Geometric Study of Architectural Designs on a Twelfth Century Structure

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Abstract

The article "*Geometric Analysis of Forumad Mosque's Ornaments*", which appeared in the Bridges Enschede Conference Proceedings, presented the analysis of some architectural designs and ornaments that adorn the walls of a 12th century structure in Iran: Friday Mosque of Forumad. All the patterns studied in that article were created based on the division of a circle into six congruent arcs. In the present paper we continue our study of the geometric patterns that have embellished this mosque but include patterns that are not necessarily related to the division of a circle into six. The most interesting ornamental patterns are those that cannot be classified in a usual manner. Rather, their geometric compositions are based on a traditional construction process that were used by the craftsmen of the past, the radial grid approach, which results in a rectangular tiling unit called *girih.* In this article we analyze some *girih* patterns as well.

1 Introduction

Friday mosque of Forumad is a relatively small building in the northeast region of Iran. It is a two-iwan mosque with no dated inscription; it is considered a 12th century structure by most historians. An iwan is a vaulted hall, walled on three sides, with one end entirely open. Various ornaments decorate the entire structure in stucco, glazed brick, and floral motifs with underlying geometric substructures, and calligraphic inscriptions which make this mosque a prototype of the medieval Iranian architecture in applying plentiful complex ornaments. Studying the design of ornaments in this mosque demonstrates the advanced knowledge of the geometry of its craftsmen who probably had been influenced by the mathematicians of their time.

Following the previous study in the geometric patterns of this mosque by the authors, this paper endeavors to detect design methods in ornamental patterns which do not follow the usual division manner of a circle into six parts. The article presents the compositions of the geometric designs in the radial grid approach for some of the patterns, which results in a rectangular tiling unit called *girih.*

2 Friday Mosque of Forumad

Friday mosque of Forumad, Figure 1, has no inscription to evidence the exact date of the structure. However, archaeologists such as Andreh Godard, a French archeologist and art historian who excavated this mosque in the 1920's, considered this building a 12th century structure, which was built during a period when the two-iwan mosques were popular in the eastern parts of Iran [2]. This mosque can be considered a valuable resource for the study of architecture and architectural ornaments of the medieval Islamic period in Iran. The mosque includes two iwans in about 6.60 meters width in the north and the south sides of the structure. The northern iwan is 12 meters and the southern iwan is about 14 meters

high. The interior area of the mosque is about 820 square meters and it is one of the masterpieces of Iranian architecture in the application of outstanding stucco ornaments [4]. It is also among the first mosques in Iranian architecture for using glazed brick in combination with brick ornaments and stucco. The tendency of the application of various ornaments is evident from the fully decorated walls inside and outside of the mosque. The patterns consist of floral motifs, geometric forms, calligraphy, and inscriptions. There is also a mihrab – a niche set into the middle of a wall in a mosque to indicate the direction of Mecca – in the southern iwan. It is decorated with stucco ornaments and outstanding stalactite vault muqarnas. Because of the weather conditions and other destructive factors, most of the ornaments of the mosque have been damaged and it is difficult to visualize and restore their entire patterns.

Figure 1: *Friday mosque of Forumad.*

3 Geometric Analyze of the Architectural Ornaments

All the designs that were studied in "*Geometric Analysis of Forumad Mosque's Ornaments*" [3] were composed according to the division of a circle into six congruent sections. Therefore, most of them inherited six-fold rotational or reflective symmetries. However, we are able to locate patterns with different symmetries in this medieval structural complex. We first present a six-fold pattern that was not analyzed in the aforementioned article. We then analyze an interesting four-fold pattern. After that the *girih* construction approach of radial grid will be presented for some patterns. The main aim of this paper is extracting and depicting the geometric substructure of design of the prominent ornaments, of which most have been destroyed. It is worth mentioning that all the analysis and step-by-step constructions of the designs in this article are original and were depicted and performed by the authors, except for the design in Figure 8 where the layout of its geometric composition was illustrated in a nineteenth century book without any explanations of the involved steps for its construction [1]. The proposed methods for geometric compositions of designs are based on traditional instructions suggested by master designers who had the most access to original artisans' repertories of the past. The methods can be considered as reliable sources for conservation and restoration of the ornaments.

3.1. A Six-Fold Design. The stuccowork in Figure 2 is on a wall that surrounded one of the entrance gates of the mosque.

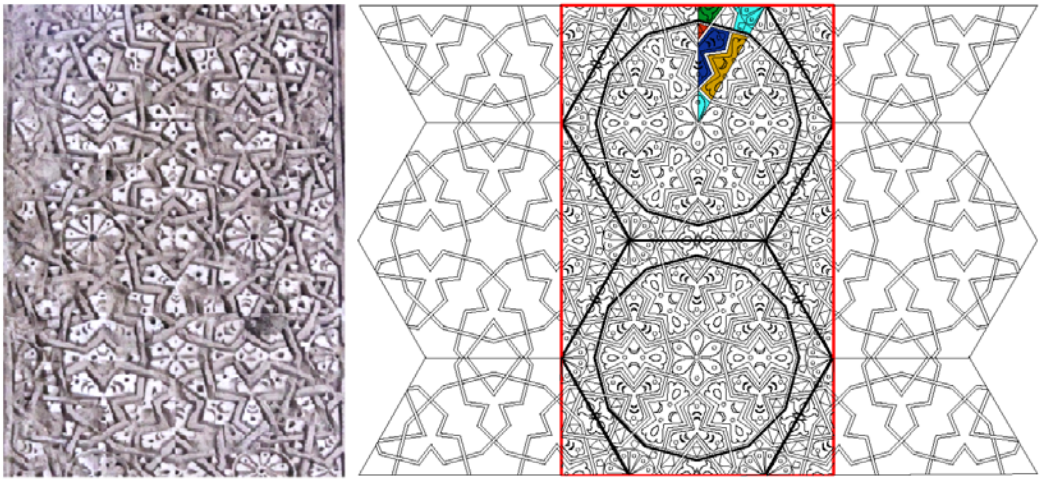

Figure 2: *The stuccowork in the entrance of an iwan, and its layout.*

To compose the underlying design for this pattern one may take the following steps in Figure 3 from the top left to the bottom right:

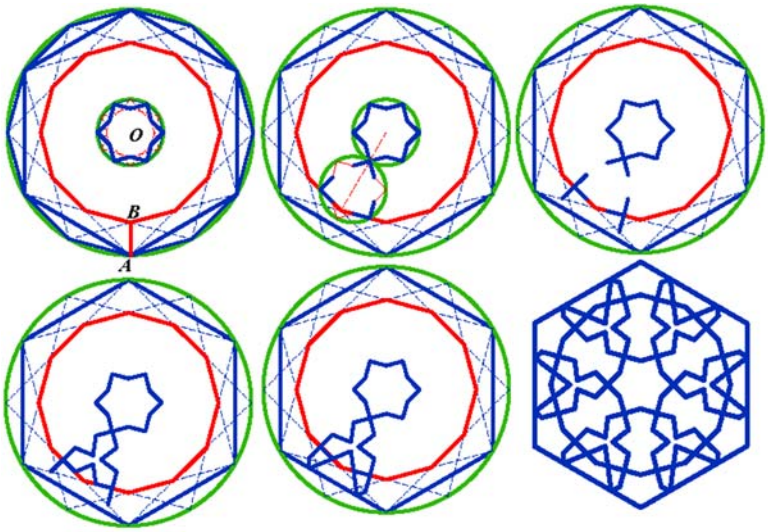

Figure 3: *The steps of the geometric construction of the stuccowork in Figure 2.*

Draw a dodecagon inside a hexagon, based on the lines of a (12, 3) star polygon – a star, which is constructed based on dividing a circle into 12 equal arcs and connecting a vertex to another one with 3 arcs distance and continuing the process to meet all the vertices in a single or multiple strokes – as is illustrated in the first image on the top left. Then, draw a circle at center O with radius congruent to segment AB and inscribe a dodecagon inside of it. After that, using the (12, 3) star polygon inside of this circle construct a six-petal star as in the first and second images. Make another circle with the same radius as AB, tangent to the previous circle, and inscribe three petals of the six-pedal star as is presented in the second top right image. Extend the two segments inside of the circle equal to them as is shown in the next image. In this stage we can complete the design by choosing an angle smaller than or equal to 90° (here in

the Figure we chose 75°) and add segments inside of the circle properly (to save its three-fold rotational symmetry) and then extend some segments to complete the design as is shown in the middle image of the bottom row. One should rotate the generated design about O with the angle of rotation 60° to complete this pattern.

3.2. A Four-Fold Design. The decorative stucco stripe in Figure 4 is on the northern iwan of a wall that surrounded one of the entrance gates of the mosque. It is created by the repetition of one square shape motif.

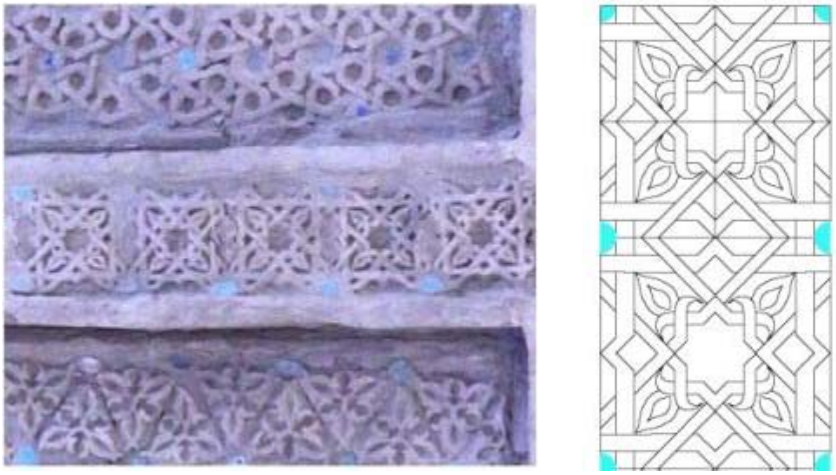

Figure 4: *The stucco and glazed brick work in the northern iwan, and its layout.*

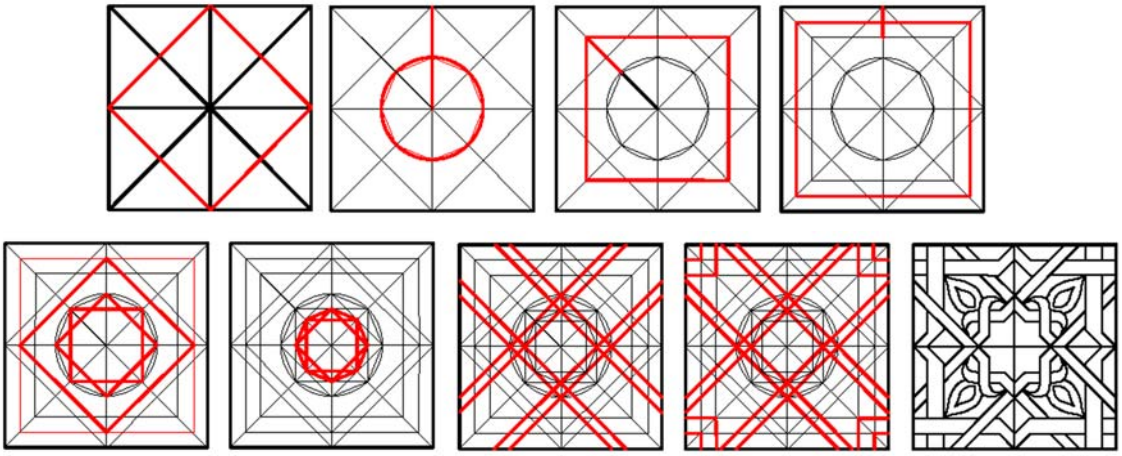

Figure 5: *The steps of the geometric construction of the stucco and glazed brick work in Figure 4.*

The geometric construction of this unit is demonstrated in Figure 5 in nine steps from top left to the bottom right. Draw a square and its reflection axes. Then using the midpoints of its sides, construct another square inside of the first one. Rotate this new square 45° to make the third square. Add stripes properly and draw a circle centered at the center of the original square with radius equal to ¼ of its sides. Make an (8, 2) star polygon with vertices on this circle. Make some new stripes by adding another (8, 2) star polygon inside the previous one. Use the sides of the star polygons to make off-diagonal stripes as are illustrated in the bottom row of the middle images in Figure 5 and then complete the motif.

3.3. More complex patterns. There are more complex patterns that adorn the surface areas of the mosque. We present their underlying compositions using some traditional methods. One method is the radial grid method that is presented in [5].

The Stuccowork in Figure 6 is a pattern that decorates a plinth of one southern arch. It is a pattern that has been partially destroyed and only a part of it has survived. In general, the process of fully restoring the underlying pattern is more complicated in such cases.

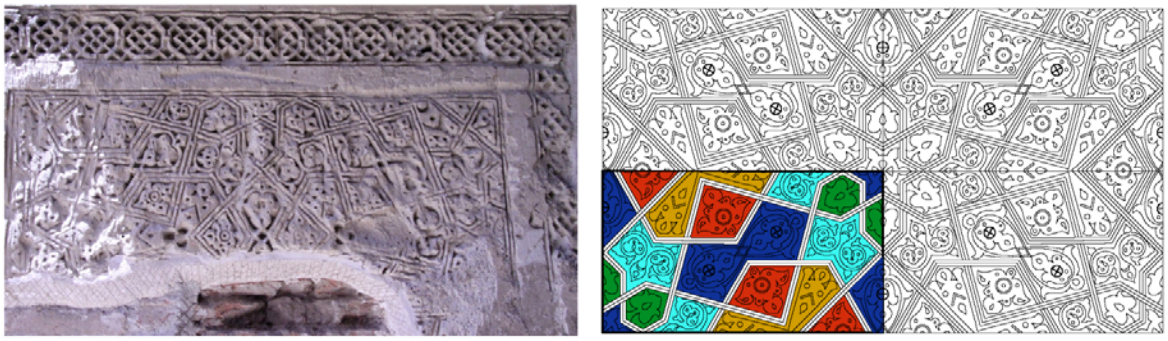

Figure 6: *The stuccowork in the southern arch and its computer generated design.*

To create this pattern we follow the steps that are taken in Figure 7 from top left to bottom right.

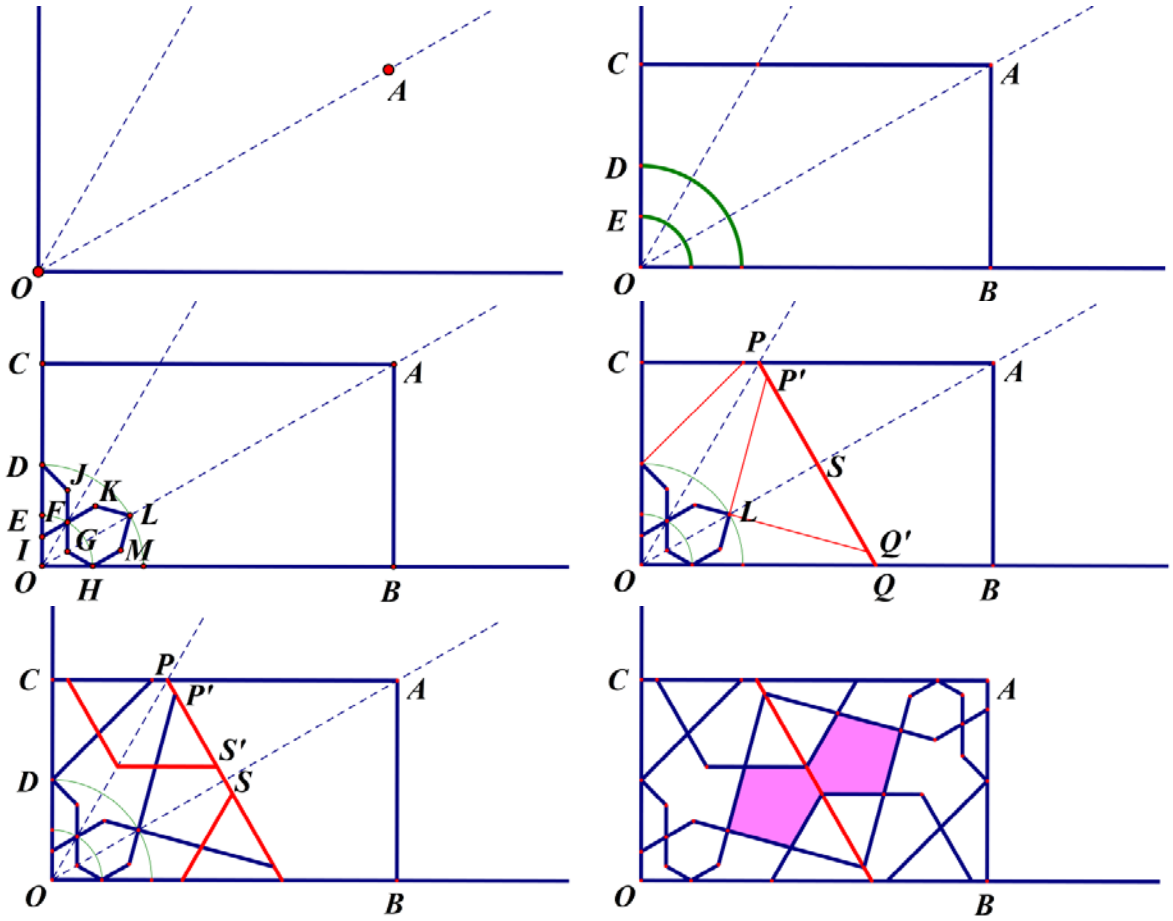

Figure 7: *The stuccowork in the southern arch and its computer generated.*

Divide the right angle *O* into three congruent angles (this is a constructible trisection using the construction of an equilateral triangle). Place an arbitrary point *A* on the first radial. Make perpendiculars from *A* to the sides of the right angle to find points *B* and *C*. The rectangle *OBAC* is the frame of the *girih*. Find the midpoint of *OC* and call it *D*. Also find E, which is the midpoint of *OD*. Then construct two arcs as in the figure. Point *F* is located in the intersection of the small arc and the second radial. From *F* make a parallel line to *OC* to find *G*. Connect *G* to *H*, which is the intersection of the small arc with *OB*. Find *I* on *OC* in a such a way that *FG* = *FI* (traditionally speaking, make a circle with center at *F* and radius *FG* to cut *OC* at *I*). Now make a segment from *D* to the other end of the big arc on *OB*. This segment will intersect the extension of *GF* on *J*. Now find *K*, *L*, and *M* using facts such as *FK* = *FJ* and *K* is on line of *IF*. *KL* = *JD*. *HM* is parallel to *OA* and *HM* = *FK*. *P* is the intersection of the second radial with *AC*. Find *Q* on *OB* in such a way that *CP* = *BQ* (the triangle *OPQ* is an equilateral triangle) and find *S*, the center of the rectangle *OBAC*. Extend *ML* and *KL* to intersect *PQ* at *P'* and *Q'*. Find the reflection of *LP'* under *OP*. Also find *S'* in such a way that *SS'* = *PP'*. From *S'* make a segment parallel to *OB* to intersect *OP* and then find the reflection of this segment under *OP* and *OA*. Complete the process by using 180° rotation of the constructed segments in *OQPC* about *S* to cover *QBAP*.

The stuccowork in Figure 8 is another pattern on this mosque that has been damaged severely. We noticed that the layout of its *girih* was illustrated in the book *Arabic Geometrical Pattern & Design* [1], which included all images from an old book published in the French edition, *Les Eléments de l'art: le trait des entrelacs, Firmin-Didot et C*^{ie}, Paris, 1879, written and illustrated by J. Bourgoin. The plate for this *girih* is numbered 110. The book does not provide step-by-step instructions for the geometric constructions but there are underlying circles and segments using thin dashed lines that are instrumental for forming such instructions.

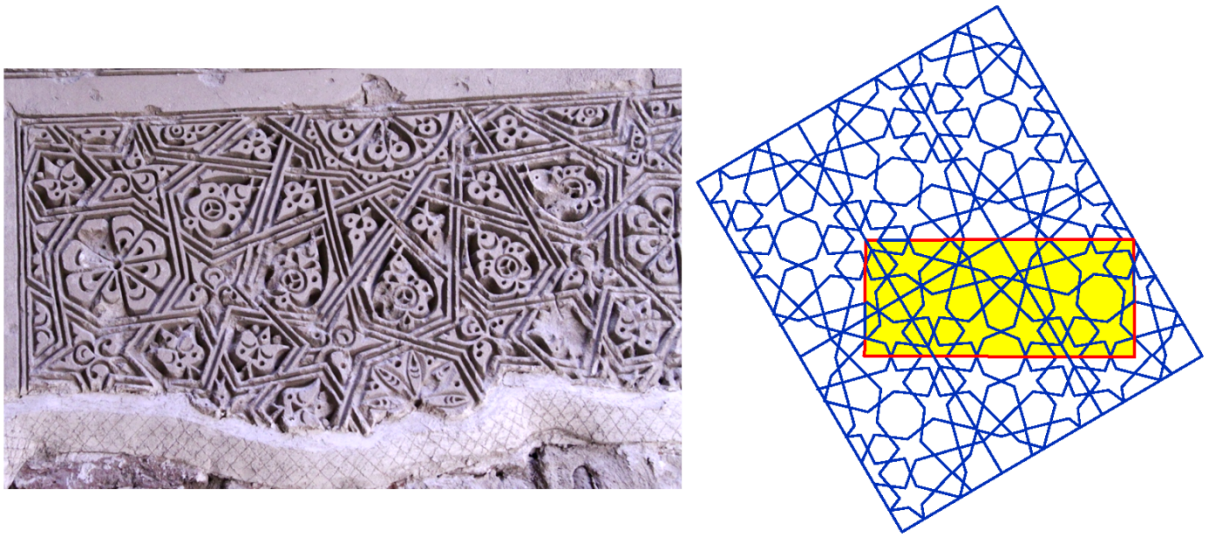

Figure 8: *The stuccowork in the southern arch and its computer generated layout.*

For constructing the layout of this geometric design divide the given 90° into 6 equal angles by 5 radials. Select an arbitrary point *P* on the fourth radial and make the rectangle for the *girih*. Make a tangent circle to the sides of the triangle *ABC*, where the center is located on the first radial. The center of this circle is the intersection point of the three angle bisectors of the triangle *ABC*. Other small circles are congruent with the first circle. The larger circle with center at *A* is tangent to the first circle. The radius of the

smaller circle with center at A is created by dropping a perpendicular from the intersection of the third radial and the second small circle. We now have a layout of several circles and arcs that are tangent to each other. We use the tangent points to connect them properly as is illustrated in the third image of Figure 9. The last image is the *girih* that creates the pattern in the aforementioned stuccowork.

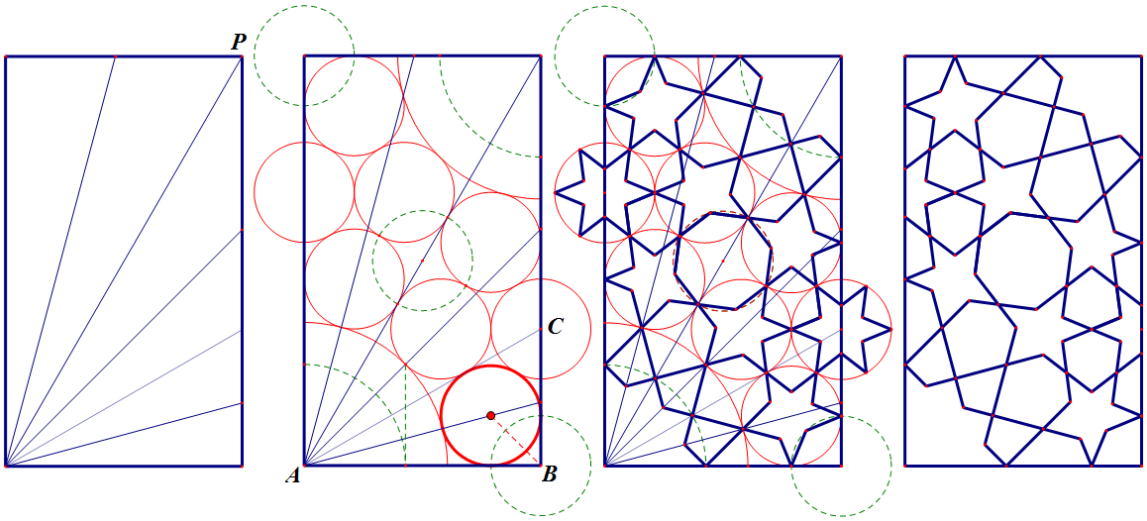

Figure 9: *The steps of the composition of the stuccowork in the southern arch.*

The stucco pattern in Figure 10 is another example of the complex designs adorning the mosque. We use a different method to illustrate the steps for the composition of the layout of this design.

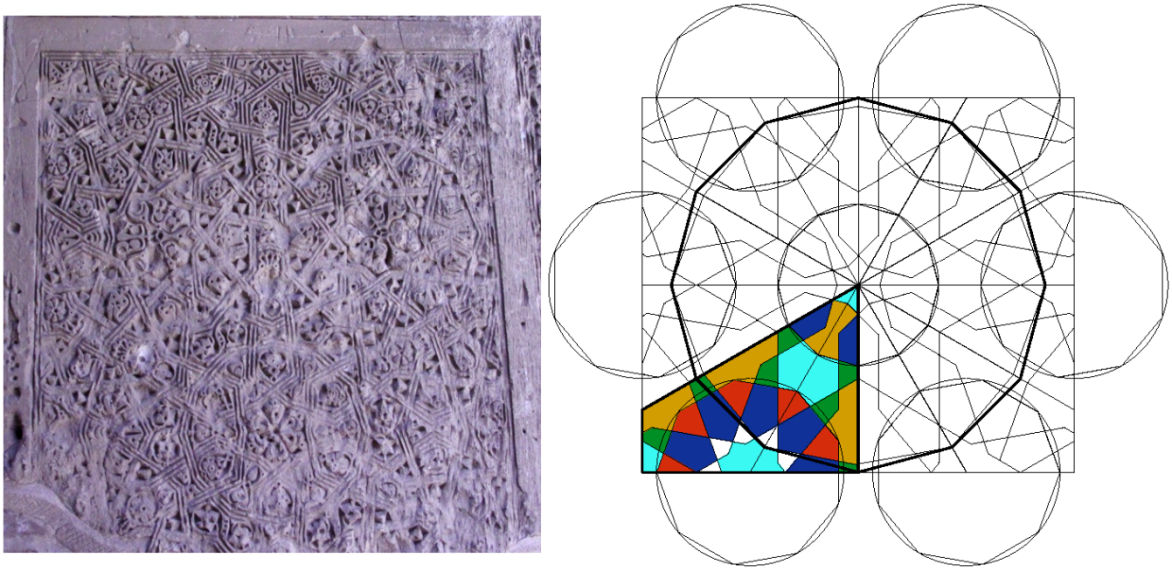

Figure 10: *The stuccowork of the plinth of one of the southern arches and its underlying pattern.*

First, we draw a right triangle OAB. At the center B by the radius AB draw a circle and make a dodecagon inside of it. Then draw two other circles at the center O with the radius equal to half of AB and then the other circle with the radius equal to half of OA and inscribe inside them two nonagons. After that, we connect the nonagons' angles every third corner. Then we extend these lines. As it is shown in the figure

we draw an orthogonal line from *B* to one of the extended lines and then draw a circle at the center *B* by the radius equal to this orthogonal line. Draw a dodecagon inside of this circle and connect every fifth vertex. Finally, draw *BC*, a chord of the large dodecagon, and transfer all lines based on this chord which are shown in Figure 11. By this way the repetitive unit is made. By transferring this pattern based on the dodecagons' chords the whole pattern is created which is shown in Figure 10.

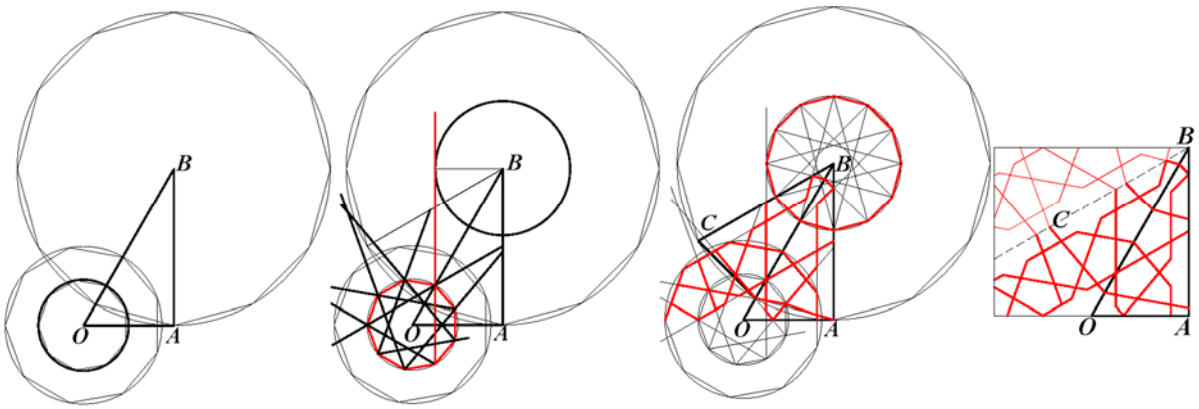

Figure 11: *The steps of the composition of the stuccowork in the southern arch.*

4. Conclusion

This paper analyzes the geometric patterns of architectural ornaments of *Friday mosque of Forumad*, a small 12th century structure known as one of the earliest examples of Iranian mosques with complicated ornaments. Craftsmen applied creative and unique forms of design in the ornaments, of which many parts of them have been destroyed. Therefore, this paper is an attempt to detect design methods for preservation purposes and also for the presentation of the prominent features of this structure. Ornamental patterns, which have not followed the usual division manner of a circle into six parts are studied in this paper. The analyses are based on traditional construction processes that might have been used by craftsmen in the past, and with common construction tools: the straightedge, and the compass. These patterns are unique forms of geometric design in the radial grid approach, which results in a rectangular tiling unit called *girih*. Therefore, these patterns could be considered as initial forms of geometric design in the forms of *girih* patterns in the history of Iranian Islamic architecture.

References

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[2] Godard, Andreh. Et al. 1992.Athar-e-Iran.Translated by Abolhasan sarvghad moghadam in Persian. Mashad: Astan-e- quds-Razavi publication.
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[5] Sarhangi R., *Interlocking Star Polygons in Persian Architecture: The Special Case of the Decagram in Mosaic Designs*, Nexus Network Journal, Architecture and Mathematics, Volume 14, Number 2, Autumn 2012, Birkhäuser Verlag, Springer, Heidelberg, Germany, pp. 345-372.