On the Question of Meter in African Rhythm: A Quantitative Mathematical Assessment

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Abstract

For a given collection, corpus, or genre of musical rhythms, the *pulse saliency histogram* lists the number of times a note onset occurs in each pulse position of the rhythm time span. Two families of rhythms may then be considered to be similar with respect to their pulse location preferences if their pulse saliency histograms resemble each other. This mathematical tool thus provides a quantitative method for assessing the similarity of music corpora, and for statistically testing musicological hypotheses. Its usefulness is illustrated in the context of measuring the similarity of African and Western music, and revisiting the issue of whether African music exhibits hierarchical meter.

Introduction

Much has been written about the similarities and differences between Sub-Saharan African and Western music. Often African music is claimed to have more complex rhythms [11]. Kofi Agawu chronicles a good deal of the literature that focuses on the purported prominence of rhythm and its complexity in African music, relative to that of Western music [1]. The rhythm in African music has also been compared, from the point of view of complexity, to the rhythm in Indian music. In comparing African and Indian music with European music, Benjamin I. Gilman (1909, p. 534) writes: "Hindu and African music is notably distinguished from our own by the greater complication of its rhythms. This often defies notation." [6]. Is it indeed the case that African and Western musical rhythms are fundamentally different? John Miller Chernoff (1979, p. 54) writes that “Western and African orientations to rhythm are almost opposite.” [4]. On the other hand, for David Temperley (2004, p. 289) “African and Western rhythm are profoundly similar.” [14]. What is one to make of such antithetical pronouncements?

African music has also been compared with Western music regarding its possession of *meter*. Martin Clayton nicely disentangles several definitions of meter [5], and Justin London clarifies the distinction between meter and *grouping* [10]. On one end of the conceptual spectrum, meter is defined as a pulsation of equally spaced (regular) beats that may be sounded or merely felt, and that functions as the railing on which rhythms ride. At the other end of this spectrum the regular beats are hierarchically arranged according to their strength within a periodic cycle. Such is the view of Lerdahl and Jackendoff (1983, p. 12) who define meter as a "regular pattern of strong and weak beats to which [the listener] relates musical sounds." [9]. According to this definition the meter furnishes the musician with a hierarchy of temporal reference points. This hierarchy referred to as the *Generative Theory of Tonal Music* (GTTM) is illustrated in Figure 1 (left) for the case of a 16-pulse measure (periodic cycle). The height of the column in each pulse position reflects the *relative* strength of each pulse. This strength of a pulse location is correlated with the degree of the expectancy of occurrence of an onset at that particular location. What is important here is the ordering (rank) of these locations in terms of their magnitudes. This is the definition that has been frequently invoked to contrast African with Western music, and that is the focus of this study. Simha Arom (1991, p. 206) writes: "The pulsation is the only temporal reference the musicians have." [2]. M. S. Eno Belinga, (1965, p. 18) dismisses meter as follows: "In African music only one thing matters: the periodic repetition of a single rhythmic cell." [3]. The consensus of these and other authors is that African music does not possess meter in the hierarchical sense as embodied by the Lerdahl-Jackendoff model.

In this paper the two questions outlined above concerning the similarity of African and Western rhythm, and whether African rhythm possesses meter, are subjected to a mathematical analysis, using *pulse saliency histograms* that yield quantitative measures of how similar African and Western rhythms are, as well as how much hierarchical meter African rhythm possesses. These calculations are made using several data sets that the reader may find in references [1]-[2], [7]-[8], and [12]-[17].

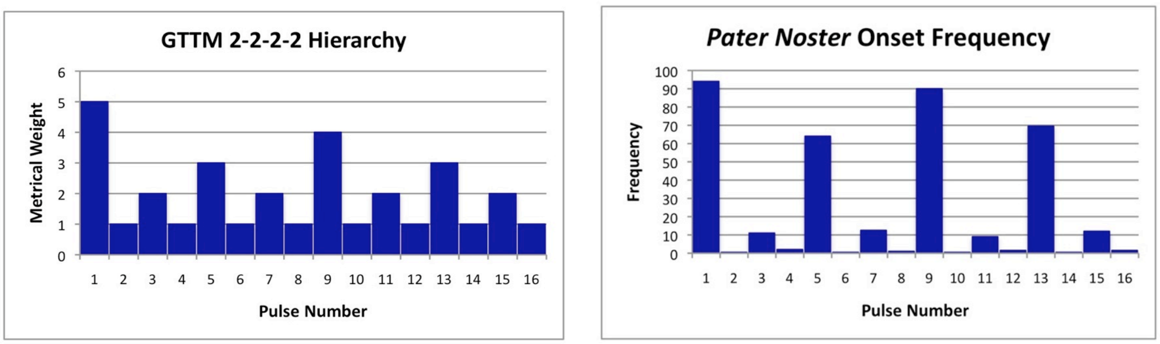

Figure 1: *The GTTM hierarchy (left), and the histogram of* Pater Noster *(right).*

Pulse Saliency Histograms

A pulse saliency histogram calculated from a given corpus of symbolically notated music resembles the GTTM hierarchy shown in the left diagram of Figure 1. The difference is that the height of a column in the pulse saliency histogram corresponds to the empirically observed frequency of occurrence of an onset in that position in the rhythmic cycle. The pulse saliency histogram of the onsets in Palestrina's Sixteenth Century motet, *Pater Noster*, compiled by Joshua Veltman [17] is shown in the right diagram of Figure 1. One way to compare these relative frequency histograms is via the Pearson correlation coefficient (ρ) of the histogram bin heights. It turns out that this correlation is quite high: $\rho = 0.632$ with $p < 0.005$. However, the Pearson correlation coefficient measures a *linear* relationship of these heights, and ignores their rank ordering. Indeed, the correspondence between this histogram and the GTTM hierarchy in terms of the ranks of the saliencies of its pulses is visually striking. A more appropriate tool for testing the meter hypothesis should use this information. In both graphs, pulses 1 and 9 contain the first and second highest columns. In both graphs, pulses 5 and 13 come next, and have approximately equal height. The same may be said of the third-level pulses 3, 7, 11, and 15, as well as the fourth-level pulses 2, 4, 6, 8, 10, 12, 14, and 16. This visual comparison is compelling enough in this case, but in order to obtain a quantitative measure of the relationship (similarity) that exists between the hierarchies (or ranks) of the two histograms, the Spearman rank correlation coefficient may be computed between the rank vectors determined by each of the sixteen ordered heights. The resulting correlation between the two histograms is $r = 0.930$ with $p < 0.00001$. It is worth pointing out that the GTTM hierarchy has received a fair share of criticism for being Western Euro-centric, with limited generalization to non-Western music. The very high correlation between these two histograms certainly provides strong evidence that the GTTM hierarchy is an accurate model of the metric hierarchy contained in the Sixteenth Century Italian motets composed by Palestrina. Experiments with other Western classical music supports similar conclusions. But how does the GTTM model fare with African music?

African Timelines and Western Songs

One corpus of rhythms that are considered to be quite different from Western rhythms, consists of the *timelines* of Sub-Saharan Africa. Much has been written about their uniqueness, complexity, and the degree of syncopation that they possess [1]-[2], [12]-[16]. Therefore intuition would dictate that these timelines provide a suitable data set to favor a conclusion that African rhythm and Western rhythm are fundamentally different. To test this hypothesis, a corpus of 34 notated timelines used in African, Afro-Cuban, and Afro-American music were collected from several published papers and books [1]-[2], [12]-[14]. These timelines all have a cycle (measure) of 16 pulses, and the number of their onsets varies between 5 and 10. The histogram computed from these 34 rhythms is shown in Figure 2 (left), where k denotes the number of onsets. At first glance this histogram appears to look quite different from those in Figure 1. In GTTM and *Pater Noster*, the most salient pulses after pulse 1 are 5, 9, and 13, whereas in the African timelines pulses 3 and 7 are preferred indicating that the latter rhythms are more syncopated than the former. Another obvious difference is that the African timelines make use of almost all pulses with a similar frequency, noticeably largely disregarding pulses 2 and 16. However, before hastily concluding that African timelines are profoundly different from Western rhythms, and lack hierarchical meter it behooves one to compare these two histograms with the Spearman rank correlation coefficient. The Spearman correlation between the African timelines and GTTM is $r = 0.793$ with $p < 0.0002$, and for *Pater Noster* $r = 0.786$ with $p < 0.0002$. The Pearson correlation coefficient (which ignores the metric hierarchy) between the African timelines and GTTM is $\rho = 0.74$ with $p < 0.0005$, and for *Pater Noster* $\rho = 0.59$ with $p < 0.009$. These are very high correlations at highly statistically significant levels. The Spearman rank correlation coefficient, measures the ordering of the histogram bins by height, and is thus more sensitive to the contour of the histogram bins than to their absolute values. Recall that the contour is a three-symbol string that assigns the symbol "+" if the value increases, "-" if it decreases, and "0" if it stays the same. Note that all three histograms have exactly the same contour: [- + - + - + - + - + - + - + - +]. Therefore, the correlation coefficient provides significant evidence that African timelines, as a family, do contain hierarchical meter, and thus are similar in this regard to the rhythms of Western music. Furthermore, the shape of the histograms provides additional information as to how they differ while respecting the hierarchy.

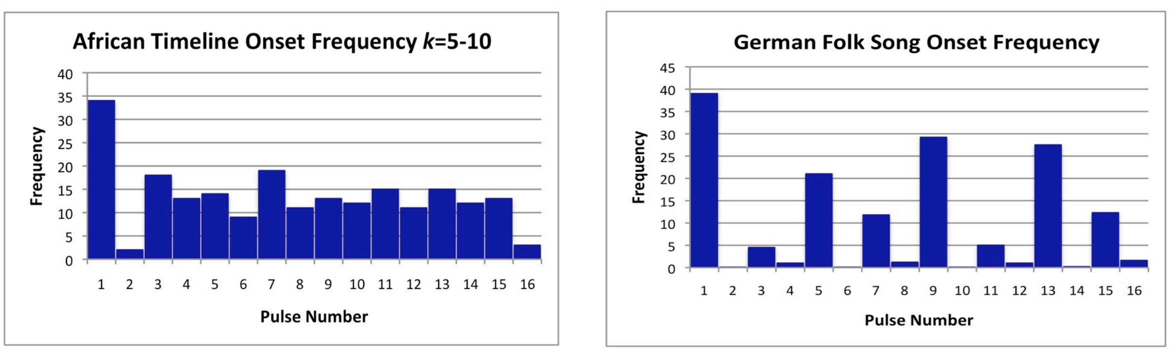

Figure 2: *The onset frequencies of the African timelines (left), and of the German folk songs (right).*

The histogram for Palestrina's *Pater Noster* has an uncanny almost perfect resemblance to the GTTM hierarchy, and one may wonder how a histogram might vary if obtained from a different European music genre from another century. Thanks to the work of Huron and Ommen [8], who calculated the onset frequencies for the well-known Essen corpus of Germanic folk songs, this question can be easily answered. The histogram for this corpus, shown in Figure 2 (right), differs somewhat from the histogram of *Pater Noster*, but like the other three above, has exactly the same contour: [- + - + - + - + - + - + - + - +]. Its Spearman rank correlation coefficient with GTTM is $r = 0.930$ with $p < 0.00001$ (for the Pearson correlation, $\rho = 0.973$ with $p < 0.000001$) and its correlation with the histogram of the African Timelines is, $r = 0.730$ with $p < 0.0007$ (for Pearson correlation $\rho = 0.68$ with $p < 0.002$).

Conclusions

The results presented here support the hypothesis put forward by David Temperley [14] that African and Western rhythm are profoundly similar. Furthermore, the Pearson and Spearman rank correlation coefficients provide quantitative measures of the degree of this similarity. In addition, the Spearman rank correlation coefficients provide mathematical evidence, that African rhythm does indeed possess hierarchical meter, along the lines of the GTTM model, thus lending mathematical support to the perspective of Kofi Agawu [1], and to Mark Hijleh's "*practical theory of world rhythm*" which "reveals that even older Western art music can have more in common rhythmically with its Asian, African and American counterparts than might be suspected." [7].

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