statistically significant results with a range in gains (keeping in mind that the number of items for each test varied- see section 2.3 for details).

		Card Rotation Test (160 points possible)			**Paper Folding Test** (20 points possible)			**Surface Development Test** (60 points possible)		
Year	**N**	**Pre**	**Post**	**Gain**	**Pre**	**Post**	**Gain**	**Pre**	**Post**	**Gain**
08'	24	133.17	135.42	2.25	12.63	13.88	1.25*	39.88	47.08	7.20**
09'	22	118.23	132.91	14.68**	12.00	13.00	1.00**	40.32	45.23	4.91**
10'	20	115.60	127.95	12.35*	10.95	13.20	3.25**	37.60	43.20	5.60*
11'	22	100.32	107.27	6.95	10.95	12.86	1.91**	34.23	43.68	9.45**
12'	18	94.67	113.94	19.27**	11.67	13.61	1.94**	35.67	45.56	9.89**
13'	22	102.91	135.59	32.68**	14.18	15.55	1.37**	47.64	53.00	5.36**
14'	26	115.04	134.54	19.50**	11.96	13.81	1.85**	39.69	46.50	6.81**
All	154	112.18	127.44	15.26**	12.08	13.71	1.63**	39.42	46.40	6.98**
Pre- and post-test scores represent the mean * *p<.005, * p<.05										

Table 2. *Spatial Abilities Test Results Overall and by Year*

3.2 Influence of Spatial Experiences on Results

The influence of pre-existing spatial abilities had the potential to influence students' performance on spatial tests given [1]. To account for this, data was gathered on spatial experience. Using the ranking system via responses to a survey (described earlier in Section 2.2), students were identified as either low spatial or mid to high spatial experience level. A one-way between-groups analysis of covariance (ANCOVA) was then utilized using the spatial experience level as the independent variable while controlling for pre-existing differences that may exist via the pre-test score (co-variate). Preliminary checks prior to running the analysis were completed to ensure there was no violation of assumptions for an ANCOVA test. Results of the analysis are presented in Tables 3 & 4. The ANCOVA run on all three spatial tests resulted in no significant difference found between the low and mid to high spatial groups on post-test scores: card rotation $F(1,151)=.06$, $p=.81$, partial eta squared=.00; paper folding $F(1,151)=.01$, $p=.93$, partial eta squared=.00; and surface development $F(1,151)=.62$, $p=.43$, partial eta squared=.00. When examining the tests individually, there was a strong relationship found between the pre- and post-test scores as indicated by the partial eta square values (see Table 4). Of the three tests, paper folding and surface development had the highest results with a large portion of variance on the post-test associated with the pre-test score.

Spatial Test	STEM N=42		Non-STEM N=112		Low spatial experience N=80		Mid to high spatial experience N=74	
	Pretest Mean (SD)	Adj. Posttest Mean (SD)	Pretest Mean (SD)	Adj. Posttest Mean (SD)	Pretest Mean (SD)	Adj. Posttest Mean (SD)	Pretest Mean (SD)	Adj. Posttest Mean (SD)
Card Rotation	119.05 (26.93)	136.43 (21.91)	109.60 (26.54)	124.06 (26.24)	104.35 (25.06)	122.58 (25.58)	120.64 (26.38)	132.69 (24.87)
Paper Folding	13.60 (2.91)	15.12 (2.68)	11.51 (3.22)	13.19 (3.29)	11.46 (3.59)	13.21 (3.61)	12.74 (2.74)	14.26 (2.73)
Surface Development	46.43 (10.69)	52.40 (8.28)	36.79 (14.36)	44.14 (13.21)	36.93 (15.11)	44.11 (14.47)	42.12 (12.46)	48.86 (9.69)

Table 3. *Descriptive Statistics by Grouping Variable & Spatial Test.*

3.3 Influence of Fields of Study on Results

Beyond spatial experiences, a student's major can also potentially have an influence on performance on spatial tests. Using data on identified majors of all participants, students were categorized into STEM and non-STEM areas (explained in Section 2.2). Similar to the analysis for spatial ability, an ANCOVA was utilized to control for any pre-existing differences that could be a result of a student's major. In this case, the grouping by major served as the independent variable. Results of this analysis are presented in Table 3 & 4.

As was the case for spatial experiences, students' area of study showed no significant influence on results when the pre-test score was taken into account: card rotation F(1,151)=3.45, p=.07, partial eta squared=.02; paper folding F(1,151)=.65, p=.42, partial eta squared=.00; and surface development F(1,151)=.78, p=.38, partial eta squared=.01. When doing an additional review of partial eta squares, there was again a strong relationship found between the pre- and adjusted posttest scores (see Table 4).

	Card Rotation	Paper Folding	Surface Development
Spatial experience			
Pre-Test	.46**	.63**	.70**
Adjusted Post-Test	.48**	.64**	.69**
Between-Groups	.00	.00	.00
STEM vs. Non-STEM			
Pre-Test	.47**	.62**	.68**
Adjusted Post-Test	.49**	.64**	.70**
Between-Groups	.02	.00	.01
**p<.005			

Table 4. *Partial Eta Scores from ANCOVA by Spatial Test*

3.4 Additional Perceived Impact of Course

A final area of analysis was included with the most recent cohort of students from 2014. At the end of the college course, a series of open ended questions were asked related to the impact of the course. Of the 26 students that term, 23 elected to respond. One question asked students to summarize their thoughts on the course by selecting three adjectives to describe the course. Frequency of words used were gathered and listed below in Table 5. Additionally, a question asked for students to describe how they feel they have changed personally as a result of this course. Sample comments are noted in Table 5 that align with words used to describe the Origami experience. While mathematics is noted in a few places, students spoke to a vast variety of benefits ranging from personal well-being to the intrinsic value of the experience.

Word Frequency	Words	Sample comments
4	Relaxing	• I feel relaxed when I fold the models and even if I was upset of anything and fold origami, it makes me happy.
3	Rewarding Complex Beautiful	• I learned the importance of applying myself and working hard. It made me appreciate the work I completed in this course far more than any other class I have ever taken. • In the beginning of the course I became frustrated very easily with the challenge of the model; however, I have learned to take my time to understand clearer rather than give up.
2	Unique Patience Mathematical Intriguing Creative	• I have learned to be more patient since these models did not come easy to me. • Origami has helped me visualize certain spatial geometries and symmetries more clearly. • This class sparked my artistic side again and made me excited for the summer when I'll have more time to practice and improve my skills.
1	Visual, useful, unfamiliar, technical, surprising, structured, skill, precise, practical, motivating, meticulous, intricate, interesting, interdisciplinary, inspirational, humbling, fun, fulfilling, expressive……	

Table 5. *Words and descriptive statements on the impact of the Origami course*

4 Conclusion

Looking first at how students did at the beginning and the end of the Origami course offer perspective on Origami's influence on spatial ability. Pre- to posttest improvement reported in Table 1 reveal significant change in mean scores from pre- to posttest. Of the three spatial tests utilized, the strongest results were found with the tests including three-dimensional visual skills. The card rotation test, for two cohorts of students, did not show significant gain in scores earned. This may be due to the fact that two-dimensional manipulation is a more commonly used skill in everyday life. When reviewing the data as a whole, results are highly significant. While this is an important finding, it is not conclusive because there is no group in which to compare against. Future study with a full experimental design would be beneficial to isolate if Origami was the major contributor to the increase in mathematical performance.

An additional measure used as part of this study was the identification of pre-existing spatial skills. This element was included to investigate if other experiences outside of the college course influence Origami's impact on students' spatial ability. Analysis related to spatial abilities included a comparison of low spatial and mid to high spatial experience groups. ANCOVA results reveal that posttest performance is not significantly influenced by pre-test scores when existing spatial abilities is taken into account. The same was true when reviewing STEM versus non-STEM groups. Thus, students' pre-existing spatial skills do not have a major impact on their resulting spatial skills as demonstrated on the posttest. However, it can be said that posttest performance can be directly associated with pre-test scores, for both spatial experience and major area of study based on a review of partial-eta square values. In terms of what this says about Origami's influence, there is evidence that Origami is beneficial to mathematical skills regardless of a student's background.

A final area worthy of noting is the newest data gathered as of 2014. The questions used were designed to capture other influences the Origami course may have beyond the focus of the study conducted. As it turns out, students perceived many benefits beyond mathematics. Comments and words shared offer an indication that Origami has value from a more personal, individualistic perspective. While this was not a major part of this study, it is a worthy avenue of investigation for future research work.

As a whole, research presented offers strong evidence of the positive influence Origami has on spatial abilities. With spatial skills serving as a valuable life and career skill, this conclusion can be of benefit to those seeking resources to develop spatial ability. While data presented is encouraging, there remains a great deal of work to be done. Additional studies are needed that offer data-based evidence of the influence Origami has on learning. There is also clear indication that the impact of Origami can fall beyond mathematics. Studies seeking to quantify behavioral or possibly attitudinal data could provide other evidence of the benefit of the regular practice and benefits of Origami art.

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