

## RaM - Recycle and Mathematics: the art of tiling for eco-design

Alessandra CAPANNA  
Dipartimento di Architettura e Progetto  
“Sapienza” Università di Roma  
Via Flaminia 359 - 00196 Roma, ITALY  
E-mail: alessandra.capanna@uniroma1.it

Marcella Giulia LORENZI  
CEL – Lab. for Scientific Communication  
University of Calabria, Campus of Arcavacata  
Ed. Polifunzionale, 87036 - Rende CS, ITALY  
E-mail: marcella.lorenzi@unical.it

### Abstract

The aim of this workshop is to construct a beautiful centered hexagonal number trivet by making use of colored recycled magazine paper. What is seemingly only a funny and very simple puzzle made with circular tiles, composing intriguing and colored artistic objects, is an engaging exercise in the field of mathematics applied to eco-design process.



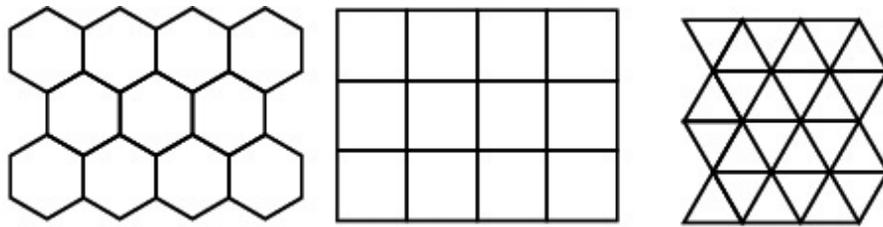
Figure 1: Recycled Paper Trivets

### Introduction

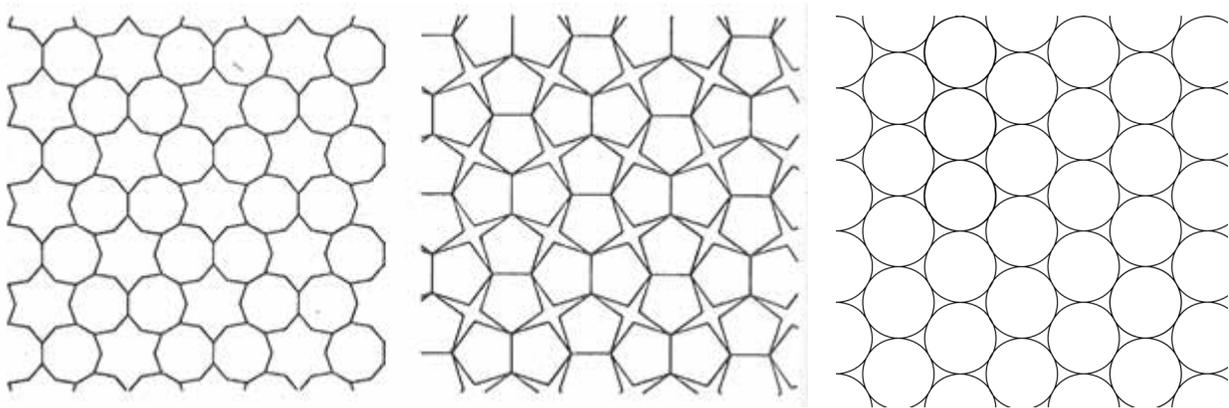
Tessellation is the process of covering a portion of horizontal plane using the repetition of regular geometric elements **with no overlaps and no gaps**. The word “tessella” in Latin means “a small square”, used to make mosaics (from the Greek word “tessera”, square, four).

A honeycomb is an example of a tessellated structure, as well as the black and white components of a chessboard. The easier example of tessellation is the pavement of our kitchen, whose surface is generally covered with square tiles [11, 12].

The condition to create regular tessellation with only one kind of regular polygon is that the interior angle must be an exact divisor of 360 degrees. This works for the triangle, square, and hexagon (Figure 2). Tessellations of the plane by *two or more* convex regular polygons, such that the same polygons in the same order surround each polygon vertex, are called semiregular tessellations, or sometimes Archimedean tessellations (Figure 3), see [6, 10].



**Figure 2:** *The only three regular tessellations composed of regular polygons symmetrically tiling the plane*



**Figure 3:** *Three possible semiregular tessellations of the plane by convex regular polygons or any closed couple of figures. The basic scheme - circles and triangles with curved sides - is the scheme of our hexagonal trivet.*

Filling a plane with a repeated pattern is a way of unifying aesthetics, elegance and simplicity, satisfying natural human “sense of order”, see Gombrich [7] and has been used over the centuries in art and architecture, reaching its maximum development in Spain in the thirteenth century, where the Moors used all seventeen symmetry groups entangled in their decorations of the Alhambra. Their preference for abstract schemes was due to the strict observance of a limitation set by Islamic law (Hadith): images of living beings, human or animal, are forbidden.

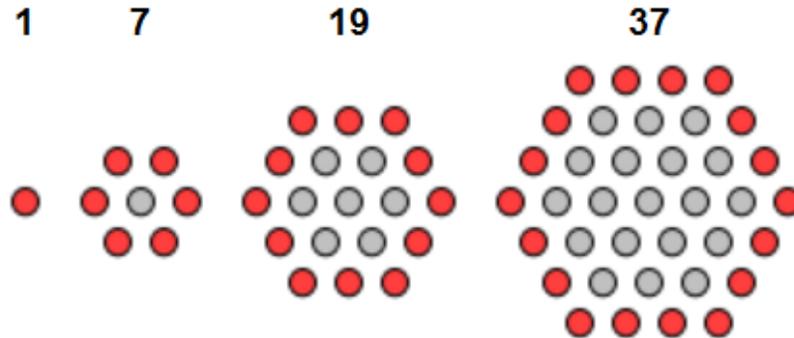
Godfrey H. Hardy said that “*a mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas*”, see [8]; on the other hand, **as architects and designers**, we are somehow concerned with the inverse process of transforming ideas and patterns into everyday items. Symbolic aspects of numbers, for example, can have a great influence on design process. The concept of “design”, as explained by Walter Gropius, embraces in general the entire orbit of what surrounds us and derives from the hand of the man, from simple domestic furnishings to the complex organization of an entire city, “from the spoon to the town”, as quoted by the Milanese architect Ernesto Rogers in 1952, see [1].

### **Centered Hexagonal numbers: Nineteen, Thirty-seven and other beautiful numbers**

The aim of our workshop is to design bright colored trivets (heat-proof mat for pot or pan). As we will see later, we will need 37 equal circles (which are actually plane spirals), to make them. Why 37?

**37** is a very particular number: it is a prime number, the twelfth in order; it is the first irregular prime and the only two-digit number whose product, when multiplied by the sum of its digit is equal to the sum of the cubes of these figures:  $37 * (3 + 7) = 3^3 + 7^3$

Among other beautiful properties, **37** is in particular a centered hexagonal number, a figurate number that represents a hexagon with a dot in the center and all other dots surrounding the center dot in a hexagonal lattice.



**Figure 4:** *Centered Hexagonal number layout*

The  $n^{th}$  centered hexagonal number, also known as Hex Number in Martin Gardner’s description of patterns made with figurate numbers, see [2], is given by the formula:

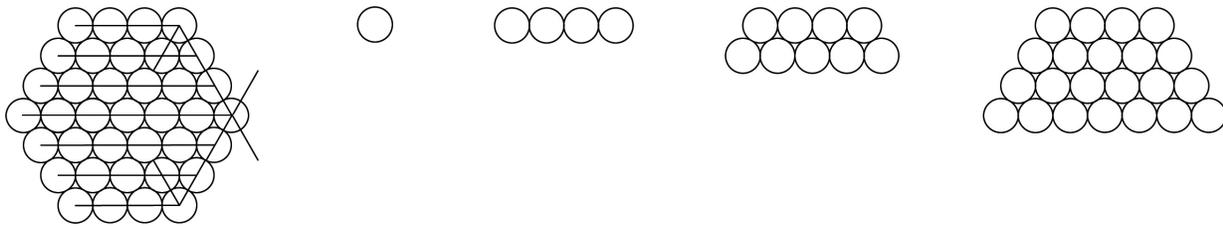
$$n^3 - (n - 1)^3 = 3n(n - 1) + 1$$

That is: each term is: Three times the number squared plus three times the number plus one.

As a matter of fact, the first few centered hexagonal numbers are:

1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, 469, 547, 631, 721, 817, 919, ... .

The layout also fit the conditions of respecting the multiple of  $60^\circ$  in the symmetry axis rotation and offset, so we can draw the outline of the trivet with all the circles tangent and alternate, beginning with a row of 4 units, see [2, 3, 4, 5, 9].



**Figure 5:** *Hexagonal recycled paper trivet layout*

Another interesting mathematical property is that Hex numbers are always equal to 1 mod 6.

In [5], we can read the chapter *Figures from figures. Doing Arithmetic and Algebra by Geometry*, a synthetic overview which can substance a sort of “mathematization” of our compositive formulation. As architects with deep interest in mathematical and geometrical constructions of projects and objects, we are concerned above all with the correspondence of the mathematical concept with its architectural configuration. At the bottom of our projects are sometimes ideas in form of a kind of mathematics without numbers, often well represented by figured numbers, integers – on the contrary – that can be represented by means of a regular geometric pattern.

So, during the craft activity, it is interesting to prove algebraically and geometrically some mathematical property with the help of easy figures shaping objects, from which to extract, or better, to understand and visualize formulas and algebraic generalizations.

We can also consider **19** as a very interesting Hex number.

In fact it is “the right one” in terms of dimension, if we need a drinks mat.

Furthermore, besides being the previous – as we have seen – of 37, in the sequence of centered hexagonal numbers,

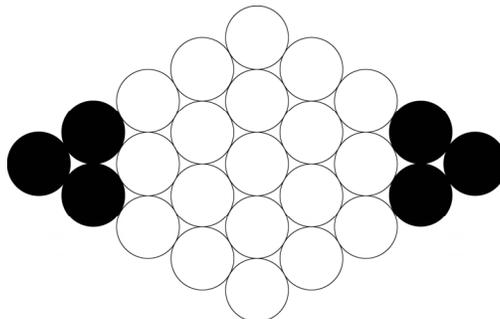
- 19 is the only non-trivial normal magic hexagon which contains 19 hexagons (the other being 1),
- $19 = 4! - 3! + 2! - 1!$ ,

If, instead, we want to construct a rhomb shaped trivet, we need  $n^2$  elements, because it is constructed with two mirrored equilateral triangles: the first numbers of the sequence are 4, 9, 16, 25, 36, 49 (Figure 6).



**Figure 6:** Rhombic recycled paper trivet

In terms of geometrical figures, it can be formed also with a central hexagon and two triangles.



**Figure 7:** Rhombic trivet layout

The triangular numbers,  $\text{Tri}(n)$ , are related with Hex numbers  $\text{Hex}(n)$  as follows:

$$\text{Hex}(n) = 1 + 6 \text{Tri}(n-1)$$

That is: each centered hexagonal number is the sum of six triangular numbers, where a triangle of side length  $n$  is equal to the sum of all natural numbers between 1 and  $n$ .

And so how that generalizes, for rhombic shaped patterns, for odd  $n$ , is

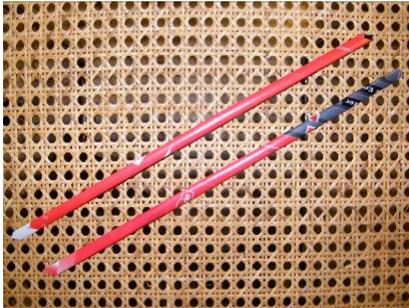
$$(2k + 1)^2 = \text{Hex}(k + 1) + 2\text{Tri}(k)$$

## Workshop: making of the trivets

So, let's see how to make a trivet using recycled magazines paper in seven steps. First of all, we need some old magazines.



1. Tear out colored pages from the magazines, grouping them by color (let's consider white as a color too, even if it actually is the sum of all the colors of the spectrum)
2. You can choose to have monochromatic or multicolored works, the only limit is fantasy!



3. Fold the paper according to a 45 degree line starting from one of the corner of the page, to make many flat sticks
4. Select by color



5. Roll each stick to obtain small wheels. We can identify the Archimedean spiral (or arithmetic spiral), the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity (each spire has a constant distance from the center)
6. Compose the hexagon gluing all the elements each other in the points of tangency and on a sheet of thicker paper (it could be the cover of one magazine or a cardboard)
7. Apply a plenty layer of vinyl glue to make the trivet waterproof and strong; let dry completely before use.

What is seemingly only a funny and very simple puzzle made with circular tiles, composing intriguing and colored artistic objects, is an engaging exercise in the field of mathematics applied to eco-design process.

### **Eco-design: RaM means Memory**

It is important to educate young people to beauty-art and mathematics, and this becomes more significant if we also prepare them to save their future world, considering recycling.

Recycling is processing used materials into new products to prevent waste of potentially useful materials, reduce the consumption of fresh raw materials, reduce energy usage, reduce air and water pollution, decrease the use of dumping grounds and fillings, by reducing the need for "conventional" waste disposal, and lower gas emissions as compared to virgin production. The 2030 Challenge, see [13] indicates a measured and achievable strategy to reduce global energy consumption. In the Architect's field it concerns, above all, to achieve best practices to reduce gas emissions in constructions by the year 2030 (and abolish them by the year 2050). It is not only a matter of producing "clean energy" and solar buildings, but also recycling materials and sites.

Recycling is a key component of modern waste reduction and is the third component of the triple-R "Reduce, Reuse, Recycle" [14] waste hierarchy. Paper, glass, aluminum can be regenerated.

RaM is the name we gave to our trivet, which re-uses paper from magazines, with no need or regenerating it. The acronym RaM means: Recycle and Math. However, RAM is also the acronym for random access memory, and in our common feeling RaM is our technological memory. So we can say that every object reusing matter and is hand-made, contains the memory of the previous.

### **References**

- [1] M. Bussagli, *Capire l'architettura*, Firenze: Giunti, 2003 (*Understanding Architecture*, Sharpe, 2005)
- [2] M. Gardner, *Mathematical Games: On the patterns and the unusual properties of figurate numbers*, Scientific American July 1974, p. 116.
- [3] M. Gardner, *Penrose Tiles to Trapdoor Ciphers: And the Return of Dr Matrix*, by W. H. Freeman, New York, The Mathematical Association of America (Incorporated), 1989, 1997.
- [4] M. Gardner, *Tilings with Convex Polygons*. Ch. 13 in *Time Travel and Other Mathematical Bewilderments*. New York: W. H. Freeman, pp. 162-176, 1988.
- [5] J.H. Conway, R.K. Guy, *The Book of Numbers*. New York: Springer-Verlag, 1996.
- [6] M. Ghyka, *The Geometry of Art and Life*. New York: Dover, 1977.
- [7] E. H. Gombrich, *The sense of order: a study in the psychology of decorative art*. London: Phaidon, 1979.
- [8] G. H. Hardy, *A Mathematician's Apology*, London: Cambridge University Press, 1940.
- [9] T. Pappas, *Tessellations. The Joy of Mathematics*, pp. 120-122. San Carlos, CA, 1989.
- [10] E. W. Weisstein, *Semiregular Tessellation*. In *MathWorld--A Wolfram Web Resource*. <http://mathworld.wolfram.com/SemiregularTessellation.html>
- [11] K. Williams, *Italian Pavements: Patterns in Space*. Houston, 1997.
- [12] K. Williams, J. Moran, *Dalla tassellatura del piano alla pavimentazione di spazi urbani: introduzione* <http://matematica.unibocconi.it/dossier/dalla-tassellatura-del-piano-alla-pavimentazione-di-spazi-urbani>
- [13] Architecture - The 2030 Challenge [http://architecture2030.org/2030\\_challenge/the\\_2030\\_challenge](http://architecture2030.org/2030_challenge/the_2030_challenge)
- [14] Recycling guide <http://recycling-guide.org.uk/rrr.html>